Nonrandom Quantization Errors in Timebases

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Abstract - Timebase distortion causes nonlinear distortion of waveforms measured by sampling instruments. When such instruments are used to measure the rms amplitude of the sampled waveforms, such distortions result in errors in the measured rms values. This paper looks at the nature of the errors that result from nonrandom quantization errors in an instrument's timebase circuit. Simulations and measurements on a sampling voltmeter show that the errors in measured rms amplitude have a non-normal probability distribution, such that the probability of large errors is much greater than would be expected from the usual quantization noise model. A novel timebase compensation method is proposed which makes the measured rms errors normally distributed and reduces their standard deviation by a factor of 25. This compensation method was applied to a sampling voltmeter and the improved accuracy was realized.

I. INTRODUCTION

Nominally uniformly spaced sample intervals are fundamental for modern sampling instruments. This is true whether the samples are taken in real time or equivalent time. The deviations away from uniform time intervals have two components: a random part called time jitter that is not the subject of this paper, and a deterministic part called timebase distortion. Uncorrected timebase distortion causes nonlinear distortion of the sampled waveforms. Several papers have been written on techniques for measuring and correcting for deterministic timebase distortion [1-6]. These correction techniques usually depend on resampling the recorded waveform to produce a new waveform that represents the signal sampled uniformly. Here we present an alternative correction method that does not rely on recalculation of the waveform if the quantity of interest is the root-mean squared (rms) amplitude of the sampled signal.

Not all instruments have the type of timebase error discussed in this paper, although most equivalent-time types do. The following describes the type of timebase error under discussion. The timebase on many instruments uses a clock circuit that runs independent of the signals being sampled. This clock circuit usually has a smallest time resolution unit that can be programmed. This time unit is the quantization resolution of the timebase. If the instrument makes a measurement that requires sample intervals that are not integer multiples of this unit, the realized sample times will have a quantization error. Such an error can occur when the measurement requires an integral number of samples over one or more periods of the signal being sampled. In this case the timebase will have quantization errors that are dependent on the frequency of the signal being sampled.

When designing sampling instruments the timebase quantization resolution is usually selected such that its effects on the accuracy of the instrument are below the random noise level of the sampling process. To do this requires estimates of the effects of such an error process. The traditional method employed is to treat the quantization as a random noise process [7-10]. For an instrument used to measure the rms amplitude of the sampled signal, such a model leads to an error estimate for the calculated rms values characterized by a normal probability density error distribution. The timebase quantization, however, violates the assumptions necessary for use of the random noise model. To use that model the errors must be independent and identically distributed (iid). As will be shown in this paper, certain timebase quantization processes can cause the quantization errors to be correlated with each other and with the signal being measured. This causes the errors in the calculated rms values to be non-normally distributed. A consequence of using the wrong error model is that it underestimates the probability of large errors.

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This paper examines the nature of the sampled waveform errors that arise from timebase quantization. A unique method of correcting the sampling process is described that significantly reduces the errors in the calculated rms amplitudes of such sampled waveforms and results in a normal error distribution. This modified sampling technique requires only a very small change in the way that sample times are calculated and no change in the basic design of the quantized timebase.

Timebase quantization errors were encountered at NIST during the development of a high accuracy sampling voltmeter [11]. The NIST Wideband Sampling Voltmeter (WSV) measures the rms amplitude of periodic signals by sampling the signal's waveform using equivalent-time sampling. The design for this timebase and how it interacts with the signals being sampled is described in section II. Results from a simulation model of the voltmeter indicated the differences between the correct error model and a random error model as described in section III. A modified quantization scheme, which significantly reduces the errors caused by this type of timebase distortion, is described in section IV. In addition to the simulation studies, measurements on the NIST WSV verified the accuracy of the simulations and the improvements possible by use of the modified quantization scheme as described in section V.

II. RAMP QUANTIZATION

The NIST WSV adjusts the sampling rate to be an integer multiple of the signal frequency. However, because the timebase is quantized, the actual sample times are not precisely uniformly spaced. The following is a simplified description of the way the sample times are generated to help the reader understand how the time quantization process interacts with the signal frequency and results in an error in the measured rms value.

The timebase generator makes use of a voltage ramp, a reference DAC, and a comparator. Because of the large frequency range covered by the voltmeter, 10 Hz to 200 MHz, multiple ramp slopes are used. Twenty-two slopes are used altogether; three are used to cover each decade of frequency. During the measurement process the frequency of the input signal is determined and used to select the fastest ramp such that for one signal period the ramp voltage changes by less than the DAC full scale voltage range. The ramp start time is synchronized with the input signal. The ideal sample interval, t_i, is calculated by dividing the signal period by the number of samples to be taken over one period. Selecting a start delay and adding multiples of the ideal sample interval generates the ideal sample times. From the known slope of the ramp the reference voltage corresponding to each sample time is determined and rounded to the nearest DAC level. This

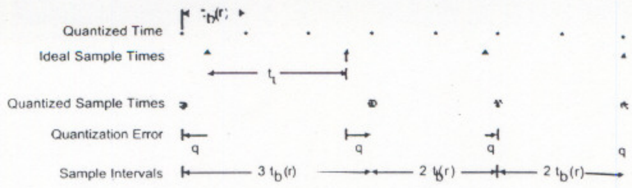

Fig. 1. Example of timing relation between quantized time, ideal sample time, quantized sample times, quantization error, and sample intervals.

rounding is the cause of the timebase quantization and from the computations the timebase quantization error for each sample time is known. For each sample time the DAC is set to the corresponding voltage level and a sample strobe is generated when the comparator detects the ramp crossing the DAC voltage. The signal amplitude at each sample time is measured and the signal's rms value is calculated as the rms of all the measured sample values.

The time intervals are quantized by the resolution of the reference DAC. The weight Q (in volts) of the least significant bit (lsb) of the DAC and the ramp slope r (in volts per second) determines the timebase resolution $t_b(r)$; therefore, the smallest time interval that can be realized for the selected slope r is given by $t_b(r)= Q/r$. The actual time intervals are limited to multiples of this unit of time. Since the ideal sample times are rounded up or down to the nearest quantized time, the quantized sample sequence will have two sample intervals $m \times t_b(r)$ and $(m+1) \times t_b(r)$, such that

$$m \le \frac{t_i}{t_b(r)} = (m+\delta) < m+1, \qquad (1)$$

where m is the integer number of quantization intervals in the ideal sample interval and δ is the fractional ideal sample-interval quantization factor. Figure 1 shows an example of the relation between the quantized times, the ideal sample times, and the quantized sample intervals.

If δ is close to 0.5 then in general the quantized sample intervals will alternate between the two quantized sample intervals. Occasionally two sample intervals of the same size will occur together. For δ close to 0.5 the sequence of quantization errors will be negatively correlated. However, if δ is very close to 0 or 1, the pattern is different. Then the quantization intervals are primarily of one size with an occasional interval of the other size. In these cases the sequence of quantization errors will be positively correlated. Thus the pattern of quantization interval is determined by δ.

When used as the timebase for measuring the rms value of a sinewave, the errors caused by this quantization process

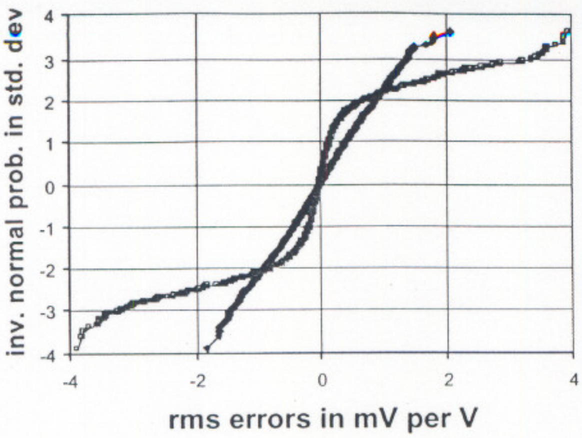

Fig. 2. RMS measurement error for ramp quantization (gray squares) is non-normally distributed, and rms measurement error for random quantization (dark diamonds) is normally distributed.

vary with the value of δ. Since the value of δ is a function of the period of the signal being measured, the errors in the rms measurement vary with the signal frequency. The rms measurement errors can become large if the quantization errors are correlated with the signal being measured. Since this timebase increases the probability of having correlated quantization errors relative to a random error process, the probability of large measurement errors is increased at select frequencies.

For purposes of this paper, the important features of the timebase being described are first that the quantization resolution of the timebase remains fixed for a range of input signal frequencies, and second that the sampling process is adjusted to the signal frequency. This type of timebase process is hereafter referred to as a ramp timebase. In examining the errors caused by this timebase error process it is important to keep in mind the differences in its behavior relative to the behavior for a random timebase error process. First, for the ramp timebase the quantization errors are not random. Rather they are a function of the measurement and signal parameters. Thus, averaging repeat measurements can not reduce the size of these errors caused by this process. They cause a bias in the measured values. Second, the probability distribution of the errors in the measured quantities is changed. These effects are examined using a simulation model.

III. SIMULATION MODEL

A simulation model of the NIST WSV was developed to examine the effects of timebase errors on the rms measurement process. The simulations allowed either a truly random timebase quantization or the nonrandom ramp-dependent quantization patterns. The random time base errors used a uniformly distributed random timebase error of amplitude $\pm 0.5\, t_b(r)$. For both timebase error types the resultant rms measurement errors were determined for a large number of input signal frequencies, phases, and other parameters.

Figure 2 shows the cumulative normal distribution plots of the rms measurement errors for both timebase error types. For this plot the rms measurement errors are sorted, the inverse normal probability of the sample number in units of standard deviation are plotted on the vertical axis and the rms error values are plotted on the horizontal axis. Thus, if the errors are normally distributed this plot will show a straight line. The plot for the random timebase errors is a straight line showing that the errors in these rms measurements are normally distributed. The line for the ramp- timebase errors is not straight. The line deviates significantly from a straight line before reaching ± 2 standard deviations. Thus, about 5 percent of the errors are significantly larger than would be expected from a normally distributed random error mechanism. Conversely, almost 95 percent have an error significantly less than expected.

Since many of the errors for the ramp-quantization timebase are less than expected, is there some way of modifying the quantization error patterns to take advantage of this and eliminate the large errors? The next section shows how this can be done and how it should not be done.

IV. MODIFIED RAMP QUANTIZATION

Looking at the timebase quantization errors associated with the largest rms measurement errors shows that these errors are associated with positively correlated quantization errors. These result from the fractional quantization factor δ being close to 0 or 1. A useful quantity for understanding how this correlation affects the measured rms values is the cumulative sum of the quantization errors from the first sample to sample j, $C_q(j)$. This quantity is given by

$$C_q(j) = \sum_{i=1}^{j} q_i \,, \qquad (2)$$

where q_i is the timebase quantization error for time sample i. This quantity plays an important role in the modified quantization scheme that reduces this effect. The ramp-quantization process holds the magnitude of each quantization error q_I to less than half the timebase resolution, i.e., $|q_I| < 0.5\, t_b(r)$. If the quantization errors were random the standard deviation of $C_q(j)$ would be proportional to $\sqrt{j}\, t_b(r)$. Figure 3 shows a plot of $C_q(j)$ for a frequency with large error where δ is close to 0. Because the quantization errors are correlated, $C_q(j)$

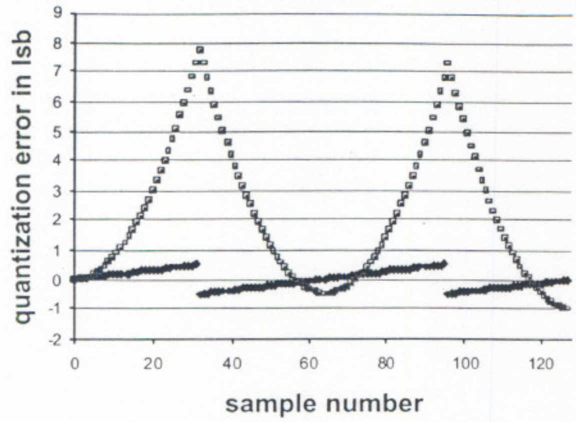

Fig. 3. Example of quantization errors (dark diamonds) and cumulative sum quantization errors (gray squares) showing correlation of quantization errors and large cumulative sum values.

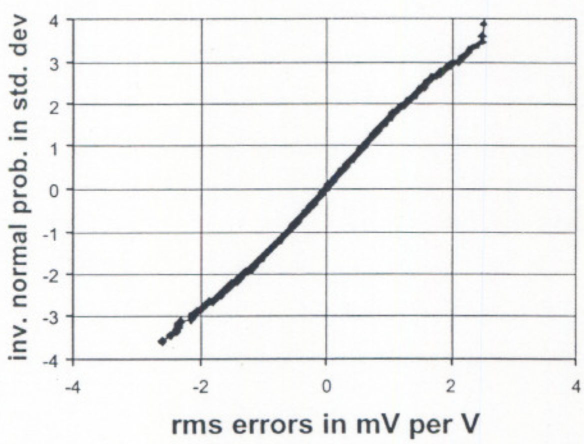

Fig. 4. . RMS measurement error for ramp quantization plus random dither of 0.5 lsb amplitude gives normal distribution.

become large compared to $t_b(r)$ much more quickly than would be expected in a random model.

One, not so good, way to break up this correlation is to add a random dither to the ideal sample times before quantizing them. Figure 4 shows the cumulative error distribution for rms measurements taken with a uniformly distributed random time dither of amplitude $\pm 0.5\, t_b(r)$ added to each ideal sample time before being quantized. The resultant distribution is now normal but at the expense of being larger.

A better way of breaking up this correlation is to restrict the size of the cumulative sum of quantization errors, $C_q(j)$. This can be accomplished by adding the cumulative sum of previous errors, $C_q(j-1)$, to the ideal sample time before quantization. The value of $C_q(0)$ is taken to be zero. The new set of values for $C_q(j)$ will be between

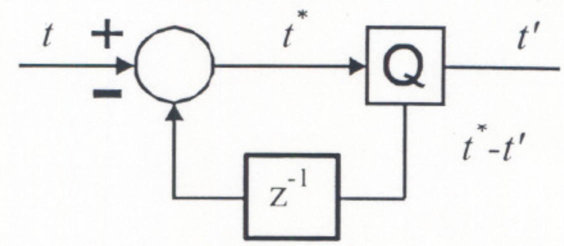

Fig. 5. Cumulative sum limited quantizer; the quantization error from quantizer Q is fed back to an adder via a unit sample delay to limit the cumulative sum of quantization errors.

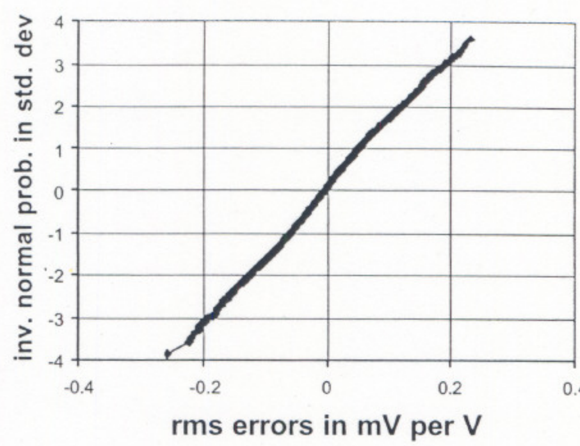

Fig. 6. RMS measurement error for CSL quantization is normally distributed and much smaller than errors shown in figs. 2 and 4.

$\pm 0.5\, t_b(r)$. Figure 5 shows a simple feedback computation that performs this operation without the need to calculate the cumulative sums. This quantization method is referred to as the cumulative-sum-limited (CSL) quantization scheme. The distribution of rms errors that results from the use of the CSL quantization scheme is shown in fig. 6. The rms errors are now normally distributed and have a standard deviation of about 1/25 of the standard deviation from the ramp timebase. The next section shows that this improvement was realized in the NIST WSV.

During the simulations each of the parameters that affect the rms error was varied to determine their effects. The primary factors that determine the rms error caused by timebase quantization are: the number of bits, b, used in the timebase DAC; the number of samples, N, used in the rms computation, the number of cycles, n, of the signal that are sampled; the quantization error for the first sample, ε_0; and the fraction of the DAC range that represents one signal period. For the data given above and in much of this paper, the parameters are often chosen as 128 samples, over one cycle of the signal, with the timebase DAC resolution set at 10 bits, the signal phase at 0 degrees, the initial quantization value as zero, and the

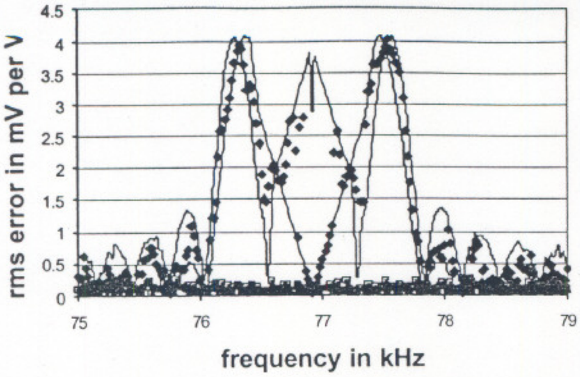

Figure 7. Predicted rms error range for ramp quantization (maximum-upper solid line, minimum-lower solid line) and measurement rms error for ramp quantization (dark diamonds), and for CSL quantization (gray squares).

fraction of the DAC range that represents one signal period between 0.5 and 1.0. Based on a large number of simulations, varying each of these factors over a wide range, the rms error data were fitted to a functional relation for each quantization method. The empirical relation for the dependence of the rms error for the ramp-quantization method is given as

$$rms_{rq} = \frac{1.7\times 2^{-b}}{\sqrt{N}} + 5\times 2^{-2b} \tag{3}$$

and for the modified CSL quantization is given as

$$rms_{CSLq} = \frac{5.3\times 2^{-b}\times n^{0.82}}{N^{1.4}} + 1.46\times 2^{-2b}. \tag{4}$$

Note how the error with the CSL quantization, rms_{CSLq}, drops much more rapidly as a function of the number of samples, and has a dependence on the number of cycles of the measured signal, which is not present in the ramp-quantization timebase induced error.

The functional dependence of the rms error versus signal phase is the same for both timebase quantization schemes. When plotted versus signal phase angle at the start of the sampling interval, the error is a sinewave with a period of 180 degrees. The amplitude of this sinewave is called the phase maximum error, PME. This is the largest error possible for a given frequency while varying the signal phase relative to the sample interval and holding the other measurement parameters constant. Because the period of this phase dependent error is 180 degrees, the PME can be determined by simulating or measuring the rms error for two signal phases separated by 45 degrees and calculating the root-sum-square of the two error values. The values of PME versus frequency were simulated and compared to PME values measured on the NIST WSV as described in the next section.

V. EXPERIMENTAL VERIFICATION

The validity of the simulation model and the value of the CSL quantization scheme compared to the traditional ramp-quantization scheme are shown with measurements taken on the NIST WSV. The DAC resolution for the WSV was reduced to 10 bits for this experiment to accentuate the errors. With the traditional ramp-quantization scheme, the rms errors become very large around certain frequencies. One such peak occurs for signal frequencies around 77 kHz. Figure 7 shows the measured and simulated PME results for frequencies from 75 kHz to 80 kHz. The two solid lines show the results of simulations of the rms error using the traditional ramp-quantization scheme. The top line shows the largest predicted PME for each frequency varying all the other parameters. The lower line shows the smallest predicted PME for each frequency. Thus, if the voltmeter performs the same as the simulation model the measured PME's for ramp quantization should fall between the two curves. The series of points with diamonds shows the measured PME for the NIST WSV using the traditional quantization scheme. All values fall between the two curves predicted by the simulation model.

The CSL quantization scheme was implemented on the NIST WSV. This was done by a simple software change to the sample time computation. The lower curve of squares in fig. 7 shows the measured PME using the CSL quantization scheme. These values are (as predicted by the simulation) much lower and do not show the presence of large deviations around certain frequencies.

VI. CONCLUSIONS

The unexpected effects of nonrandom timebase quantized errors on the measurement accuracy of the NIST WSV was modeled and verified. A new scheme for quantizing the timebase, CSL quantization, was described that decreases the quantization-related errors by a factor of 25. The CSL quantization scheme was demonstrated on the NIST WSV and the results showed that the instrument's accuracy could be improved significantly using this easy-to-implement procedure. In the present NIST WSV design, the instrument's timebase related errors were reduced to less than the noise level by using a higher resolution newly designed timebase [12].

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