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Code Probability Distributions of A/D Converters with Random Input Noise

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ABSTRACT
The specific architecture of an A/D converter influences the code probability distributions that result from random input noise. In particular, the ouput codes of successive approximation A/D converters have a spiked distribution, and its variance is half that of the corresponding input noise. In addition, the distribution has a small bias. These and other related results are derived, and are qualitatively supported by measurement data on a real 16-bit A/D converter.

I. INTRODUCTION

Architecture dependent effects are generally ignored when considering A/D converter noise models [1]. In most analyses, both the equivalent input noise and the corresponding digital output codes are assumed to be normally distributed. However, the influence of a converter's specific architecture on the noise-related probability distribution of output codes can be quite significant, especially when the equivalent input noise exceeds a least significant bit (LSB) or so. As will be seen, the consequences of these effects can be important. In a closely related area, much work has been reported on quantization of dithered signals (see for example, [2] and its references); however, in those studies the dither signal is usually added directly to the input signal, external to the A/D converter (i.e., pre-sampler noise). These analyses ignore the effects of post-sampler noise, internal to the converter.

Architecture dependent effects are particularly striking for successive approximation A/D converters, as is illustrated in fig. 1. Fig. 1-a shows a recording of the output noise from a 16-bit successive approximation A/D converter [3] that has approximately 6 LSB's of internal noise (1-σ), and fig. 1-b shows the probability distribution function (PDF) of the output codes. (We will use PDF here to mean probability ***distribution*** function, rather than probability ***density*** function, since we will be dealing primarily with the probability of occurrence of codewords that are discrete random variables.) It is readily apparent that the PDF differs significantly from that of a normal distribution (see overlaid curve), and is characterized by sharp

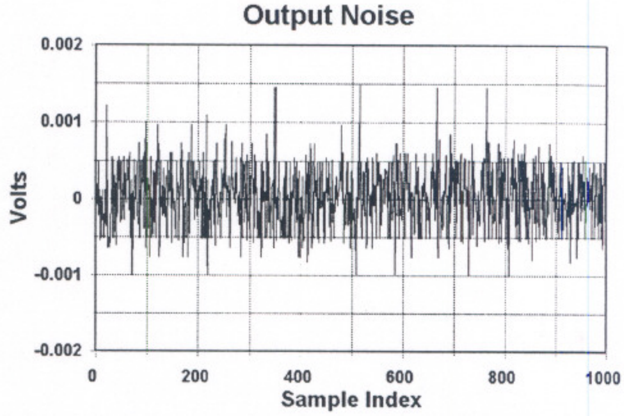

Fig. 1a Output noise from a 16-bit successive approximation A/D converter.

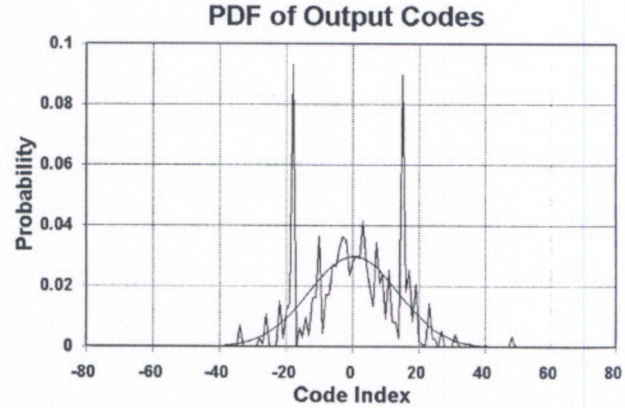

Fig. 1b Probability distribution of output codes from data of fig. 1a. A normal distribution is overlaid for comparison.

spikes. This behavior is the result of post-sampling noise, i.e., noise that appears at the comparator input and varies during the successive approximation process. By modeling this process, the resulting PDF is easily computed in closed form, as will be described later. Two significant consequences of this behavior are: 1) the variance of the output value is actually on average about 50 percent smaller than that of the input noise, regardless of noise level; and 2) the process produces a small signal-dependent bias. For Gaussian input noise, this bias limits the

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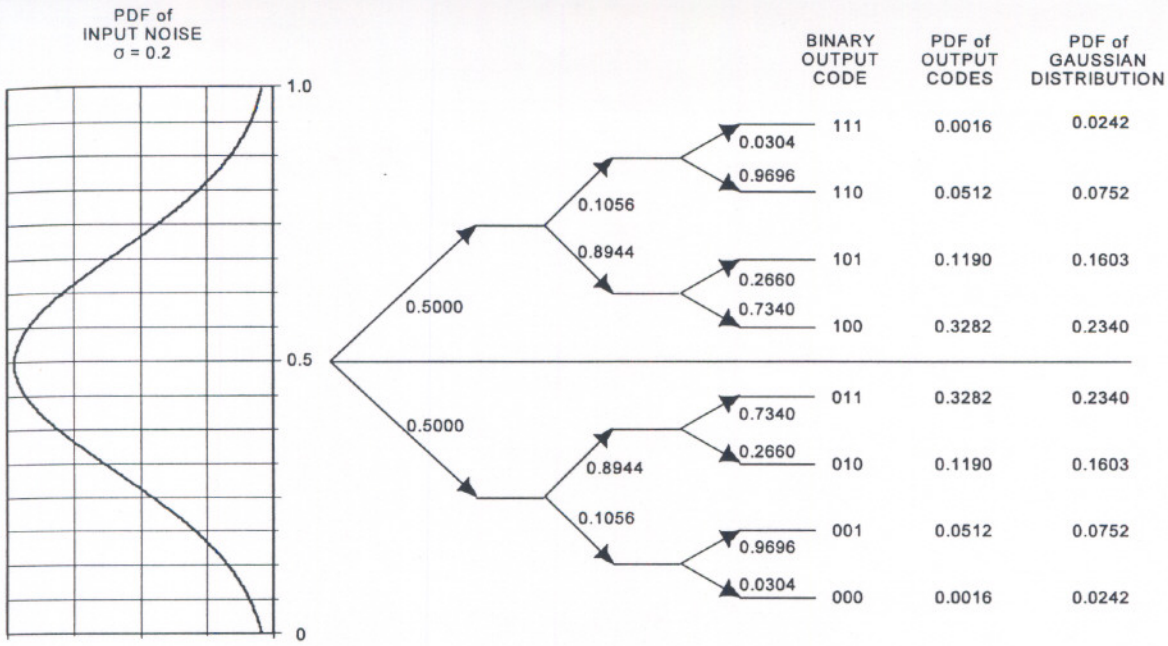

Fig. 2 Successive approximation decision tree, showing decision probabilities for Gaussian input noise with $\sigma = 0.2$.

improvement that averaging or digital filtering can achieve to about 5.7 effective bits. This is in contrast to pre-sampling noise or dither that can improve the effective bits without limit, given enough averaging [2]. For post-sampling noise having a uniform distribution, the improvement in effective bits is yet smaller. While the bias is sufficiently small to be insignificant for most applications, the variance reduction could be important.

II. SUCCESSIVE APPROXIMATION NOISE MODEL

Successive approximation A/D converters perform an N-step sequential, binary search to encode the signal value [4]. The process begins with a reference level set to the mid-level of the input signal range. At each step, the reference level is incremented or decremented by $2^{-(i+1)}$FSR, where i is the step index ranging from 1 to N, and FSR is the full scale range of the converter. The decision to increment or to decrement is determined by a comparator that decides if the input signal is greater than (output=1) or less than (output =0) the reference level at that step in the process. The result of this process is a decision tree as illustrated in fig. 2, that gives 2^N unique classification intervals, each of which is assigned a binary code. The i^{th} bit in the binary code is given by the i^{th} output of the comparator. (This process results in a mid-riser transfer characteristic in which the midrange occurs at a code-transition; offsetting the process by -0.5 LSB gives the more common mid-tread characteristic.) When noise is present at either of the comparator inputs, its decision at each step becomes a stochastic process. If the noise distribution is known, the probabilities of occurrence of each decision in the tree can be readily computed, and the probability of occurrence of each output code is the product of the conditional probabilities of the N decisions that led to that code. Fig. 2 illustrates the decision tree for a 3-bit A/D converter with the input signal level set at 0.5 (midscale) and with added Gaussian noise having zero mean (μ) and a standard deviation (σ) of 0.2. The probability of occurrence of each decision and of each output code is included in the figure for this particular signal and noise distribution; the PDF for ideally quantized pre-sampler Gaussian noise is also included. (The probabilities for the first and last codes do not include the significant probabilities of under and overflow that this large amount of noise causes.)

Having calculated the PDF, p(k), of output codes, the mean and variance of the output code distribution are given as

$$\mu = \sum_{k=0}^{2^N-1} \left[\frac{k}{2^N} \, p(k) \right]$$

and

$$\sigma^2 = \sum_{k=0}^{2^N-1} \left[\frac{k}{2^N} - \mu \right]^2 p(k)$$

where k is the codeword index, ranging from 0 to 2^N-1.

III. RESULTS

Fig. 3 shows the calculated PDF of output codes of an 8-bit successive approximation A/D converter with an input signal range of 0 to 1. The input noise is centered at 0.5 with σ of 0.05,

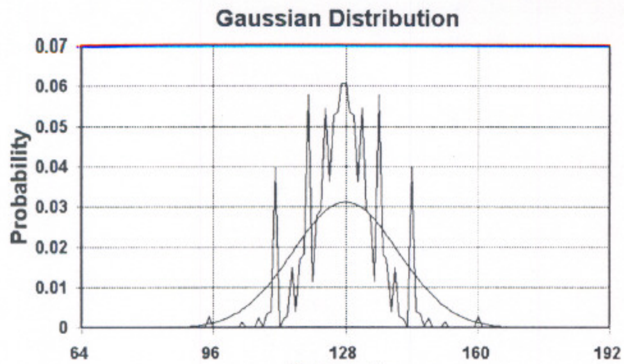

Fig. 3a PDF of output codes with Gaussian input noise ($\mu = 0.5$, $\sigma = 0.05$). For comparison, the PDF of the input noise is overlaid.

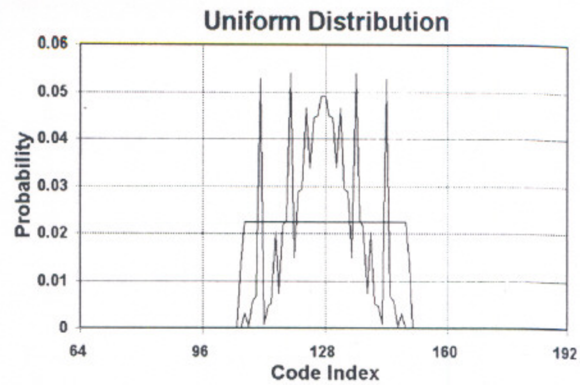

Fig. 3b PDF of output codes with uniform input noise ($\mu = 0.5$, $\sigma = 0.05$). For comparison, the PDF of the input noise is overlaid.

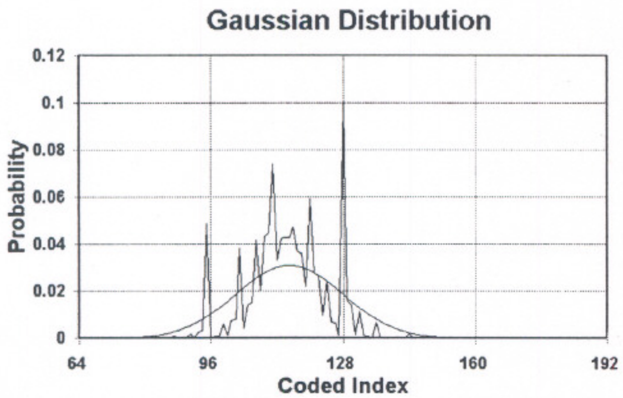

Fig. 4a PDF of output codes, with Gaussian input noise ($\mu = 0.45$, $\sigma = 0.05$). For comparison, the PDF of the input noise is overlaid.

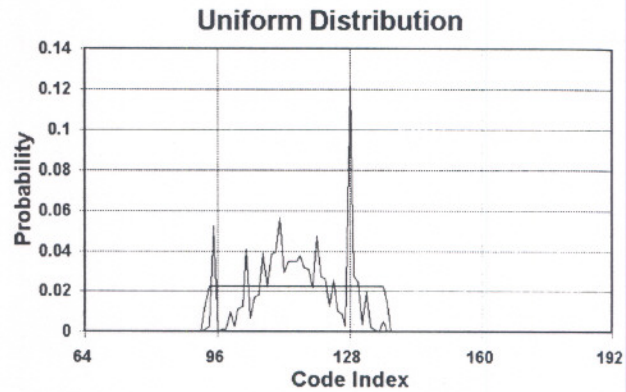

Fig. 4b PDF of output codes with uniform input noise ($\mu = 0.45$, $\sigma = 0.05$). For comparison, the PDF of the input noise is overlaid.

with both Gaussian and uniform distributions shown. In both plots, the PDF of the input noise is laid over the PDF of output codes, for comparison. Note that the output distributions are narrower than the input distributions. The tails of the input distributions are surpressed because each time an intermediate decision causes an excursion into the tails, the next decision almost certainly brings the excursion back toward the center. However, this same process tends to favor some moderate excursions, causing the sharp spikes that show up in these plots. Note that these processes produce an output PDF for the uniform distribution that is similar in appearance to that of the Gaussian distribution. In the case of Gaussian noise, the ratio of the variances of output codes to input noise is 0.43; for uniform noise, the ratio is 0.48. In both cases, the bias, i.e., the mean of the output distribution minus the mean of the input signal-plus-noise, is zero, since for these cases the distributions are symmetrical. When the input signal is offset from midscale, the output distribution becomes skewed as shown in fig. 4 (an offset of 0.05), and the bias is nonzero. In this example, the offset is -13 LSB's or -0.05078, and the bias is discussed below. The bias varies cyclically with offset as shown in fig. 5. Thus, as the input offset is swept through a range of 2^x, where $x = \mathrm{int}[\log_2(4\sigma)]$ and int[*] designates the integer part of [*], the bias goes through one cycle. From this, it is apparent that the cycle width is approximately proportional to σ, and is not dependent on the number of digitized bits, so long as σ is significantly greater than one LSB.

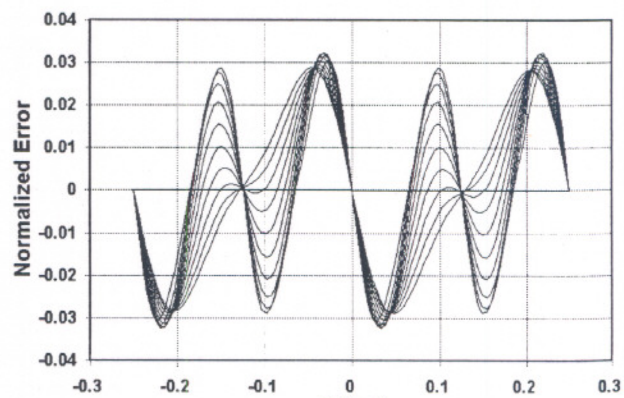

Fig. 5 Normalized bias vs. offset. The input noise is Gaussian with σ ranging from 0.03125 through 0.0625.

The functional curve describing a cycle however, varies according to the remainder part of $[\log_2(4\sigma)]$. In fig. 5, the bias, normalized by the standard deviation of the input noise, is plotted versus offset (ranging from -64 LSB's to +64 LSB's), for Gaussian input noise for 10 values of σ ranging from 0.03125 to 0.0625. The maximum and minimum values of the mean error (normalized by σ) vary between $\pm$ 0.028 and $\pm$ 0.032, while the rms value of the normalized bias varies between 0.017 and 0.021. Over this same range, the ratio of variances (output code divided by input noise) ranged from 0.42 to 0.56. For the specific case shown in fig. 4, with Gaussian noise the normalized bias is 0.025, and for the uniform noise case, it is 0.073. The ratios of variances for the two distributions are 0.53 and 0.61, respectively.

From the data in fig. 5, we can compute the maximum increase in effective bits that can be achieved for such a converter, by averaging its output data. The effective bits, E, of a converter is given by [5]:

$$E = \log_2\left(\frac{FSR}{rms\ noise \cdot \sqrt{12}}\right)$$

Assuming that the input noise is greater than one LSB so that quantization error can be neglected, and that FSR = 1, this expression can be rewritten in terms of the normalized bias as

$$E = -\log_2 \mu(\sigma) - \log_2\left(\sigma \cdot \sqrt{12}\right)$$

where $\mu(\sigma)$ is the rms value (over all offset values) of the normalized bias for input noise σ. The first term in this expression is the amount by which the effective bits can be improved with unlimited averaging. For Gaussian noise, this amount ranges from 5.6 to 5.9 bits depending on σ, i.e., on which of the traces in fig. 5 is appropriate. Averaged over all values of σ (all of the data in fig. 5), the improvement is 5.74 bits.

It is interesting to note that the application of external (pre-sampler) dither can be used to reduce the bias even further. The result is calculated by convolving one cycle of the bias produced by post-sampler noise (fig. 5) with the PDF of the external dither. For example, when the external dither has a standard deviation equal to that of the internal noise, the bias is reduced by 0.28 (an increase of 1.8 effective bits), and if the external noise is twice as large as the internal noise, the reduction factor is 0.012 (6.4 effective bits). However, to approach these levels of reduction would require much more averaging.

IV. CONCLUSION

This work suggests that architecture should not be ignored when considering the effects of internal noise on the performance of A/D converters. While only one architecture was studied in this paper, it is likely that other architectures will exhibit different noise transformations. For successive approximation converters, the most important findings are that the PDF of output codes is characterized by spikes showing marked preference for certain specific codes, and the variance of the output codes is only about half that of the input noise.

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