

# DETERMINATION OF KERR CELL PARAMETERS WITH COMPARATIVE DIGITIZED MEASUREMENTS

G.J. FitzPatrick  
Member

J.E. Lagnese  
Nonmember

National Institute of Standards and Technology  
Gaithersburg, Maryland 20899 USA

**Abstract** – Kerr cell measurement systems consist of polarimetric devices that modulate a beam of light according to the voltage applied to them. The output consists of a series of intensity oscillations or “fringes”. Calibration of Kerr cells has been performed in the past through simultaneous measurements of the Kerr cell and voltage divider outputs using analog oscilloscopes. The evaluation of the Kerr cell constant was then made by comparison of the peak voltages as measured by the two systems. Digitized data is ideal for the application of curve-fitting techniques and are now used for the evaluation of the Kerr cell constant (or alternatively, for the determination of the ratio of the voltage divider). The fitting techniques utilize the entire waveforms rather than comparing them at the peak voltage only. This paper describes the application of curve-fitting techniques to digitized waveforms for the evaluation of the Kerr cell constant. Results are presented for Kerr cells used to cover the range from 10kV-300kV. Cell constants for the same cell geometry but with different Kerr liquids are also reported. The uncertainties of the evaluated Kerr cell parameters and their dependence on fringe number are discussed. The effects on the evaluated cell constants produced by segmenting the digitized Kerr waveforms are also examined.

**Keywords:** Kerr cell measurements, curve-fitting, high voltage, impulse measurements, digital signal processing, impulse testing, comparative measurements.

## INTRODUCTION

Kerr cells are electro-optic transducers whose optical properties change when high voltage is applied to them. They have been used at NIST and elsewhere for many years for measurement of high electric fields and high voltages[1,2,3,4,5]. The uncertainty in the voltage determined by Kerr measurements depends upon the measuring system uncertainties and on the uncertainty in the parameter  $V_m$ , known as the Kerr cell constant. This cell constant is usually determined through simultaneous measurements made with a calibrated voltage divider connected in parallel with the Kerr cell. This paper describes the application of curve-fitting techniques to the evaluation of the Kerr cell constant. Previously, such measurements were made with

analog oscilloscopes and the cell constant evaluated by comparing the output intensity of the Kerr measurement system with the voltage divider output at the peak voltage only. The measurements that are reported here are now made with digital recorders which make the comparison of the two measurements over the entire waveform much easier than those formerly done with analog measurements. The results indicate that the fitted values of  $V_m$  have much less scatter than the single-point comparisons do. The following sections contain a description of the Kerr measurement and curve-fitting technique, a presentation of the results, and a discussion of their significance.

## KERR CELL MEASUREMENTS

A typical Kerr cell and major components of the optical detection system are shown in Figure 1. The system consists of a light source (stabilized laser), a Kerr cell with polarizers at its input and output, a light detector for optical/electrical, and a voltage recorder (digitizer) to measure the detector output. The Kerr cell itself is essentially a parallel plate capacitor connected to the high voltage circuit at the point where the voltage is to be measured. When high voltage is applied to the electrodes of the cell, the electric field between them induces a birefringence in the Kerr liquid in the direction parallel to the field and that perpendicular to it.

The relation of the measured output light intensity,  $I$ , to the applied voltage,  $V$ , for an ideal Kerr measurement system having polarizers oriented at  $\pm 45^\circ$  to the direction of the field within the Kerr cell is given by [6]:

$$I/I_m = \sin^2(\pi/2)(V/V_m)^2 \quad (1)$$

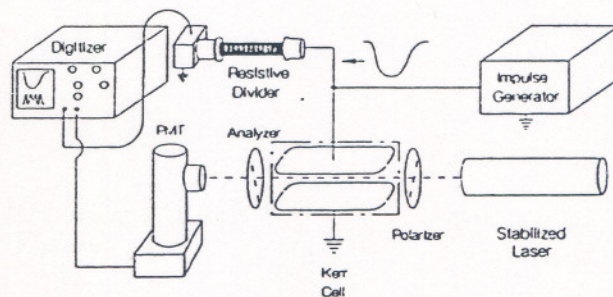


Figure 1. Kerr cell measurement system including laser, polarizers, photomultiplier tube (PMT), and digitizer. A resistive voltage divider is shown connected in parallel to the Kerr cell.



$I_m$  is the light intensity at maximum transmission. The cell constant,  $V_m$ , is defined as:

$$V_m = d/\sqrt{2B}l' \quad (2)$$

where  $B$  is the electro-optic Kerr coefficient,  $d$  is the electrode spacing, and  $l'$  is the effective electrode length. The output intensity of the Kerr measurement system for an applied impulse voltage is shown in Figures 2a and 2b. The output intensity shown in the figure and described by Equation (1) passes through a number of maxima and minima called fringes as the applied voltage changes. The fringe number,  $n$ , at any voltage,  $V$ , is defined as the ratio of  $(V/V_m)^2$ . If the cell constant is known, then the voltage can be determined from the Kerr cell intensity measurement by solving Equation (1). This can then be used to calibrate a voltage divider through simultaneous measurements made with the Kerr cell connected in parallel to the divider.

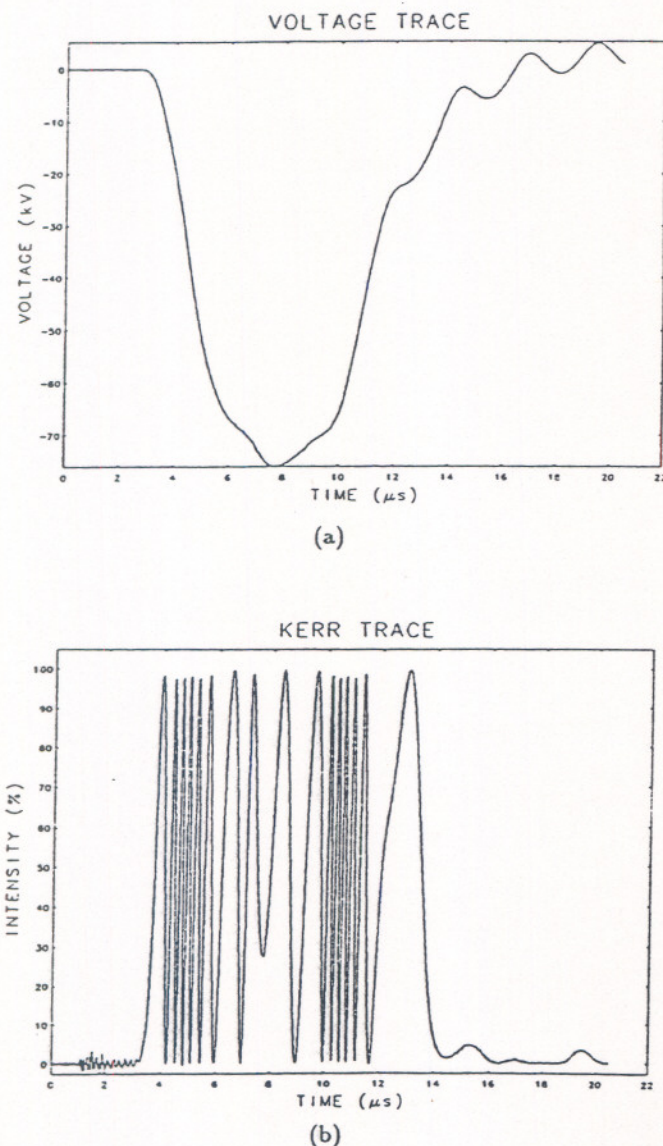


Figure 2. Simultaneous measurements made with (a) a resistive voltage divider and (b) the corresponding Kerr intensity measurement.

## KERR CELL CONSTANT DETERMINATION

The Kerr cell and voltage divider are connected in parallel to the high-voltage impulse generator as shown in Figure 1. The voltage divider provides a scaled replica of the high-voltage waveform applied to it. The high voltage,  $V$ , is related to the divider output,  $v_{out}$ , by:

$$V = D \cdot v_{out} \quad (3)$$

where  $D$  the divider ratio, which is ideally a constant. Equation (1) is then fitted to the measured Kerr output intensity using the voltage determined from the divider output voltage from (3). Analog measurements have previously been used to determine Kerr cell constants with an estimated relative uncertainty of 0.3% in the measurement of the peak voltage[7].

For the digital measurements, the following method is used for the fitting the measured curves. Given data pairs  $(V_i, I_i)$  for  $i=1, \dots, M$ , we consider the problem of fitting a model  $y(V; p)$  to the data;  $p$  is the vector of the unknown parameters. The problem is to compute estimates of those parameters which minimize the quantity  $Q$  over a suitable region, where  $Q$  is given by:

$$Q = \sum_{i=1}^M \{I_i - y(V_i; p)\}^2 \quad (4)$$

The algorithm employed to estimate the optimal values of the parameters is a modification of the Levenberg-Marquardt algorithm, which is an iterative search procedure for finding the minimum value of the sum of squares of  $M$  nonlinear functions in  $N$  variables. It is essentially a combination of the Gauss-Newton and steepest descent methods, and is designed to avoid both the divergence problems associated with the Gauss-Newton algorithm and also the progressively slower convergence frequently encountered in the steepest descent method. The starting point in the search is determined by initial estimates of the optimal parameters. A new search direction is chosen by a suitable interpolation between the search directions provided by the two algorithms. At points where the objective function,  $Q$ , is rapidly changing (usually these are far from the optimal solution), the search direction is close to that provided by the method of steepest descent. Near the optimal solution, where the gradient of the objective function is small and the function is approximately linear, the search direction is close to that provided by the Gauss-Newton method.

Another feature of the Levenberg-Marquardt algorithm, as implemented in the public-domain software package employed in this study[8], is the use of implicitly-scaled variables in order to achieve scale invariance of the method and to limit the size of the correction in any direction where the objective function is changing rapidly. Under reasonable conditions on the objective function, this optimal choice of the correction enhances global convergence from starting points far from the solution and has a fast rate of convergence for problems with small residuals.

The available data consisted of 2048 measurements of  $V_i$  and  $I_i$ , with each data pair simultaneously measured at equally-spaced time samples taken at 10 ns intervals. The  $V_i$  are the voltages measured with the voltage divider and the  $I_i$  are the corresponding intensities measured with the Kerr cell system connected in parallel with the divider.



To eliminate the effects of random noise in the  $V_i$  measurements, these data were smoothed using a least squares cubic spline fit with 100 equally-spaced knots. The smoothed voltages were those used in Equation (5) to obtain the fits shown in Figures 3 and 4. An examination of the residuals between the measured and smoothed voltages showed that the fitting procedure had the desired effect of removing the random noise without distorting the voltage waveform. Figure 5 demonstrates the result of using the noisy voltage data when fitting Equation (5) to intensity data under the conditions shown in Figure 4. The rms error of the difference of the fitted and measured curves is reduced by about half when the voltage data are smoothed.

The model used in this study for fitting of the measured Kerr waveforms is a modification of Equation (1):

$$I = I_m \sin^2((\pi/2)(V - V_o)/V_m)^2 + \phi) + I_o \quad (5)$$

The phase angle,  $\phi$ , in this equation is used to account for constant birefringences in the Kerr measurement system, such as those produced by stress in the glass windows of the Kerr cell. Equation (5) also contains a term,  $I_o$ , to account for background light intensity in the measurements. This term arises from room light entering the photodetector and from nonideal polarizers that may pass a small amount of light even when their axes are orthogonally aligned.

The single-point estimate that has been used in the past to calculate the cell constant from the peak voltage,  $V_p$ , using analog measurements of the Kerr intensity and divider output is:

$$V_m = (V_p - V_o)/\sqrt{n_p} \quad (6)$$

The peak voltage,  $V_p$ , is determined from the voltage divider measurement and from  $n_p$ , which is the Kerr fringe number corresponding to the voltage peak. The voltage offset,  $V_o$ , is found from the divider data and is usually taken as the average of several hundred data points preceding the rise of the voltage impulse. In this study, the value of  $V_o$  found from fitting Equation (5) to the Kerr waveform was used for the single-point calculation, Equation (6).

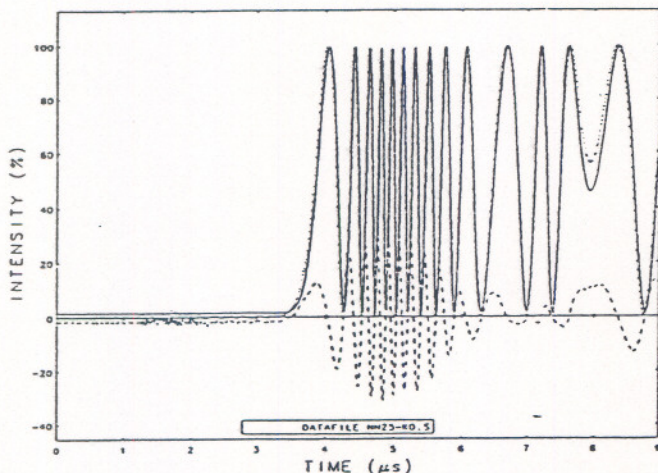


Figure 3. Measured and fitted Kerr intensities for  $\phi$  and  $V_o$  set to zero. The measured intensity appears as individual points. The fitted intensity curve is the solid curve. The difference of the two intensities is shown as the dashed curve.

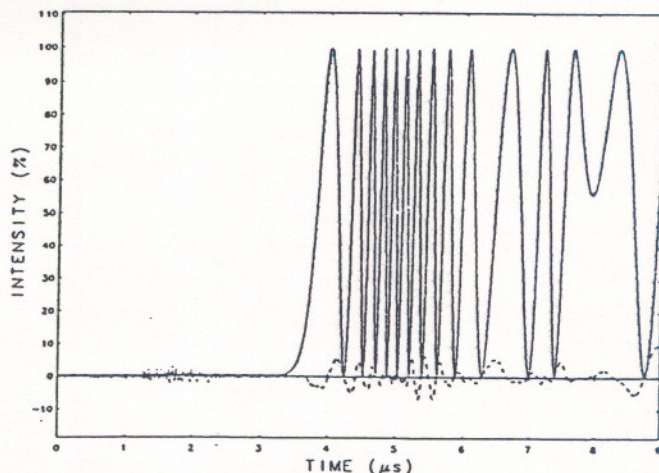


Figure 4. Measured and fitted Kerr intensities for fitted  $V_o$ . The measured intensity appears as individual points; the fitted intensity curve is the solid curve. The difference between the two intensities is shown as the dashed curve.

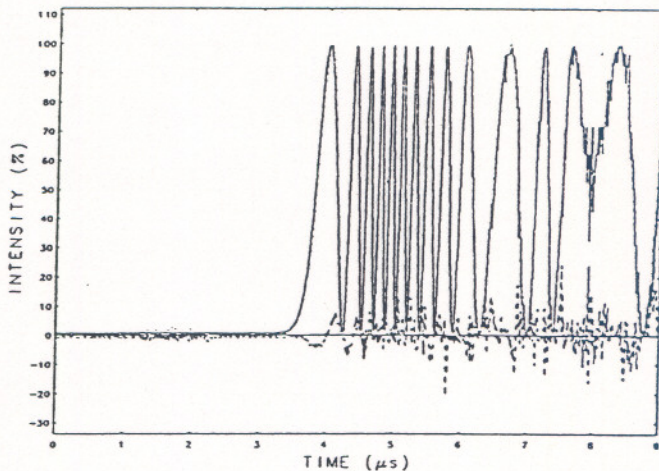


Figure 5. Fitted Kerr intensity curve found using unsmoothed voltage data. The measured intensity appears as individual points; the fitted intensity curve is the solid curve. The difference between the two intensities is shown as the dashed curve.

## RESULTS AND DISCUSSION

The function given in Equation (5) was used to fit the measured intensity curves to a point slightly beyond the peak voltage. Initially, the curve-fitting was performed with only the adjustable parameters for the intensity maximum,  $I_m$ , intensity offset,  $I_o$ , and cell constant  $V_m$ ; the phase angle,  $\phi$ , and offset voltage,  $V_o$ , were set to zero. A typical curve fit using only these parameters is shown in Figure 3. Fitting the phase angle,  $\phi$ , resulted in fits that were somewhat better: the mean differences between fitted and measured curves and the total rms error were reduced. The values determined for the fitted phase angle were small, as was expected since this parameter accounts for fixed birefringences in the optical measurement system. These birefringences, such as stress birefringence in the cell windows, have been found in other investigations to be small[9]. When the offset voltage,  $V_o$ , was fitted and  $\phi$  was set to zero, further im-



provements in the fits were realized, as shown in Figure 4. The differences between the fitted and measured curves were in the worst case less than 5% of  $I_m$  and the rms error was reduced to less than half of that when  $V_0$  was not a fitted parameter.

It is useful to compare the fitted values of  $V_m$  to those estimated from the single point  $V_p$  as described in the previous section, Equation (6). The results given in Table 1 show that the scatter in the value of  $V_m$  is reduced by an order of magnitude over that resulting from the calculated single-point values. These values were determined as the average of  $V_m$  determined from fits of impulse voltage waveforms having peak voltages over a range from  $1.73V_m$  (3.5 fringes) to  $5.96V_m$  (35.5 fringes). The error reported in the table represents three standard deviations ( $\pm 3\sigma$ ). It should also be noted that the comparison of  $V_m$ 's also includes a correction for temperature, since the data were taken at slightly different temperatures and the cell constant varies with temperature. The temperature correction method has been reported elsewhere[10]. The  $V_m$ 's reported in Table 1 are all corrected to 22.6°C. The largest differences between the fitted and the measured curves, as shown in Figure 4, typically occur at the points midway between the minimum and maximum intensities, where the intensity changes most rapidly with the voltage.

Table 1 - Kerr cell constants found with curve-fitting technique and calculated at a single point.

Polarizer Position	$V_{m,fit}(V)$	$V_{m,calc}^*(V)$
Orthogonally aligned	$19530 \pm 30$	$19470 \pm 360$
Misadjusted	$19520 \pm 40$	$19583 \pm 470$

All numbers reported are  $3\sigma$

Values are temperature-corrected to 22.6°C

\* corrected for voltage offset

The values of the fitted and calculated cell constants for fringe numbers,  $n_p$ , between 3.5 and 35.5 are shown in Figure 6. The fringe number  $n_p$  is an indicator of the peak voltage, since they are related through Equation (6). The  $V_m$ 's determined from the fitted curves are essentially independent of the fringe number. The corrected single-point estimates of  $V_m$  fall below the fitted values at low fringe number, but the two curves converge to nearly the same value for fringe numbers greater than 25. This dependence of the values of the calculated  $V_m$ 's on fringe number is in agreement with theory, which predicts that the random error  $\delta V_m/V_m$  is:

$$\delta V_m/V_m = [(\delta V/V)^2 + (\delta n/n)^2]^{1/2} \quad (7)$$

The relative error  $\delta V/V$  in the measured output voltage from the divider is approximately constant while the relative error  $\delta n/n$  in fringe number decreases with increasing  $n$ . According to Equation (7), the random error in the calculated  $V_m$  also decreases as the fringe number increases. This is characteristic of Kerr cell measurements: their sensitivity increases with increas-

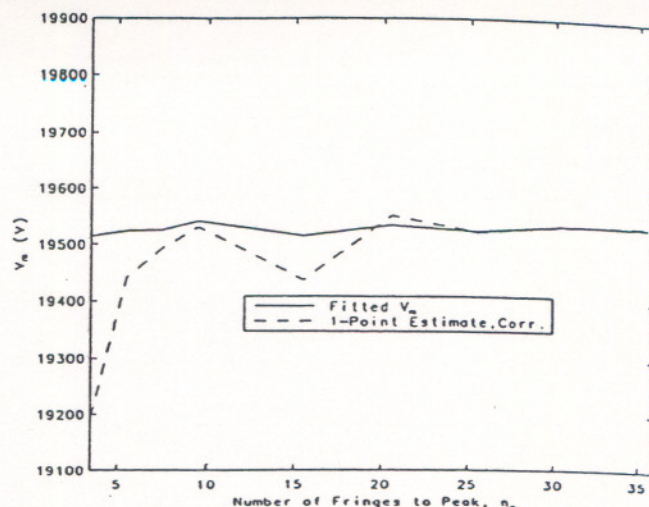


Figure 6. Dependence of fitted and calculated values for the Kerr cell constant  $V_m$  on fringe number corresponding to the peak voltage,  $n_p$ .

ing voltage. The sensitivity of the fitting technique to changes in the relative position of the axes of the polarizers was also investigated and the results are given in Table 1. For this test, the polarizers were rotated relative to each other so that they were no longer oriented orthogonally, but misaligned by approximately  $9^\circ$ . There is excellent agreement between the values of the fitted  $V_m$  determined from measurements with orthogonally-aligned polarizers and with slightly misaligned polarizers. The scatter is small for both cases. The change in the calculated single-point estimate is greater than for the fitted value of  $V_m$ , but the differences fall within  $1\sigma$  for both the fitted and calculated  $V_m$ 's. The results indicate that the fitting technique is insensitive to small misalignments of the polarizers.

In another test, the same Kerr cell was filled with a different Kerr liquid, o-dichlorobenzene, and the cell constant  $V_m$  was again evaluated using the curve-fitting software. With nitrobenzene,  $V_m$  reported in Table 1 is  $19530 \pm 30$  V. The fitted  $V_m$  with o-dichlorobenzene was  $43470 \pm 780$  V. These cell constants are evaluated by fitting a section of the Kerr output waveform, typically from the time at the start of the pulse to a time slightly greater than that corresponding to the peak voltage, as shown in Figure 4. The  $V_m$  found from fitting the first half of the waveform, from start to the voltage peak was found to differ from that found from the latter half of the waveform (from voltage peak to zero) by as much as 1-2% for the o-dichlorobenzene data. The scatter is also considerably greater in the fitted  $V_m$ , but this may be due to the range of fringes over which this average is taken, which for the o-dichlorobenzene data extended from 3.2 to 7.5 fringes. The significance of the difference in  $V_m$ , depending upon which segment of the waveform is fitted, is also unclear due to the large scatter in the data. Further measurements at higher fringe number are required to reduce this variation.

The reduced scatter in the fitted  $V_m$ 's was achieved by fitting the voltage offset  $V_0$  in Equation 5. The best-fit offsets were, however, considerably higher than expected. The voltage offset is ordinarily removed from the voltage waveform during preprocessing by subtracting the mean of several hundred data points prior to the start of the voltage pulse from the entire waveform. It was therefore expected that the fitted  $V_0$  should be small, perhaps of the order of  $0.02V_m$ . The actual values were



0.08V<sub>m</sub> to 0.1V<sub>m</sub>, and at the highest voltages, 0.01V<sub>p</sub> to 0.02V<sub>p</sub>. The single-point estimates, when corrected for voltage offset according to Equation (6), were in excellent agreement with the fitted V<sub>m</sub>'s, as shown in Table 1. However, the uncorrected single-point estimates differed from the fitted V<sub>m</sub>'s by as much as 1-2%, but this difference still falls within the scatter. The voltage correction term, V<sub>o</sub>, in addition to accounting for any residual offset in the voltage waveform, may also compensate for small timing delays between the voltage and intensity waveforms. For an ideal ramp voltage, a voltage offset is equivalent to a time delay. The voltage pulses used in this study rise monotonically to V<sub>p</sub> and can be approximated by a voltage ramp. Also, the effect of including a voltage offset in Equation (5) is the same as introducing a voltage-dependent phase angle (with or without the fixed phase angle  $\phi$ ). This can be seen if the  $(V - V_o)^2$  term is expanded:

$$I = I_m \sin^2((\pi/2)(V/V_m)^2 + \alpha V + \beta) + I_o \quad (8)$$

with:

$$\alpha = -2V_o/V_m^2$$

$$\beta = (V_o/V_m)^2 + \phi$$

Further studies are being undertaken to determine the significance of the correction term, V<sub>o</sub>, and also to investigate the systematic errors in the determination of the cell constant with both the curve-fitting and single-point estimates.

### CONCLUSIONS

Preliminary results using the curve-fitting technique described in this paper indicate that this approach may reduce the uncertainties in the determination of the Kerr cell constants using comparative digitized measurements. The Kerr-cell constants resulting from the curve-fitting technique had much smaller variation with peak voltage than did the corresponding single-point estimates; the standard deviation of the fitted cell constants was an order of magnitude smaller than that for the calculated single-point values over a wide range of voltages. The best fits to the measured Kerr waveforms were attained when a constant correction term to the voltage in the Kerr equation was included. The correction term was larger than expected and the significance of this correction will be further investigated. In the past, analog measurements have been limited to evaluating the cell constant only at single points, typically at the peak of the voltage impulse. It is only with digitized measurements that the use of curve-fitting techniques such as the one described in this paper are possible.

### ACKNOWLEDGEMENT

Partial support for this work was provided by the U.S. Department of Energy, Office of Energy Storage and Distribution.

### REFERENCES

- [1] E.F. Kelley and R.E. Hebner, Jr., "Electro-Optic Measurement of the Electric Field Distribution in Transformer Oil," 1978 Annual Report, Conference on Electrical Insulation and Dielectric Phenomena, ISBN #0-309-02861-2, pp. 206-212
- [2] R.E. Hebner, Jr., E.C. Cassidy, and J.E. Jones, "Improved Techniques for the Measurement of High-Voltage Impulses Using the Electro-Optic Kerr Effect," *IEEE Trans. Inst. Meas.*, vol. IM-24, no. 4, pp. 361-366, December, 1975
- [3] M. Zahn, and T. Takada, "High voltage electric field and space-charge distributions in highly purified water," *J. Appl. Phys.*, vol. 54, no. 9, pp. 4762-4775, September, 1983
- [4] J.D. Cross and R. Tobazeon, "Electric Field Distortions Produced by Solid Dielectric Spacers Separating Uniform Field Electrodes in Nitrobenzene," 1972 Annual Report, Conference on Electrical Insulation and Dielectric Phenomena, Buck Hill Falls, Pa, pp. October, 1972
- [5] D.C. Wunsch and A. Erteza, "Kerr cell measuring system for high voltage pulses," *Rev. Sci. Instrum.*, vol. 35, pp. S16-S20, 1964
- [6] R.E. Hebner, Jr., Richard A. Malewski, and E.C. Cassidy, "Optical methods of electrical measurement at high voltage levels," *Proceedings of the IEEE*, vol. 65, no. 11, pp. 1524-1548, November 1977
- [7] Van Brunt, R.J., ed., *Research for Electric Energy Systems - An Annual Report*, NIST Interagency Report, NISTIR 4339, pp. 60-70, December 1989
- [8] More, J.J., "The Levenberg-Marquardt Algorithm: Implementation and Theory," *Lecture Notes in Mathematics V630 Numerical Analysis* (Ed. Watson, G.), pp. 105-116. Springer-Verlag, Berlin, 1978
- [9] R.E. Hebner, Jr., R. Sojka, and E.C. Cassidy, "Kerr coefficients of nitrobenzene and water," *NBS Interagency Report NBSIR 74-544*, Washington, D.C., August 7, 1974
- [10] R.E. Hebner, Jr. and M. Misakian, "Calibration of high-voltage pulse measurement systems based on the Kerr effect", *NBS Interagency Report NBSIR 77-1317*, Gaithersburg, MD, September 16, 1977