A Result from the NIST Watt Balance and an Analysis of Uncertainties

Richard L. Steiner, David B. Newell, and Edwin R. Williams

***Abstract*— An improved determination of the ratio of power, measured in terms of the Josephson and quantum Hall effects, and also the meter, kilogram, and second, has been completed. The result is expressed as: $W_{90}/W = 1 + (8 \pm 87) \times 10^{-9}$. This is an order of magnitude improvement from the last NIST (formerly NBS) determination. The Type A relative standard uncertainty (statistical) is 30 nW/W, and the Type B relative standard uncertainty is 82 nW/W. Type B uncertainty components are listed and discussed within the context of experimental procedure descriptions. Improvements for the next version of this experiment are also mentioned.**

***Index Terms*— Measurement units, SI power units, SI watt measurement, watt balance, watt ratio measurement.**

I. Introduction

The PRESENT National Institute of Standards and Technology (NIST) watt balance is the latest in a series of experiments based on an idea by Kibble [1]. The goal is to determine the ratio of power measured in terms of the Josephson and quantum Hall effects versus power measured in terms of the meter, kilogram, and second. For brevity, we must refer to earlier papers containing greater details of this experiment [2] and recent improvements [3]. Another paper presents just the watt results reported here, calculating several fundamental physical constants obtained from this determination [4]. Here we briefly describe the list (Table I) of Type B uncertainties associated with various aspects of the experimental procedures, and the Type A uncertainty (statistical), all expressed as relative standard uncertainty (RSU) in equivalent units of power (W). Our improved uncertainty is mainly from better procedures and reference calibrations that are a direct result of extensive tests for systematic error sources.

The power measured in this experiment is virtual, meaning that no actual power is generated in either of two separate modes, and thus no dissipative forces are involved. Both modes involve an induction coil interacting with a radial magnetic field of 0.1 T magnetic flux density, produced by two opposing superconducting solenoids along with two smaller coils for fine adjustment. This field is trimmed to obtain two symmetries relative to the surface of a cylindrical region, defined by the coil radius of 35 cm and its travel path of 10 cm. An even symmetry along $\hat{z}$ through the $z = 0$ center plane minimizes noise for measurements at the center, and radial symmetry $B_r(z)/r\,\hat{r}$ at the center cancels to first order any dependence upon the induction coil's dimensions. In the first measurement mode (velocity), the coil is moved vertically, tracing out the cylindrical surface at 2 mm/s, servoed to induce a voltage near 1.018 V. The average voltage U corresponding to an average velocity v from position and time measurements provides a ratio U/v proportional to $B_r(z)$. With the induction coil at the center for the second mode (force), a current I is applied to create an electromagnetic force that balances an additional mechanical force F from Earth's gravity g on a kilogram mass standard m. This mode provides a second ratio F/I also proportional to $B_r(z)$. The combined ratio of these two mode results (1) eliminates the dependence on $B_r(z)$. Reordering the variables results in a convenient ratio of two measurements of power, which can be written as numerical values $\{mgv\}_{\mathrm{SI}}$ in SI unit W and $\{UI\}_{90}$ in assigned unit W_{90}. W_{90} is derived from the adopted values for the Josephson constant $K_{\text{J-90}}$ and the von Klitzing constant $R_{\text{K-90}}$ [4]. Because of the equivalence statement, W_{90}/W is thus determined from the experimental data $\{mgv\}_{\mathrm{SI}}/\{UI\}_{90}$

$$\frac{(F/I)}{(U/v)} = \frac{(F \bullet v)_z}{(UI)} = \frac{\{mgv\}_{\mathrm{SI}} \mathrm{W}}{\{UI\}_{90} \mathrm{W}_{90}} \equiv 1 \quad \text{in SI.} \qquad (1)$$

TABLE I
Relative Standard Uncertainties in the NIST Watt Experiment

Standard Uncertainty Source	nW/W
Reference Units	
Voltage	30
Mass	20
Resistance	8
Length	5
Frequency	5
Gravity	7
External Effects	
Refractive index	43
Mass buoyancy	23
Alignments	40
Leakage resistance	20
Magnetic flux z-profile fit	20
Knife edge Hysteresis	20
RF noise offsets	10
RSS sub total	**82**
Statistical Type A	**30**
Combined standard uncertainty	**87**

Manuscript received July 2, 1998.

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Publisher Item Identifier S 0018-9456(99)02904-6.

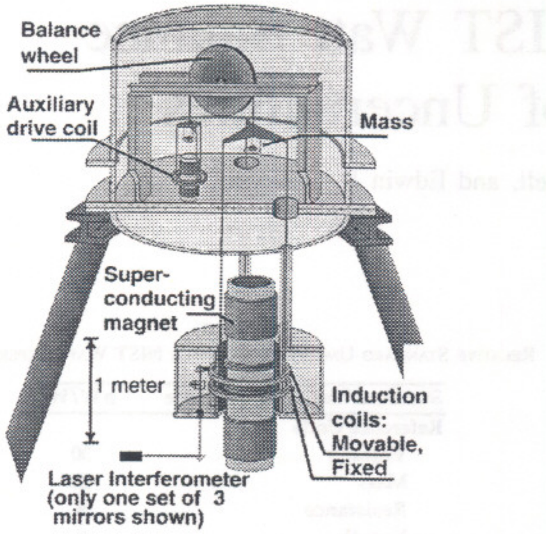

Fig. 1. Schematic diagram of the NIST watt balance, showing new vacuum chamber.

II. Experimental Procedures and Uncertainties

A. Alignments

A schematic diagram of our experiment is shown in Fig. 1. Since gravity defines the force vector, the alignment of the magnetic field and induction coil to it are critical in keeping the velocity and force vectors coincident. Setup procedures also align the coil's center of mass with the electromagnetic center, and the optical center of the interferometry system with the center of mass. A detailed analysis with an estimate for this RSU was reported earlier [5], but a larger value of 40 nW/W is now assigned to account for inconsistencies in recent tests, realignments less frequent than changes in the coil support structure, and limited resolution in the alignment instruments.

B. Voltage Measurements

All voltages are measured against a voltage drop across a resistor with a constant current source, hereafter referred to as HgV, since it is based on an op-amp with feedback to a thermally-stabilized mercury battery. HgV is calibrated against a Zener reference standard in 30 min intervals at the start and end of each measurement mode. The Zener is in turn calibrated daily with a Josephson array voltage standard, located in the same laboratory. Hourly variability in the Zener and HgV contribute to the random noise within each run. Thermal emf's in the wires are regularly measured, but their unpredictability is the main uncertainty in the voltage component. An RSU of 30 nW/W arises from two times the uncertainty of 15 nV in the volt transfer scheme and Josephson system. The DVM's are linear and regularly recalibrated to within 1 μV/V gain error at 1 V, so reading signals of 1 mV or less contributes insignificant error.

C. Velocity Mode

1) Field Profile Calculation: It is seen from (1) that only the value of U/v corresponding to F/I at $z = 0$ is required. However, these ratios are a) proportional to a nonlinear function $B_r(z)$ that varies in magnitude by 0.03% along the range of 8 cm and slowly varies in time. The U/v measurements are also b) limited in resolution and c) noisy at tens of hertz and 0.002% of the signal from externally induced vibrations. To address item a), each velocity pass records about 650 values U/v in 40 s, and a profile function $B_r(z)/B_r(z = 0)$, normalized to the center, is established by averaging all of the curves generated during a day's run of typically 300 passes. Calculable as an eighth-order orthogonal polynomial, this curve adequately represents spatial variations ($<$100 μm) of the magnetic field caused by daily changes in the trim currents or liquid helium level (filled every four days). This normalized function is then subtracted from each array of ratio data, so that the residual differences reflect the fit over a very large sample but applicable to a short time frame b). A final averaging of these residuals filters out the vibration noise c). For a 0.5 h set of 40 averaged U/v ratios the uncertainty is usually within 80 nW/W, a factor of three better than a simpler procedure fitting each curve alone. We tried to account or test for several effects that might affect this routine. The solenoid control can briefly lose constant-current due to RF pulses, so passes that grossly deviate from an earlier profile are ignored. Considering the long-term scatter in these tests, we estimate an RSU of 20 nW/W for the velocity data analysis.

2) Interferometry: The lasers are vertically aligned to within about 0.05 mrad of vertical. Recalibrated at the same time is the Abbe offset between the optical center and the center of mass of the coil, which produces a false $\hat{\mathbf{z}}$ velocity if the coil is swinging. To maintain this alignment on the corner cubes, the coil rotation about $\hat{\mathbf{z}}$ is electrostatically controlled to the angle of alignment, compensating for torque from the suspension bands. The interferometry is done in air, so a refractive index correction is applied from the modified Edlen formula [6]. The RSU for the index correction is 43 nW/W, from the uncertainties in the pressure, temperature, and humidity sensors; CO_2 and He content; and temperature gradients across the laser path. A refractometer built at NIST [7] to check this correction agreed with the Edlen formula to this same uncertainty. Calibrations of the laser frequency (affecting length) and a crystal frequency reference (affecting timing and Josephson voltages) indicate an RSU of 5 nW/W each.

D. Force Mode

1) Current Measurements: In the force mode, the experiment operates similarly to a force balance. The wheel, balancing on a Tantun-G alloy knife-edge, is kept at a constant angle by servoing the current in the induction coil. The coil repositioning is programmable to 300 nm with several microns instability over minutes, and about 150 μm daily variation relative to the profile minimum, the effect of which is minimal. When the force position was intentionally selected far off-center to a steeply sloping portion of the profile, the short-term noise increased with no significant change in watt value. With -10 mA current in the induction coil already balancing a 0.5 kg mass set onto the countermass side, a 1 kg reference mass is then placed on the coil side, requiring $+10$ mA to maintain balance. This current reversal eliminates constant electrical or mechanical offsets, e.g., thermal emf, balance offset current, etc. The coil current is measured as a voltage drop across a 100 Ω reference resistor in a 25°C oil bath. The RSU associated with calibration and drift of the resistor is 8 nW/W. An RSU of 20 nW/W for the reference mass is estimated from changes in the mass between calibrations, arising from unpredictable accumulation of surface contaminants and/or mass loss from wear during heavy use.

2) Knife-Edge Hysteresis: A single F/I average is based on five pairs of on and off balance measurements performed in 30 min, with an RSU within 50 nW/W. Much of this noise and even larger variations between sets now appear to be from irreproducible forces occurring at the knife edge when it rotates during mass placements. Normally, the maximum vertical displacement is within 300 μm, or about 1 mrad. A test offset of this amount lasting 5 s caused a severe hysteresis effect of 100 nW/W. However, recording the coil position for several weeks revealed that during normal mass placements, the servo-control tends to overshoot then undershoot the center position, and these overshoots frequently reverse pattern. Numerically integrating the deflection over the elapsed time for positioning the mass indicated 1) a correlation of the individual areas with force mode scatter and 2) a very small value for the average of several hundred integrals over weeks of data with typical force scatter. Taken together, they suggest that hysteresis is a possible reason for short-term variations in watt data, but the long-term RSU is 20 nW/W.

3) Gravity: The force calculation requires a value of local gravity. A gravimeter is used to monitor gravity and check for uncalculated tidal effects. The values of g were within five parts in 10^9 for four measurements over 15 months. A gravity transfer measured -13.36 μm/s^2 change from the U.S. Geological Survey point under the gravimeter to the 4.18 m relative altitude of the standard mass. The RSU for g is 7 nW/W, dominated by uncertainty in the single transfer.

4) Buoyancy Correction: A force correction for the buoyancy of air is also calculated from the environmental conditions [8]. Temperature control and structural modifications to the mass mover reduced the temperature gradient to less than 5 mK over 10 cm around the mass, and kept it constant during and between weighings. The room air was checked in July 1997 and showed no helium contamination, but a leak from the Dewar apparently occurred soon after, affecting some months of data. A ventilation system was installed and tests with a residual gas analyzer in April 1998 gave an estimated upper limit for He of 30 parts in 10^6. CO_2 was also monitored and found to average about (450 ± 50) parts in 10^6. Evaluating all of the uncertainties from environmental measurements gives an RSU for the buoyancy correction of 23 nW/W.

5) Nonlinear Field Effect Tests: Ferromagnetic, paramagnetic, or diamagnetic forces caused by ferrous or superconducting materials could violate earlier assumptions in the theory. We tested for these forces with a low field (superconductor off) and measured the force for the induction coil current of +10 mA, −10 mA, and zero. No ferro- or dia-magnetic forces were found. To first order, small ferro- or dia-magnetic forces will not cause an error. We expect these effects are negligible when the magnet is energized. This low field test generated a considerably different U/v profile that was in excellent agreement with the F/I data. The results also showed no systematic errors for the averaging of powerline noise (60 Hz), which is always large (mVs), or from coupling of the auxiliary drive coil field to the inductive coil.

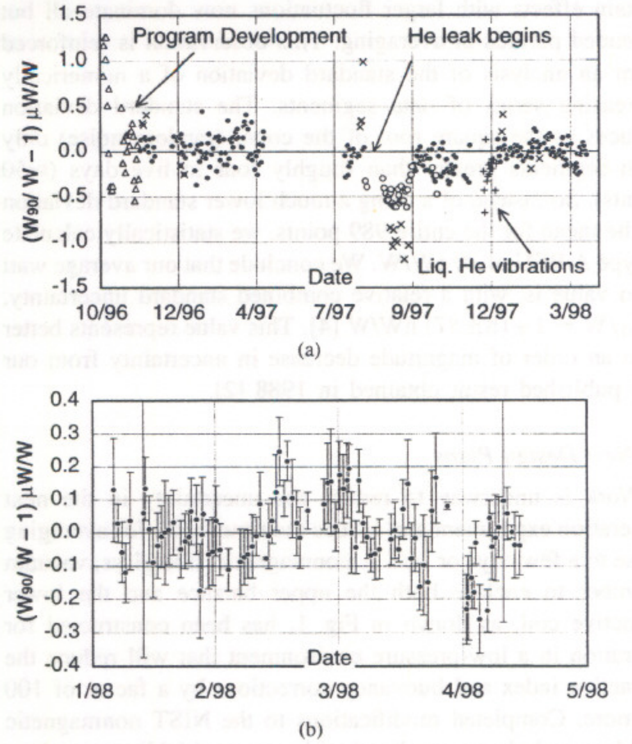

Fig. 2. (a) Deviation of W_{90}/W from 1 versus time. Each value is an average of one continuous run of normally 8–25 points. Legend: • = points with no identifiable control problem, Δ = program development, o = He gas leak, + = excessive noise from liquid He vibrations, and X = test or control problems. The data period of 10/97 to 12/97 is suspected of lesser, intermittent He contamination. (b) Expanded scales showing the 1998 data in the watt determination. Error bars are the standard deviation for each run.

III. Results and Future Plans

Our experiment has been best characterized since early 1998, with 989 points taken over four months. Fig. 2(a) summarizes all of our data as separate runs since October 1996, each run averaging generally 8–25 points. Fig. 2(b) is an expanded scale for data beginning January 1998, which is used in the final determination. Earlier periods of data are still consistent with the most recent data, although a helium leak into the experiment's enclosures clearly affects some data. The standard deviation of a single day's average is presently about 100 nW/W. Although day-to-day scatter has gradually decreased over the last year, the standard deviation of any longer period is about 140 nW/W. Over this time the combined Type B RSU was reduced by more than a factor of two to 82 nW/W (Table I). The average value and standard deviation for the last four months of data are not largely different from all acceptable data accumulated over the last 17 months. Visual inspection of Fig. 2(b) suggests nonrandom, perhaps periodic, week-long fluctuations. This suggests that the short-term precision of our experiment has reached its limit, where certain effects with larger fluctuations now dominate all but extended periods of averaging. This observation is reinforced from an analysis of the standard deviation of a numerically increasing series of data segments. The standard deviation reduces as the square root of the count (random noise) only with segments greater than roughly four to five days ($\approx$50 points). So instead of a using a much lower standard deviation of the mean for the entire 989 points, we statistically calculate a Type A RSU of 30 nW/W. We conclude that our average watt ratio value is, with a relative combined standard uncertainty, $W_{90}/W = 1+(8\pm87)$ nW/W [4]. This value represents better than an order of magnitude decrease in uncertainty from our last published result obtained in 1988 [2].

A. New Design Plans

Work is underway to reduce the uncertainty in the next generation experiment and reduce the time frame for averaging noise to a few days or less. A nonmagnetic, fiberglass, vacuum chamber to enclose both the upper balance and the lower inductive coil, as drawn in Fig. 1, has been constructed for operation in a low-pressure environment that will reduce the refractive index and buoyancy corrections by a factor of 100 or more. Completed modifications to the NIST nonmagnetic building include a room lined with copper-shielding to reduce RF interference. The concrete floor and stiffened, insulated walls are physically isolated from contact with the rest of the building to reduce vibrations. Improvements in the apparatus will address the volt measurement routine, coil vibration noise, alignment reproducibility, and knife-edge hysteresis. Rebuilding the experiment with these alterations will require at least two years before our goal of monitoring the stability of a one kilogram mass standard to 10 μg is reached.

ACKNOWLEDGMENT

The following have directly affected the results of this experiment, and the authors are indebted to them: P. T. Olsen, G. Stenbakken, A. Gillespie, K. Fujii, and A. Picard. The following have made essential calibrations: R. Davis, Z. Jabbour, J. Keller, C. Tilford, D. Vaughn, A. Miiller, R. Dziuba, J. Sims, and J.-H. Kim.

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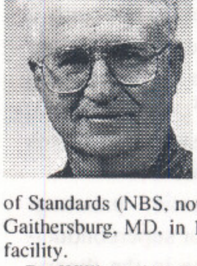

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