Digital Impedance Bridge

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***Abstract*— An impedance bridge that compares two-terminal standard inductors to characterized ac resistors in the frequency range of 10 Hz to 100 kHz is described. A dual-channel, digitally-synthesized source and sampling digital multimeter are used to generate and measure relevant bridge signals. A linear interpolation algorithm is used to autocalibrate the bridge to a 1 nF gas dielectric capacitor. An intercomparison of the new bridge with existing measurement standards conducted in the low audio frequency range shows agreement of 50 to 200 parts in 10^6 for inductors for 1 mH to 10 H.**

I. Introduction

THIS paper describes an automatic impedance bridge, composed mostly of commercially available instruments, that is designed to automatically compare a standard ac resistor to a test inductor in the frequency range of 10 Hz to 100 kHz. The bridge is similar to the one described in [1], and is a refinement of the one described in [2]. It is relatively simple, requiring only a computer controller, tuned detector, digital signal generator, sampling digital multimeter (DMM), and measurement probe.

Impedance bridges employing dual-channel digital sinewave synthesis techniques are well-known [3]–[5]; most of these bridges achieve their high accuracies by relying heavily on inductive voltage dividers (IVD's) to measure the required voltage ratios because of their small in-phase and quadrature error components. The impedance bridge discussed here relies on the ratiometric voltage measurement accuracy of a sampling DMM in conjunction with a least-mean of squares (LMS) sine fit algorithm to extract amplitude and phase information of the fundamental signal component. While the impedance bridge discussed here does not quite approach the measurement uncertainty levels of the IVD-based bridges, it does serve as a valuable tool in the development of fully automated, wideband, sampling-based impedance analyzers.

II. Hardware

A. Digital Signal Generator

A simplified schematic of the digital impedance bridge (DIB) is shown in Fig. 1. The bridge is supplied with test signals V_R and V_V by a commercially-available, dual-channel signal generator similar to the one described in [6]. The generator must have amplitude stability of about 2 μV/V per minute, amplitude resolution corresponding to at least 12 bits, and phase resolution and stability of about 0.001 degrees. Amplitude and phase accuracy are not critical, although linearity of these two quantities allows for better convergence of the balancing algorithm.

The generator chosen for this bridge produces a synchronization signal output, as shown in Fig. 1. This TTL-level signal is used to synchronize the sampling DMM to the two test signals, V_R and V_V.

Manuscript received July 1, 1994; revised October 15, 1994.

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IEEE Log Number 9409140.

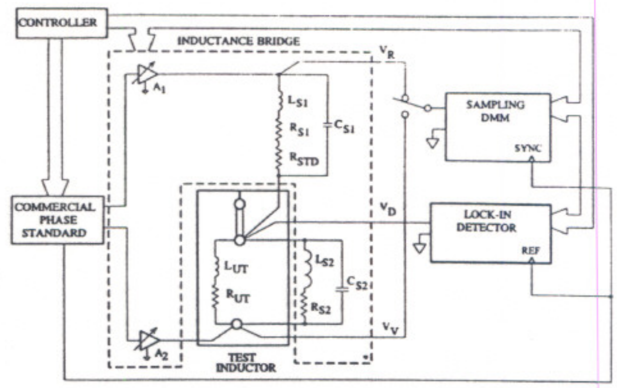

Fig. 1. Impedance bridge simplified diagram.

B. Measurement Probe

The bridge consists of seven standard ac resistors from 100 Ω to 100 k Ω. For simplicity, Fig. 1 shows only one of the standard resistors, R_{STD}, along with its associated parasitic components. In order to supplement the generator's amplitude resolution, the bridge incorporates programmable voltage amplifiers A_1 and A_2, which increase amplitude resolution from 12 bits to approximately 20 bits. To minimize interconnection impedances, the voltage amplifiers, standard resistors, and switching circuitry are housed in a small, shielded probe (the dotted line of Fig. 1) that is connected directly to the inductor under test.

C. Sampling DMM

The commercially-available sampling DMM quantizes signals V_R and V_V with 16-bit amplitude resolution using an equivalent-time sampling method capable of 10 nanosecond timing resolution. Since subsequent analysis of the sampled data extracts only the test signals' amplitude ratio and phase relationships, the DMM's ac voltage measurement linearity is much more critical than its absolute accuracy. Although not shown in Fig. 1, the DMM is also used to perform *in situ*, 4-terminal dc resistance measurements of the seven standard ac resistors.

III. Measurement Technique

Referring to Fig. 1, the bridge operates by comparing a known standard ac resistor to the two-terminal inductor under test, shown as L_{UT}, along with its associated series resistance, R_{UT}. The test inductor can also be modeled as an inductance and equivalent parallel resistance, whichever model is more appropriate. The signal generator is adjusted to produce a null signal, V_D, using an autobalancing algorithm. When operating at a null, the ratio of the unknown total impedance, termed collectively as Z_{UT}, to the standard impedance, termed collectively as Z_{STD}, is proportional to the ratio of the two voltages by:

$$\frac{V_V}{V_R} = \frac{Z_{UT}}{Z_{STD}}, \quad \text{or} \quad Z_{UT} = \frac{Z_{STD} V_V}{V_R}. \tag{1}$$

The sampling DMM is used to measure the ratio of V_V to V_R, and the phase angle between them using a 4-parameter sine fit algorithm as described in [7], [8]. The L_{UT} and R_{UT} values are then computed by removing the stray measurement probe open-circuit impedances, shown as L_{S2}, R_{S2}, and C_{S2}.

IV. Bridge Calibration

Even with careful attention to minimizing lead and stray impedances linked to the standard and test impedances, it became necessary to develop a means to measure the residual stray impedances present in the bridge. A nonlinear least-squares parameter estimation method using the Newton-Raphson Algorithm was used for this purpose.

A. Parameter Estimation Method

As stated earlier, the dc resistance component, R_{STD}, of the seven standard ac resistors is measured using the sampling DMM. This measurement has a two-sigma relative uncertainty of 5×10^{-6}. The DMM is routinely calibrated at these cardinal resistance values, since the accuracy to which these values are known directly affects bridge accuracy.

The five stray bridge parameters, L_{S1}, C_{S1}, L_{S2}, R_{S2}, and C_{S2} are determined by placing a 1 nF gas dielectric capacitor in place of the impedance under test. This capacitor is first calibrated at 1 kHz using a precision, 3-terminal capacitance bridge. With the circuit adjusted for a null at V_D, the ratio of the V_V to V_R amplitudes and their relative phases are measured at n different frequencies. A $2n \times 5$ sensitivity matrix (Jacobian matrix), ΔV, is then computed by estimating the system's V_V to V_R ratio and phase sensitivity to small changes in the Fig. 1 circuit model's individual element values, over the n different frequencies, using reasonable initial estimates of circuit element values. At 5-element vector of the deviations of the stray impedances from their assumed values, $\hat{x}$, is then calculated by computing a least-squares solution of the matrix equation

$$y_M = \Delta V \hat{x}, \quad \text{thus} \quad \hat{x} = [\Delta V^T \Delta V]^{-1} \Delta V^T y_M \tag{2}$$

where

$y_M =$ a $2n$-element vector containing the difference between measured and predicted behavior of the V_V to V_R ratio and phase information over the same n frequencies.

The above process is then used to adjust the five bridge parameters until the coefficient vector, $\hat{x}$, is sufficiently small or stable.

The use of the gas dielectric capacitor in place of an open-circuit places a large known admittance in parallel with the small unknown open-circuit admittance, which increases the bridge sensitivity over the n frequencies and allows for better convergence of the interpolation algorithm. The consequence of this substitution is that the correction coefficients of the bridge, and thus its accuracy, to first order, are based on the assumption that the capacitor's capacitance and dissipation factor are flat to 100 kHz. Previous investigations of gas dielectric capacitors support this assumption [9].

B. Building the Matrices

For the above procedure, the following matrices are computed:

1)

$$\nu_{VN} = \begin{bmatrix} |V_{VN_1}| \\ |V_{VN_2}| \\ \vdots \\ \Theta_{VN_{n-1}} \\ \Theta_{VN_n} \end{bmatrix} =$$

$2n \times 1$ vector where $|V_{VN_i}|$ and Θ_{VN_i} are the magnitude ratios and relative phases of the signals (V_V and V_R) in Fig. 1 computed over the n chosen frequencies using the model's initial R, L, and C estimates.

2) For each of the five stray bridge parameters in the circuit model of Fig. 1:

$$\nu_{CS1} = \begin{bmatrix} |V_{CS1_1}| \\ |V_{CS1_2}| \\ \vdots \\ \Theta_{CS1_{n-1}} \\ \Theta_{CS1_n} \end{bmatrix} =$$

a $2n \times 1$ vector containing new computed values of ratio and phase when the element in question (in this case C_{S1}) has changed, i.e., $C_{S1} = C_{S1} + \Delta C_{S1}$.

3) $V = [v_{LS1} \quad v_{CS1} \quad v_{RS2} \quad v_{LS2} \quad v_{CS2}]$.

4) $V_{NN} = [v_{VN} \quad v_{VN} \quad v_{VN} \quad v_{VN} \quad v_{VN}]$.

5) $\Delta V = V_{NN} - V$.

6) From the series of bridge balances over the n frequencies:

$$y = \begin{bmatrix} |V_{V_1}| \\ |V_{V_2}| \\ \vdots \\ \Theta_{V_{n-1}} \\ \Theta_{V_n} \end{bmatrix} =$$

a $2n \times 1$ vector where the $|V_{V_i}|$ and Θ_{V_i} values are the ratio and phase at each balance.

7) $y_M = y - v_{VN}$

TABLE I

Digital Impedance Bridge - Maxwell Wien Bridge
(differences in parts in 10^6)

Frequency	1 mH	10 mH	100 mH	1 H	10 H
100 Hz	160	60	160	50	20
1 kHz	20	15	30	35	125

Maxwell Wien Bridge Uncertainties
(present calibration uncertainties in parts in 10^6)

Frequency	1 mH	10 mH	100 mH	1 H	10 H
100 Hz	200	200	200	200	200
1 kHz	200	200	200	500	2000
10 kHz	500	500	500	*	*

Digital Impedance Bridge Uncertainties
(typical 2σ values in parts in 10^6)

Frequency	1 mH	10 mH	100 mH	1 H	10 H
100 Hz	125	100	75	65	65
1 kHz	100	75	65	75	100
10 kHz	75	65	65	*	*

* Above the resonant frequency of existing standard inductors.

V. PERFORMANCE

Two independent methods were devised to partially evaluate the performance of the digital impedance bridge (DIB). An intercomparison was performed at 100 Hz and 1 kHz between the DIB and the existing measurement standard, the NIST Maxwell–Wien Inductance Bridge [10]. In addition, an impedance bridge based on the three-voltmeter method [11] (described below) was constructed and used to perform an intercomparison with the DIB using a 10 mH inductor at 1 kHz.

A. Intercomparison with the Maxwell–Wien Bridge

The results of an intercomparison of the DIB with the present standard (Maxwell–Wien bridge) at 100 Hz and 1 kHz are given in Table I. The inductance values were chosen for the intercomparison so that most of the DIB's standard resistors would be used in order to check the performance of the bridge correction scheme. The results, along with an analysis of uncertainties present in both the DIB and the Maxwell–Wien, seem to suggest a basic uncertainty of 50 to 200 parts in 10^6 for measurements made with the DIB in this inductance and frequency range.

B. Intercomparison with the Three-Voltmeter Bridge

Although primarily used for power and energy measurements, the three-voltmeter method [11] is also well-suited for impedance measurements, provided that a suitable reference impedance, three highly-accurate ac voltmeters, and a means to measure the voltmeters' input impedance are available.

A simplified diagram of the three-voltmeter bridge constructed for this intercomparison is shown in Fig. 2. The digital generator used in the DIB is used to generate signals V_R and V_V, which are then adjusted in relative amplitude and phase angle to bring the center point between Z_{REF} and Z_{UT} to a

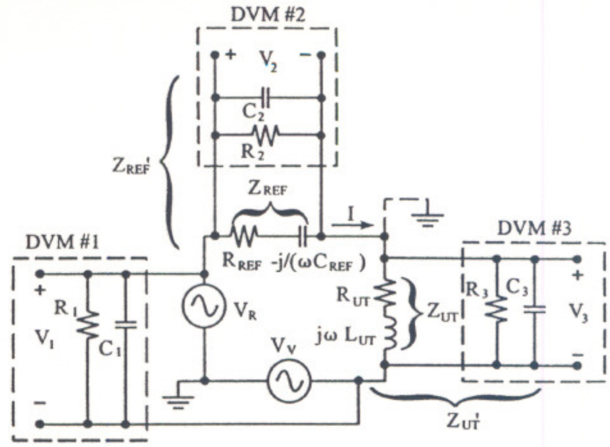

Fig. 2. Simplified diagram of three-voltmeter method bridge.

virtual ground. Then, by the law of cosines

$$V_1^2 = V_3^2 + V_2^2 + 2V_3V_2\cos\theta \tag{3}$$

and, assuming V_2 to be of zero phase, thus $I = |V_2|/Z_{REF}$, then

$$V_1^2 = V_3^2 + V_2^2 + 2V_3IZ'_{REF}\cos\theta. \tag{4}$$

Then it follows that P'_{ZUT} = the power dissipated in Z'_{UT} can be given by

$$P'_{ZUT} = I|V_3|\cos\theta = \frac{(V_1^2 - V_3^2 - V_2^2)}{2Z'_{REF}} \tag{5}$$

therefore

$$\theta = \cos^{-1}\left(\frac{P'_{ZUT}Z'_{REF}}{|V_2|\,|V_3|}\right) = \cos^{-1}\left(\frac{V_1^2 - V_3^2 - V_2^2}{2|V_2|\,|V_3|}\right). \tag{6}$$

Since θ = the angle of V_3 is known, Z'_{UT} may be expressed by the complex equation:

$$Z'_{UT} = \frac{V_3}{I} = \frac{V_3Z'_{REF}}{V_2}. \tag{7}$$

Z_{UT} may then be calculated from Z'_{UT} by correcting for the shunt voltmeter impedances, R_3 and C_3.

For the three-voltmeter bridge described above, the ac voltmeters used were first calibrated to a two-sigma relative uncertainty of 2×10^{-5} at the test voltage of approximately 1 Vrms at 1 kHz. The input impedance of each voltmeter (approximately 300 k Ω in parallel with 20 pF) was measured *in situ* using a commercially-available, precision LCR meter, so that interconnection impedances between the voltmeter and the test apparatus would not affect the measurement. The reference impedance chosen for the test was a 100-Ω precision ac resistor designed to have negligible ac-dc difference at 1 kHz.

The major source of error in the impedance measurement using a three-voltmeter bridge in this configuration results from the shunt impedance of the voltmeters. The LCR meter used to measure the voltmeters' input impedance was adequate to reduce the level of this source of error to less than 5×10^{-5} of the measured impedance.

The three-voltmeter bridge was then used to make a 1 kHz measurement of a 10 mH inductor that had been previously measured with the DIB and the Maxwell–Wien bridge. The results of this intercomparison, including allowances for uncertainties of the three measurement methods, indicates an agreement of approximately 7.5×10^{-5} among the three bridges.

VI. Conclusion

A digital impedance bridge has been described that can be used to intercompare any two-terminal impedances. It employs digital waveform synthesis and sampling, and signal processing to determine an unknown test impedance in terms of a known reference impedance. A probe for the bridge was specifically designed to automate the calibration of standard inductors over a 10 Hz to 100 kHz frequency range. Probe errors have been modeled and are corrected for using an autocalibration scheme that reduces the total relative bridge uncertainty to 5×10^{-5} in the mid-frequency range. However, as with other inductance bridges, the uncertainty degrades considerably for low value inductors and at frequency extremes. Measurements made using the new digital bridge have been compared to those made using the present standard; an aging, manually operated Maxwell–Wien bridge. Agreement is typically better than the present calibration uncertainty of the Maxwell–Wien bridge and the digital bridge will eventually be used as a reference to calibrate standard inductors. Work is underway on a second-generation digital bridge that will have an extended frequency range (to 1 MHz), improved accuracy, and the ability to intercompare two-, three-, and four-terminal impedances.

Acknowledgment

The authors thank T. M. Souders, G. N. Stenbakken, and A. D. Koffman for their suggestions, M. Chang and A. Sepulvada-Lugo for intercomparison results, and R. H. Palm for bridge construction.

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