

High-Temperature Guarded Hot Plate Apparatus – Optimal Locations of Circular Heaters

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ABSTRACT

The National Bureau of Standards (now the National Institute of Standards and Technology (NIST)) pioneered the use of circular line-heat-sources in guarded hot plate (GHP) apparatus, the most common type of absolute apparatus for measurement of the thermal transmission properties of insulation. The prototype 305 mm GHP apparatus used one circular line-heat-source in the meter plate and one in the guard plate. The later one-meter GHP apparatus used a single circular line-heat-source in the meter plate and had two heaters in the guard plate. NIST is now completing the fabrication of a 500 mm GHP apparatus, designed to cover a much broader temperature range than that achievable by the previous designs, that utilizes multiple line-heat-sources in the meter plate, the guard plate, and the cold plates. The purposes of the present paper are to (1) describe strategies for locating these heaters in order to obtain the desired (uniform) temperature distribution on the plates, (2) provide analytical solutions for computing the circular heater locations for the various strategies, (3) provide tabulated values for the desired circular heater locations, (4) compare computed temperature variations for a specific circular heater layout as obtained using both an analytical solution and a finite element method (FEM), and (5) provide a representative temperature variation, obtained using FEM, for a heater layout that is more complex than circular line-heat-sources.

INTRODUCTION

Traditionally, most guarded hot plate (GHP) apparatus have used hot plates based on a laminated design in which an electrical heater is sandwiched between electrically insulating sheets which in turn are sandwiched between metal surface plates. At higher temperatures, or under vacuum conditions, such designs can lead to problems with warping, contact resistance between the layers, departures from isothermal conditions, or electrical leakage. For over four decades, the National Institute of Standards and Technology (NIST) in Gaithersburg has focused on

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circular guarded hot plates using solid thick metal plates with embedded line heat sources in the meter plate and in the guard plate, rather than a laminated sandwich design. The advantages of such an approach have been described previously, as have been the designs of the NIST GHP apparatus (see ASTM C 1043 [1] and the references cited in that standard).

The earlier NIST line-heat-source apparatus, intended for use over the temperature range of interest to the building industry, used a single circular heater in the meter plate and either one or two circular heaters in the guard plate. The “optimal” locations for those heaters have been well documented. At higher temperatures where the thermal resistance of the test specimens and the thermal conductivity of the meter and guard plates may both be lower than would be the case for apparatus operating near room temperature, a larger number of heaters may be required in order to obtain sufficiently isothermal plates. In 1996, ASTM C 1043 [1] was revised to give recommended heater locations for cases where up to six circular line heat sources could be used in the meter plate and also in the guard plate. The general analysis procedures used to obtain the recommended heater locations are summarized in ASTM C 1043, but the detailed analyses have not previously been published in the open literature. In the present paper, these analyses are presented and tables are given showing the recommended heater locations for up to ten circular heaters in the meter plate and up to ten circular heaters in the guard ring. While in these analyses each heater is assumed to have the same resistance per unit length and to carry the same electrical current, it would be quite straightforward to generalize the analyses to have different source strengths for different heaters.

In determining the radial locations of the circular heaters, it is only necessary to consider the radial temperature distribution in the meter plate and the guard ring. In practice, it is desirable to know the radial and axial temperature distribution in the plate. An analytical solution and some computed results are given that show how the temperature distribution depends upon the number of heaters, the thermal resistance of the test specimen(s), and the thickness and thermal conductivity of the hot plate.

The above analyses are based on a series of concentric circular heaters, with each heater covering an angle of 360° . Usually one does not wish to use a number of discrete heaters with each forming a complete circle, because of the large number of leads and the difficulties of dealing with heat generation in those leads and heat conduction along them. For the new 500 mm GHP apparatus being fabricated at NIST [2,3], a single heater is used for the meter plate, with the basic shape of that heater being obtained by morphing a set of concentric circular heaters into a single serpentine heater. This design uses the equivalent of five circular heaters by having a single heater that reverses direction eight times, with the lengths of the reversing loops being the same as the lengths of the segments of circular heaters that are omitted, so that, while there will be small local perturbations in the temperature distributions near the reversing loops, the large-scale temperature distribution will be very similar to that for five circular heaters.

A similar approach was taken to select the guard-plate heater layout. For the case of circular heaters, the mathematical analyses given in the paper allow determination of those locations where temperature sensors will measure a local temperature that is the same as the mean temperature of the meter plate or guard

ring. For the case of a continuous serpentine heater, those same locations are suitable for temperature measurements provided that the sensors are not placed near the reversing loops. Alternatively, analytical or numerical analyses can be used that provide the local temperature distribution as a function of both radius and angle.

For the new 500 mm NIST GHP apparatus, long-stem platinum resistance thermometers (SPRTs) are being used as the primary temperature sensors in the meter plate and in the two cold plates. These SPRTs are too large in diameter for the morphed meter-plate heater layout described above to be used. Accordingly, that heater layout was further modified to accommodate the SPRT. Finite-element analysis was used to refine this heater layout, resulting in the prediction of a very uniform surface temperature that very closely matches the temperature measured by the SPRT. Some results of the thermal modeling have been published previously [4], but the analytical solutions presented here were not included in that paper.

METER-PLATE LOCATIONS FOR CIRCULAR HEATERS

A simplified drawing of a line-heat-source guarded hot plate, with one circular heater in the meter plate at radius a , and one circular heater in the guard plate at radius c , is shown in Figure 1. The guarded hot plate is of thickness $2m$ and outer radius d . For this analysis, it is assumed that the guard gap, of infinitesimal width, is located at radius b . There are two identical insulation test specimens of thermal resistance R , one on each side of the hot plate. The two cold plates are isothermal at a temperature of $v = 0$. For the analyses in this paper, it is assumed that the boundaries $r = b$ and $r = d$ are adiabatic.

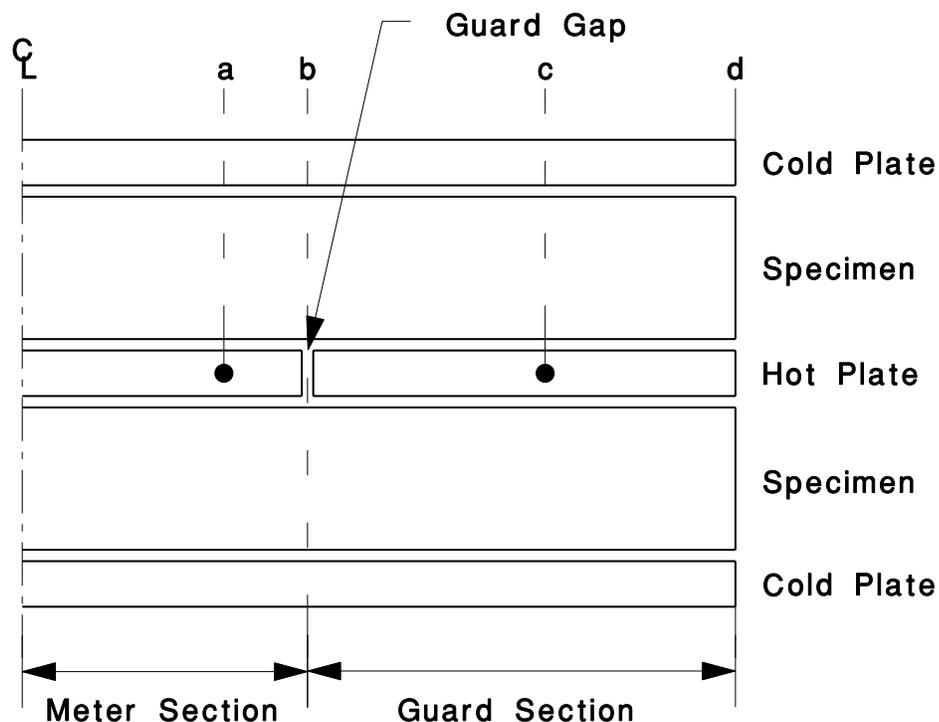


Figure 1. Line-heat-source GHP apparatus with one meter-plate heater and one guard-plate heater.

Single Line-Source Heater

In order to determine the “optimal” radius at which a heater should be located, it is only necessary to consider the radial temperature distribution in the meter plate. Axial temperature variations in the meter plate will not affect the optimal heater locations. Also, the surfaces of the meter plate, when it is properly designed, will be sufficiently isothermal that the heat flux from the meter plate into the specimens can be taken as constant everywhere over the plate surface, *i.e.*, the flux from each surface of the meter plate is simply equal to V/R , where V is the mean temperature of the surface of the meter plate. By symmetry, it is only necessary to consider one half of the meter plate, with its mid-plane being an adiabatic surface except at the location of the heat source, which is assumed to be infinitesimal in size in the radial and axial directions. Under these conditions, the radial temperature distribution in the meter plate is described by the differential equation [5]

$$\frac{d^2 v}{dr^2} + \frac{1}{r} \frac{dv}{dr} - 4E = 0 \quad , \quad (1)$$

where $v = v(r)$ is the meter-plate temperature relative to the cold-plate temperature, $E = V/(4m\lambda_p R)$, and λ_p is the thermal conductivity of the plate material. The solution of Eq. (1) is of the form $v(r) = A + Er^2 + C \ln(r/b)$ for radii outside the heater location, a , but the logarithmic term drops out for $r < a$. The values of A and C follow from the requirements that the radial temperature gradient is zero at $r = b$ and that the average temperature of the meter plate is V . The solution is

$$\frac{v(r) - V}{V} = \frac{b^2}{4m\lambda_p R} \left(\frac{a^2}{b^2} - \frac{3}{2} + \frac{r^2}{b^2} - 2 \ln \frac{r_{>}}{b} \right) = \frac{b^2}{4m\lambda_p R} \cdot F_1 \left(\frac{r}{b} \right) \quad , \quad (2)$$

where $r_{>}$ is the larger of r or a (*i.e.*, $r_{>} = a$ when $r < a$ and $r_{>} = r$ when $r > a$) and the subscript on F_1 indicates that it is the function for the case of one heater. The temperature at the guard gap, $r = b$, is

$$\frac{v(b) - V}{V} = \frac{b^2}{4m\lambda_p R} \left(\frac{a^2}{b^2} - \frac{1}{2} \right) \quad . \quad (3)$$

For the first two line-heat-source guarded hot plates designed and built at NIST, the single circular line heat source in the meter plate was located such that $a/b = \sqrt{2}/2 = 0.7071$. As seen by inspection of Eq. (3), the meter-plate temperature at the guard gap is then equal to the mean temperature of the meter plate, a feature that enables the temperature sensors to be located on the guard-plate side of the gap, rather than to have sensor leads cross the gap, thus increasing its thermal conductance. Note that this selection of the heater location does not depend upon the thermal conductivity or the thickness of either the meter plate or the specimens. The meter plate can be made more isothermal if the heater location is moved inward such that the mean temperature of the region inside the heater radius is made equal to the mean temperature of the region outside the heater, so that both regions have the same mean temperature as that of the entire meter plate.

Computation of the mean temperatures from Eq. (2) shows that the desired condition is achieved if

$$\frac{a}{b} = \exp \left[-\frac{3}{4} \left(1 - \frac{a^2}{b^2} \right) \right] = 0.6459... \quad (4)$$

Multiple Line-Source Heaters

For a meter plate with n circular heaters, each having the same power output per unit length, located at radii a_1, a_2, \dots, a_n , the radial temperature distribution is, by superposition of equations of the form of Eq. (2), with appropriate weighting for the circumference of each heater, is described by

$$\frac{v(r) - V}{V} = \frac{b^2}{4m\lambda_p R} \sum_{i=1}^n \frac{a_i}{p} \left(\frac{a_i^2}{b^2} - \frac{3}{2} + \frac{r^2}{b^2} - 2 \ln \frac{r_{i>}}{b} \right) = \frac{b^2}{4m\lambda_p R} \cdot F_n \left(\frac{r}{b} \right), \quad (5)$$

where V is the mean temperature of the surface of the meter plate due to all of the heaters, $r_{i>}$ is the larger of r or a_i , and $p = \sum_{i=1}^n a_i$ is the sum of the n heater radii. The temperature at the guard gap is given by

$$\frac{v(b) - V}{V} = \frac{b^2}{4m\lambda_p R} \sum_{i=1}^n \frac{a_i}{p} \left(\frac{a_i^2}{b^2} - \frac{1}{2} \right). \quad (6)$$

For multiple heaters, there is no unique solution giving the heater locations that will result in the temperature at the gap being equal to the mean temperature of the meter plate. In order to obtain a satisfactory set of heater locations, the requirement is introduced that half of the power input to each heater flows radially inward and half flows radially outward. With this constraint, and if the heaters have the same power output per unit length, the temperature at the gap will be equal to the mean temperature of the meter plate when the heaters are located at locations defined by

$$\frac{a_i}{b} = \frac{i}{\sqrt{n^2 + n}}, \text{ for } i = 1, 2, \dots, n. \quad (7)$$

These values can be substituted into Eq. (5) to compute the temperature distribution. For this particular set of heater locations, it is not necessary to know Eq. (5) in order to compute those locations. If one simply requires that half of the power input to each heater flows inward and half flows outward, and further requires that the plate area associated with, or serviced by, each heater is proportional to the circumference of that heater, Eq. (7) follows without any need to know the actual radial temperature distribution. However, in order to have a quantitative knowledge as to how isothermal the meter plate is, it is desirable to know that temperature distribution. The radial temperature variations in the meter plate can be made smaller than they are for the case just discussed by requiring that each Aregion@ of the meter plate has the same mean temperature. Letting region i lie between a_i and a_{i+1} , the mean temperature in that region is

$$\bar{v}_j = \frac{2}{a_{i+1}^2 - a_i^2} \int_{a_i}^{a_{i+1}} r v(r) dr, \quad (8)$$

where $v(r)$ is given by Eq. (5), $a_0 = 0$, and $a_{n+1} = b$, the outer radius of the meter plate. Carrying out the integration for the $n + 1$ regions results in a series of simultaneous equations that can be solved to obtain the heater locations for which the mean temperature in each region is equal to V . Those results and a corresponding computer program are available [6,7]. The output from that program is given in Table 1, for $n = 1$ to 10. Radial temperature distributions for up to four circular heaters are plotted in ASTM C 1043 [1] (See Figure A2.2).

Radial and Axial Temperature Distribution

When radial and axial temperature variations are considered, the temperature $v(r, z)$, relative to the cold plate temperature, must satisfy $\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2} = 0$.

The boundary conditions for the temperature v_i due to the i -th heater were taken as:

$$\begin{aligned} r = b & & 0 \leq z \leq m & & \frac{\partial v_i}{\partial r} = 0 \\ 0 \leq r \leq b & & z = m & & \frac{\partial v_i}{\partial z} = -\frac{V_i}{\lambda_p R} \\ 0 \leq r \leq b & & z = 0 & & \frac{\partial v_i}{\partial z} = -\frac{b^2}{2a_i} \frac{V_i}{\lambda_p R} \delta(r - a_i), \end{aligned}$$

Table 1. Radial locations for line heat sources in the meter plate, selected such that each region of the meter plate has the same mean temperature.

n	a_1/b	a_2/b	a_3/b	a_4/b	a_5/b	a_6/b	a_7/b	a_8/b	a_9/b	a_{10}/b
1	.6459									
2	.3535	.7909								
3	.2464	.5480	.8529							
4	.1888	.4203	.6530	.8864						
5	.1531	.3408	.5296	.7183	.9074					
6	.1288	.2867	.4454	.6042	.7629	.9219				
7	.1111	.2474	.3844	.5213	.6583	.7953	.9325			
8	.0977	.2175	.3380	.4585	.5790	.6994	.8199	.9405		
9	.0872	.1941	.3017	.4092	.5167	.6242	.7317	.8392	.9468	
10	.0788	.1753	.2724	.3694	.4665	.5636	.6606	.7577	.8548	.9519

where now V_i is the average temperature of the hot plate, due to the i -th heater, at the side $z = m$ that is in contact with the specimen and δ is the Dirac delta function. The solution to this set of boundary conditions is easily extended to that of multiple heaters by superposition over the n heaters. Assuming equal power per unit length for all of the heaters, the mean temperature at the surface of the plate, due to the i -th heater is $V_i = (a_i / p)V$ where V is the mean temperature at the surface due to all of the heaters. The temperature due to all n heaters is given by

$$\frac{v(r, z) - V}{V} = \frac{1}{\lambda_p R} \left\{ m - z + \frac{b}{p} \sum_{k=1}^{\infty} \left[\frac{\cosh \beta_k (m - z) / b}{\sinh \beta_k m / b} \frac{J_0(\beta_k r / b)}{\beta_k J_0^2(\beta_k)} \sum_{i=1}^n a_i J_0(\beta_k a_i / b) \right] \right\}, \quad (9)$$

where β_n are the roots of $J_1(\beta_k) = 0$, J_0 and J_1 being the Bessel functions of the first kind, of order zero and one, respectively. A computer program corresponding to this analysis has been written and extensive calculations have been carried out [6,7]. Figure 2 shows the relative temperature variations at different positions inside a 200 mm diameter ($b = 100$ mm) meter plate with a half thickness of 8 mm, for the case where there are five circular heaters located at the mid-plane and at the radii corresponding to the case for $n = 5$ in Table 1. The uppermost curve shows the radial temperature variation in the plane that is 2 mm from the mid-plane of the plate. The lowest curve (8 mm) shows the temperature variation at the plate surface that contacts the specimen. As would be expected, there is a systematic decrease in temperature and in the temperature variations as the distance from the mid-plane increases. These computations were carried out for the case of a nickel hot plate at 900 K, with a thermal conductivity of 70 W/(m·K) and test specimens having a thermal resistance of 0.1 m²·K/W, so that $\lambda_p R = 7$. The temperature at the surface of the meter plate is seen to oscillate around its mean value over a range of less than ± 0.015 %, except for a dip of 0.035 % at the center of the plate.

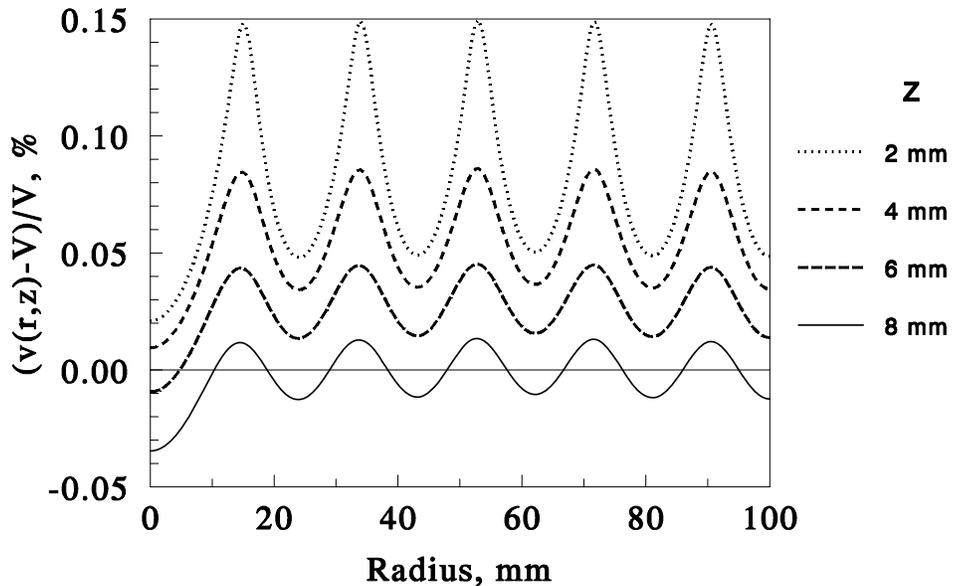


Figure 2. Radial temperature profile in the meter plate at different planes.

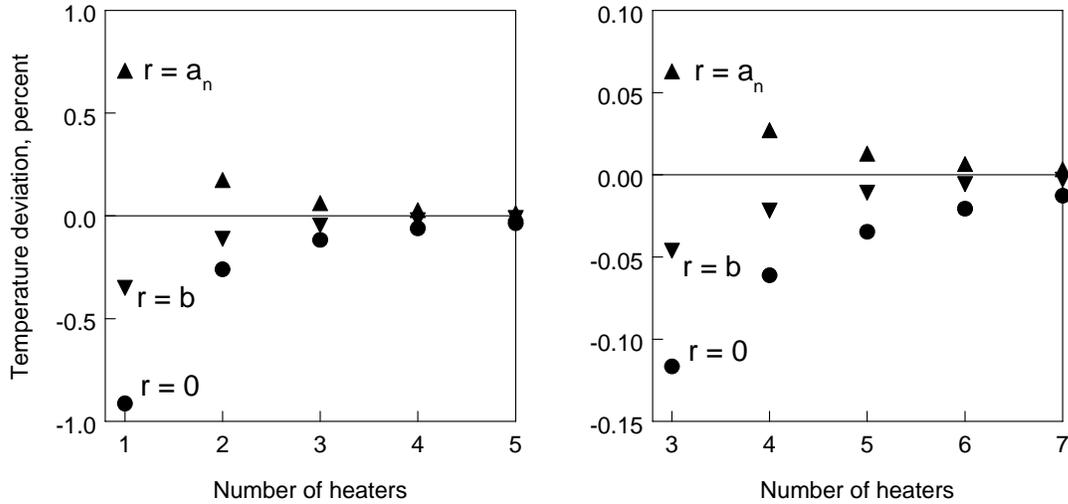


Figure 3. Extremes in surface temperature as functions of the number of circular heaters.

In order to show the effect of the number of circular heaters on the extremes in the surface temperatures, the values of $[v(r,m) - V]/V$, in percent, are shown in Figure 3 for the cases of 1 through 7 heaters for the same conditions as described for the computations of the curves in Figure 2. In Figure 3, the solid triangles represent the surface temperature at the radius of the outermost heater, which, as was shown in Figure 2, is essentially the same as the temperature at the radius of each of the heaters. The solid inverted triangles represent the surface temperature at the outer radius of the meter plate, $r = b$. The solid circles correspond to the surface temperature at the center of the meter plate, $r = 0$. The left-hand graph shows these computed values for $n = 1$ through 5, while the right-hand graph, with a very different vertical scale, shows computed values for $n = 3$ through 7. The points plotted for $n = 5$, correspond to the solid curve, labeled 8 mm, in Figure 2. For the conditions described in the previous paragraph, it is seen that the use of a single heater results in the surface temperature at the radius where that heater is located being 0.71 % hotter than the mean surface temperature, the surface temperature at the guard gap being 0.35 % colder than the mean surface temperature, and the surface temperature at the center of the meter plate being 0.91 % below the mean. Increasing the number of heaters to four would reduce temperature variations to well below 0.1 %. However, for the new GHP being built at NIST, it is preferable to use a heater design derived from an odd number of heaters (in order to allow unimpeded access for the long-stem platinum resistance thermometer) and therefore that design was derived from one using five circular heaters.

GUARD-PLATE LOCATIONS FOR CIRCULAR HEATERS

Consider a coplanar, guard plate of internal radius b and external radius d with a single circular line heat source located at a radius c . The radial temperature distribution in the guard plate, relative to the mean temperature V , is, assuming that Eq. (1) is valid and that there are adiabatic boundaries at both $r = b$ and $r = d$,

$$\frac{v(r)-V}{V} = \frac{b^2}{4m\lambda_p R} \cdot G_1\left(\frac{r}{b}\right) , \quad (10)$$

where

$$G_1\left(\frac{r}{b}\right) = C + \frac{r^2}{b^2} + 2 \ln \frac{r_{<}}{b} + \frac{2d^2}{b^2} \ln \frac{r_{>}}{b} , \quad (11)$$

$$C = -\frac{1}{2} \left(3 - \frac{2c^2}{b^2} + \frac{3d^2}{b^2} + \frac{4d^2}{b^2} \frac{\ln b/c + (d^2/b^2) \ln c/d}{d^2/b^2 - 1} \right) , \quad (12)$$

and $r_{<}$ ($r_{>}$) is the lesser (greater) of r or c , with the last term in Eq. (11) equal to zero for $r < c$. For a guard plate with multiple circular heaters, the radial temperature distribution follows by summing over all of the heaters. For a guard plate with n heaters of equal heat output per unit length located so that half of the input to each heater flows inward and half outward, the temperature at the guard gap, $r = b$, will be equal to the mean temperature of the guard plate if the heaters are located at radii c_1, c_2, \dots, c_n given by

$$c_i/b = (c_1/b) \left[1 + (i-1)(1 - b^2/c_1^2) \right] , \quad (13)$$

where c_1/b is the real positive root of

$$(n^2 + n)(c_1^4/b^4) - (d^2/b^2 + 2n^2 - 1)(c_1^2/b^2) + (n^2 - n) = 0 . \quad (14)$$

Since Eq. (14) is quadratic in c_1^2/b^2 , the root is easily obtained. An analysis also has been made and a computer program written [6,7] to obtain the guard heater locations that result in all of the regions in the guard plate being at the same average temperature as the entire guard plate. Table 2 gives the corresponding heater locations for the case where the diameter of the guard is 2.5 times the diameter of the meter plate, which is the value that was selected for the 500 mm GHP apparatus being completed at NIST.

DESIGN OF METER-PLATE HEATERS FOR NIST 500 mm GUARDED-HOT-PLATE APPARATUS

The general approach taken in the design of the hot plate heaters for the 500 mm NIST GHP has been described previously [4,6], along with the results of finite element analyses carried out to examine the uniformity of temperatures across the meter and guard plates, and the design approach is also summarized in the last three paragraphs of the introduction to the present paper. The fabrication of the hot plate is described in some detail in another paper presented at this conference [3].

The analysis described above (Eq. (9)) for computation of the radial and axial temperature distribution in the meter plate was used to provide guidance in arriving at the decision to use a meter plate heater corresponding to five circular heaters at radii corresponding to those listed in Table 1 for $n = 5$. The same heater layout was examined using a finite element analysis to compute the radial temperature distribution and compare the results with those shown in Figure 2 and, as reported earlier [4], the agreement was excellent. As described in the Introduction section,

Table 2. Radial locations for line heat sources in the guard plate, which has 2.5 times the diameter of the meter plate, selected such that each region of the guard plate has the same mean temperature.

n	c_1/b	c_2/b	c_3/b	c_4/b	c_5/b	c_6/b	c_7/b	c_8/b	c_9/b	c_{10}/b
1	1.833									
2	1.398	2.140								
3	1.262	1.757	2.256							
4	1.194	1.567	1.941	2.316						
5	1.154	1.453	1.753	2.052	2.352					
6	1.128	1.377	1.627	1.877	2.126	2.376				
7	1.109	1.323	1.537	1.751	1.966	2.180	2.394			
8	1.096	1.283	1.470	1.657	1.845	2.032	2.220	2.407		
9	1.085	1.251	1.418	1.584	1.751	1.917	2.084	2.251	2.417	
10	1.076	1.226	1.376	1.526	1.676	1.826	1.976	2.126	2.275	2.426



the layout of five circular heaters was transformed into a single heater, as shown in the inset, that reverses direction eight times, with the lengths of the reversing loops being the same as the lengths of the segments that are omitted, so that, while there will be small local perturbations in the temperature distributions near the reversing loops, the large-scale temperature distribution will be very similar to that for five circular heaters.

The heater layout just described would be very good for use with small temperature sensors such as thermocouples but, as mentioned previously, does not provide sufficient room for the long-stem platinum resistance thermometers (SPRT). Accordingly, the heater layout was further modified to obtain the shape shown in the left-hand drawing in Figure 3. First, the radial locations of circular heaters (for $n = 5$ in Table 1) were modified slightly so that the temperature read by the SPRT would correspond to the mean temperature of the meter plate. Finite-element analyses were then used to refine this heater layout, resulting in the prediction of a very uniform surface temperature, as depicted in the right-hand drawing in Figure 4. These computations were carried out for the case of a meter plate with a thermal conductivity of $70 \text{ W}/(\text{m}\cdot\text{K})$ and test specimens having a thermal resistance of $0.125 \text{ m}^2\cdot\text{K}/\text{W}$. The power to the meter plate was set so that the average surface temperature of that plate was 20.000 K above the cold plate temperature. For these computations, the temperature over the volume occupied by the sensitive portion of the SPRT was 12 mK higher than the mean surface temperature, corresponding to 0.06% of the temperature difference across the specimen.

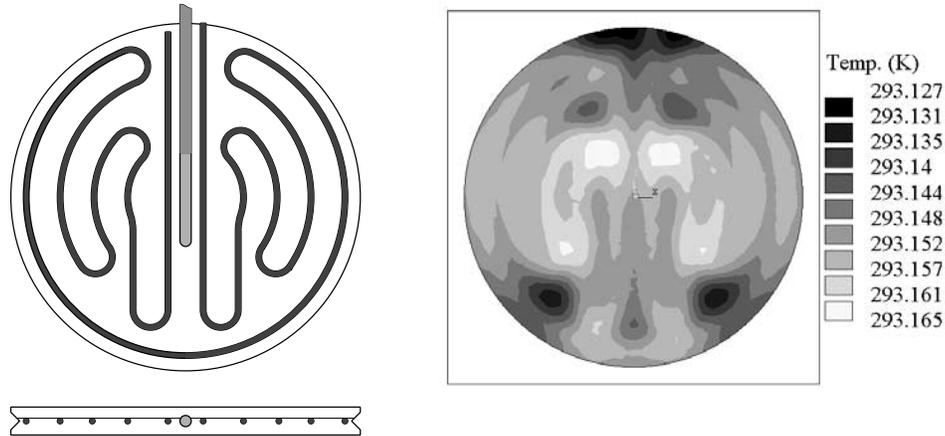


Figure 4. Meter plate heater layout for the NIST 500 mm GHP apparatus, and the computed temperature variation over the surface of the plate

The heater layout for the guard ring consisted also of five circular heaters, transformed into a single heater with switchbacks analogous to those shown in the inset to the previous paragraph. A surface temperature profile for the guard ring has been published previously [4].

CONCLUSIONS

In the 1970s and 1980s, NIST designed and built two circular GHP apparatus that used a guarded hot plate fabricated from solid, rather thick, metal plates with embedded circular line heat sources in the meter plate and in the guard plate. NIST is now completing the fabrication of a 500 mm GHP apparatus of similar conceptual design, but with more complex heater layouts to promote the guarded hot plate being quite isothermal even at high temperatures with specimens having a rather low thermal resistance. This paper has addressed the selection of optimal locations for multiple circular heaters in a meter plate and provided formulae and tables for such locations. Analytical and finite-element analyses have shown that the meter plate and the guard plate can be made very isothermal using circular heaters at these optimal locations. It also has been shown that the multiple-heater designs can be modified into a single continuous heater layout, suitable for use with small temperature sensors, and that these designs also provided excellent isothermal conditions on the plates. Finally, it has been shown how meter-plate heater designs can be further modified to allow the use of large temperature sensors, such as standard, long-stem platinum resistance thermometers, while still providing very isothermal plates.

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