# Autonomous Migration with Admission Control for Mobiles Affected by Access Network Failures 

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#### Abstract

The loss of a set of wireless network access resources causes every mobile user that is connected through the failed resources to perform an unanticipated hard handover, resulting in disrupted connections. The users connected to the failed resources can be expected to migrate en masse to other access points (APs). Those other APs, if already heavily loaded, may restrict the number of incoming users using call admission control (CAC) algorithms. Rejected users will experience loss of connectivity for longer periods of time as they are forced to try to connect to other available APs. This paper quantifies the effect of limited capacity in target APs when multiple displaced users attempt to connect to them. We first derive expressions for handover success probability and the mean number of displaced users that are successfully admitted on their first attempt, given that displaced users randomly choose an AP from a set of available ones. This leads to a simple probabilistic scheme that displaced users can use in a decentralized manner to minimize crowding effects at target APs. We also determine a performance bound by developing an optimal reassignment scheme using linear programming, and compare both to an algorithm that chooses an AP based on received signal strength (RSS).


Index Terms-failure recovery, integer linear programs, call admission control (CAC)

## I. Introduction

When a network access point (AP) fails, all of the mobile users that were connected to the failed AP experience an immediate loss of service that forces them to try to connect to an alternate access point as quickly as possible in order to minimize the length of the service interruption. When the AP supports public safety communications, the loss of connectivity can pose serious threats to the safety of both emergency personnel and the civilians they are trying to assist.
Several studies have examined mechanisms to mitigate AP failures, such as backup link multiplexing using multiple wireless interfaces at the mobile node (MN) [2], or my changing the power level of surviving APs in the network to include the maximum number of affected users in their coverage area [3]. If the APs that are on the receiving end of the migration implement some form of call admission control (CAC), they will not necessarily admit every displaced user that attempts to make a connection. This will result in a subset of the displaced users being turned away and forced to make secondary connection attempts with alternative APs. In this paper, we investigate the effect of CAC on displaced users by considering APs with finite resources that have to absorb a certain number of simultaneously arriving requests for connection from displaced users and developing

[^0]a simple probabilistic scheme that displaced users can use in a distributed fashion to choose the target AP they will first try to reach. We assume that the displaced users have knowledge of the target APs' capacities; this information can be obtained in advance by using for example the Information Service (IS) that is part of the IEEE 802.21 Media Independent Handover (MIH) standard [1]. We also obtain a performance bound by developing an integer linear program (ILP) that optimizes handover processing delay and can be used by a network controller in a centralized fashion to assign displaced users to alternate APs.

The rest of this paper is organized as follows. In Section II, we produce a theoretical model for the effects of CAC at the target APs and obtain expressions for the probability that a displaced user is accepted by its first choice of AP as well as the expected number of users that are not rejected due to CAC. We also develop an integer linear program (ILP) that seeks to maximize the number of users that are accepted by their first choice of AP. In Section III we evaluate the performance of the ILP and the probabilistic approach by comparing them to an algorithm in which each displaced mobile node attempts to connect to the AP with the highest received signal strength (RSS). We summarize our results in Section IV.

## II. Theoretical Models

We assume that there is a single failure event that causes the loss of service to at least one AP. If more than one AP is lost, we assume that all of them are lost simultaneously. $\mathcal{A}$ is the set of $N_{a}$ APs, $\left\{A_{1}, A_{2}, \ldots, A_{N_{a}}\right\}$, that are contained within the coverage area of the lost APs and that each contain at least one displaced user. In most practical situations, a displaced user can connect to only a subset of the networks in $\mathcal{A}$. For the optimization that we are considering, we need to identify groups of displaced users by the APs that they can reach. For instance, if there are two APs available to the set of displaced users and their coverage areas overlap, then the displaced users in the overlap area can migrate to either AP, while those outside the overlap have only one AP available to which they could move.

## A. Models for Migration with CAC Awareness

We compute two metrics: the probability that a randomly chosen displaced user is able to connect to the first AP that it chooses; and the expected number of displaced users that are able to connect to an AP on the first attempt. We want to maximize both of these metrics; users that are not able to connect to an AP on the first try must attempt to connect to an alternative AP, which increases the duration of interruption of
service. In order to produce simple expressions, we consider the simple case where two APs are available to a population of $N_{u}$ displaced users. We assume that each user chooses AP $A_{i}$ with probability $p_{i}$. The number of displaced users that APs $A_{1}$ and $A_{2}$ can accept are $m_{1}$ and $m_{2}$, respectively.

1) Success Probability: For a randomly chosen displaced user, $\pi_{S_{i}}$, the probability that the user is successfully accepted by $A_{i}$, can be expressed as follows by conditioning on the number of displaced users that attempt to migrate to $A_{i}$ :

$$
\begin{equation*}
\pi_{S_{i}}=\sum_{k=0}^{N_{u}} \pi_{S_{i} \mid k} \alpha_{k} \tag{1}
\end{equation*}
$$

where $\pi_{S_{i} \mid k}$ is the probability that a randomly chosen user is accepted given $k$ displaced users that attempt to migrate to $A_{i}$, and $\alpha_{k}$ is the probability that $k$ displaced users that attempt to migrate to $A_{i}$. If $k \leq m_{i}$, then all the migrating displaced users are accepted by $A_{i}$, and $\pi_{S_{i} \mid k}=1$. If $k>m_{i}$, then a user will be accepted by $A_{i}$ if it is one of the first $m_{i}$ out of $k$ displaced users to be admitted, so $\pi_{S_{i} \mid k}=m_{i} / k$. Since we assume that each user picks an AP independently, the $\alpha_{k}$ terms follow a binomial distribution, and we have

$$
\begin{align*}
\pi_{S_{i}}= & \sum_{k=0}^{m_{i}}\binom{N_{u}}{k} p^{k}(1-p)^{N_{u}-k} \\
& +\sum_{k=m_{i}+1}^{N_{u}} \frac{m_{i}}{k}\binom{N_{u}}{k} p^{k}(1-p)^{N_{u}-k} \\
= & (1-p)^{N_{u}}\left[\sum_{k=0}^{N_{u}}\binom{N_{u}}{k} \rho^{k}-\sum_{k=m_{i}+1}^{N_{u}}\binom{N_{u}}{k} \rho^{k}\right. \\
& \left.+\sum_{k=m_{i}+1}^{N_{u}} \frac{m_{i}}{k}\binom{N_{u}}{k} \rho^{k}\right] \\
= & 1-(1-p)^{N_{u}} \sum_{k=m_{i}+1}^{N_{u}} \frac{k-m_{i}}{k}\binom{N_{u}}{k} \rho^{k} \tag{2}
\end{align*}
$$

where $\rho=p /(1-p)$ and $p=p_{1}$.
In Fig. 1, we plot $\pi_{S_{i}}$ versus $p$ for various values of $N_{u}$ and $m_{i}$. As $p$ increases, $\pi_{S_{i}}$ decreases, until $\lim _{p \rightarrow 1} \pi_{S_{i}}=m_{i} / N_{u}$. Of course, if we were to have a situation where $m_{i} \geq N_{u}$ then $\pi_{S_{i}}=1 \forall p \in[0,1]$. Decreasing $m_{i}$ causes a reduction in $\pi_{S_{i}}$, although the size of the reduction decreases for a given value of $p$ as $N_{u}$ increases.
2) Expected Success Rate: We can find the expected number of displaced users that are accepted by an AP on their first attempt to connect, using the same approach that we used to simplify our expression for $\pi_{S_{i}}$. We find that

$$
\begin{align*}
\mu_{S_{i}}= & \sum_{k=0}^{m_{i}} k\binom{N_{u}}{k} p_{i}^{k}\left(1-p_{i}\right)^{N_{u}-k} \\
& +\sum_{k=m_{i}+1}^{N_{u}} m_{i}\binom{N_{u}}{k} p_{i}^{k}\left(1-p_{i}\right)^{N_{u}-k} \\
= & N_{u} p_{i}+\sum_{k=m_{i}+1}^{N_{u}}\left(m_{i}-k\right)\binom{N_{u}}{k} p_{i}^{k}\left(1-p_{i}\right)^{N_{u}-k} \tag{3}
\end{align*}
$$



Fig. 1. Plot of $\pi_{S_{i}}$, success probability for an arbitrary displaced mobile node, for various values of $m_{i}$, the available number of places at AP $A_{i}$, and $N_{u}$, the size of the population of displaced mobiles.


Fig. 2. Plot of $\mu_{S}$, the expected number of handovers that succeed on the first attempt, versus $p$ and $m_{1}$ for $N_{u}=100$ users and (a) $m_{2}=20$ users and (b) $m_{2}=80$ users.
where $p_{i}$ is the probability that a randomly chosen displaced user attempts to connect to $A_{i}$. From this expression, we can determine the expected number of displaced users that are accepted by an access point on the first try for the two AP case. In this case we let $p$ be the probability that a displaced mobile attempts to connect to AP $A_{1}$ (i.e. $p=p_{1}$ ) and we let $1-p$ be the probability that a displaced mobile attempts to connect to AP $A_{2}$. The total number of displaced users that successfully handover to the first AP they try, is $\mu_{S}=\mu_{S_{1}}+\mu_{S_{2}}$.

We show a contour plot of $\mu_{S}$ versus $p$ and $m_{1}$ for two values of $m_{2}$ in Fig. 2, where we let $N_{u}=100$ displaced users. Examining the shape of the surface reveals that the values of $p$ (and $1-p$ ) that maximize $\mu_{S}$ for a given pair of AP capacities $m_{1}$ and $m_{2}$ are well approximated by

$$
\begin{equation*}
p^{*}=\frac{m_{1}}{m_{1}+m_{2}}, \quad 1-p^{*}=\frac{m_{2}}{m_{1}+m_{2}} \tag{4}
\end{equation*}
$$

In Fig. 3, we plot the percentage deviation from the maximum value of $\mu_{S}\left(m_{1}, m_{2}\right)$ when we use $p^{*}$. The plot shows that the maximum deviation is less than $1 \%$, and that this occurs only for cases where one AP has much more spare capacity than


Fig. 3. Mesh plot of the percentage error in $\max \left(\mu_{S}\right)$ when $\left(p_{1}^{*}, p_{2}^{*}\right)$ are used versus $m_{1}$ and $m_{2}$ for $N_{u}=100$.
the other. For many other values of $m_{1}$ and $m_{2}$, the deviation from the optimal value is extremely small, indicating that the heuristic in Eq. (4) is reasonable. We can also infer that for the more general case where $N_{a}$ APs are visible, each user should choose the $i$ th AP with probability

$$
\begin{equation*}
p_{i}^{*}=\frac{m_{i}}{\sum_{j=1}^{N_{a}} m_{j}} \tag{5}
\end{equation*}
$$

## B. Integer Linear Program

The ILP seeks to maximize $\mu_{S}$. Letting $x_{i}$ be the number of displaced users that choose AP $A_{i}$, we need to maximize $\sum_{i} x_{i}$. We must constrain the $x_{i}$ 's so that they do not exceed the number of displaced users. To do this, we partition the set of displaced users $\mathcal{U}$ into subsets based on the sets of APs that they can reach. We use the power set of $\mathcal{A}, \mathcal{P}(\mathcal{A})$, which is the set of all subsets of $\mathcal{A}$, including the empty set. The cardinality of $\mathcal{P}(\mathcal{A})$ is $N_{\mathcal{P}}=2^{N_{a}}$. We order the elements of $\mathcal{P}(\mathcal{A})$ using the set of $N_{\mathcal{P}}$ indicator vectors $\mathcal{I}=\{\mathbf{i}(\mathcal{S})\}_{\mathcal{S} \in \mathcal{P}(\mathcal{A})}$, where each element of the set is a length $-N_{a}$ Boolean vector $\mathbf{i}(\mathcal{S})=$ $\left[i_{N_{a}}(\mathcal{S}), i_{N_{a}-1}(\mathcal{S}), \ldots, i_{1}(\mathcal{S})\right]$, where

$$
i_{k}(\mathcal{S})= \begin{cases}0, & A_{k} \nsubseteq \mathcal{S}  \tag{6}\\ 1, & A_{k} \subseteq \mathcal{S}\end{cases}
$$

and $\mathcal{S} \subseteq \mathcal{A}$. For example, if $\mathcal{A}=\left\{A_{1}, A_{2}\right\}$, then $\mathcal{I}$ contains the following four indicator vectors:

$$
\begin{aligned}
\mathbf{i}_{0}(\emptyset) & =[0,0] & \mathbf{i}_{1}\left(\left\{A_{2}\right\}\right) & =[1,0] \\
\mathbf{i}_{2}\left(\left\{A_{1}\right\}\right) & =[0,1] & \mathbf{i}_{3}(\mathcal{A}) & =[1,1] .
\end{aligned}
$$

We write the power set as $\mathcal{P}(\mathcal{A})=\left\{\mathcal{G}_{j}\right\}_{j=0}^{N_{\mathcal{P}}-1}$, where $\mathcal{G}_{j}$ is the $j$ th subset of $\mathcal{A}$, and the elements of $\mathcal{P}(\mathcal{A})$ are ordered such that $j=\sum_{k=1}^{N_{a}} i_{k}\left(\mathcal{G}_{j}\right) 2^{N_{a}-k}$ is the decimal representation of the Boolean indicator vector $\mathbf{i}\left(\mathcal{G}_{j}\right)$. In our previous example, $\mathcal{G}_{0}=\emptyset, \mathcal{G}_{1}=\left\{A_{2}\right\}, \mathcal{G}_{2}=\left\{A_{1}\right\}$, and $\mathcal{G}_{3}=\mathcal{A}$. Let $\mathcal{U}_{j}$ be the set of displaced users such that each user lies within the coverage area of at least one of the APs that are elements of the set $\mathcal{G}_{j}$, and lies outside the coverage areas of all APs networks


Fig. 4. Illustration of the partitioning of the set of displaced users into non-overlapping subsets, for the case where $N_{a}=3$.
that are not elements of $\mathcal{G}_{j}$. More formally, we represent the space occupied by the displaced users and the network APs as a plane $\Omega$, where $p_{u} \in \Omega$ is the point occupied by displaced user $u$, and we define the set of all points in the plane that lie within the coverage area of AP $A_{i}$ to be $\mathcal{C}_{i}, i=1,2, \ldots, N_{a}$. Then $\mathcal{U}_{j}$ is the set of all displaced users $u$, each located at a corresponding point $p_{u}$ in the plane $\Omega$, such that

$$
\begin{equation*}
p_{u} \in\left(\Omega \cap \bigcap_{k: A_{k} \in \mathcal{G}_{j}} \mathcal{C}_{k}\right) \backslash \bigcup_{k: A_{k} \notin \mathcal{G}_{j}} \mathcal{C}_{k} \tag{7}
\end{equation*}
$$

We can define a set of indicator functions to identify APs that are visible to a group of displaced users $\mathcal{U}_{j}$ as follows:

$$
f_{k}\left(\mathcal{U}_{j}\right)= \begin{cases}0, & \mathcal{U}_{j} \subset \mathcal{C}_{k}  \tag{8}\\ 1, & \mathcal{U}_{j} \nsubseteq \mathcal{C}_{k}\end{cases}
$$

where $\mathcal{U}_{j} \subset \mathcal{C}_{k}$ means that all the displaced users belonging to $\mathcal{U}_{j}$ are within AP $A_{k}$ 's coverage area $\mathcal{C}_{k}$. For example, $\mathcal{U}_{0}$ is the set of displaced users that lie outside the coverage areas of all the APs in $\mathcal{A}$. For the case where $N_{a}=3$, we show the regions that contain the sets of displaced users $\left\{\mathcal{U}_{\ell}\right\}_{\ell=0}^{7}$ in Fig. 4. As can be inferred from the figure, it is simple to prove that $\left\{\mathcal{U}_{i}\right\}_{i=0}^{N_{\mathcal{P}}-1}$ is a covering of $\mathcal{U}$, i.e. $\bigcup_{i=0}^{N_{\mathcal{D}}-1} \mathcal{U}_{i}=\mathcal{U}$ and $\mathcal{U}_{i} \cap \mathcal{U}_{j}=\emptyset$ if $i \neq j$.

As we noted above, every displaced user outside the coverage of $\mathcal{A}$ is an element of $\mathcal{U}_{0}$; each displaced user not in $\mathcal{U}_{0}$ can connect to at least one AP in $\mathcal{A}$ and thus must belong to at least one set in $\left\{\mathcal{U}_{j}\right\}_{j=0}^{N_{\mathcal{P}}-1}, j \neq 0$. If a displaced user $u$ is a member of $\mathcal{U}_{j}$, then $u$ can communicate with only those APs that belong to $\mathcal{G}_{j}$. Then if $u \in \mathcal{U}_{j}, u$ lies in $C\left(\mathcal{G}_{j}\right)$, the intersection of coverage areas of the networks in $\mathcal{G}_{j}$ :

$$
\begin{align*}
p_{u} \in C\left(\mathcal{G}_{j}\right) & =\Omega \cap \bigcap_{i: A_{i} \in \mathcal{G}_{j}} \mathcal{C}_{i} \backslash \bigcup_{i: A_{i} \in \mathcal{G}_{j}^{\mathrm{C}}} \mathcal{C}_{i} \\
& =\Omega \cap \bigcap_{i: A_{i} \in \mathcal{G}_{j}} \mathcal{C}_{i} \cap \bigcup_{i: A_{i} \notin \mathcal{G}_{j}} \mathcal{C}_{i}^{\mathrm{C}} \\
& =\Omega \cap \bigcap_{i: A_{i} \in \mathcal{G}_{j}} \mathcal{C}_{i} \cap \bigcap_{i: A_{i} \in \mathcal{G}_{j}^{\mathrm{C}}} \mathcal{C}_{i}^{\mathrm{C}} \tag{9}
\end{align*}
$$

where $\mathcal{G}_{j}^{\mathrm{C}}$ is the complement of the set $\mathcal{G}_{j}$. Given $u \in \mathcal{U}_{j}$, suppose $\exists j^{\prime} \neq j$ such that $u \in \mathcal{U}_{j^{\prime}}$ also. Then the coverage areas of the APs in $\mathcal{U}_{j}$ and $\mathcal{U}_{j^{\prime}}$ must overlap, i.e. their
intersection is non-empty, which is

$$
\begin{align*}
C\left(\mathcal{G}_{j}\right) \cap C\left(\mathcal{G}_{j^{\prime}}\right)= & \left(\Omega \cap \bigcap_{i: A_{i} \in \mathcal{G}_{j}} \mathcal{C}_{i} \cap \bigcap_{i: A_{i} \in \mathcal{G}_{j}^{\mathrm{C}}} \mathcal{C}_{i}^{\mathrm{C}}\right) \\
& \cap\left(\Omega \cap_{i: A_{i} \in \mathcal{G}_{j^{\prime}}} \mathcal{C}_{i} \cap \bigcap_{i: A_{i} \in \mathcal{G}_{j^{\prime}}^{\mathrm{C}}} \mathcal{C}_{i}^{\mathrm{C}}\right) \\
= & \Omega \cap \bigcap_{i: A_{i} \in \mathcal{G}_{j} \cap \mathcal{G}_{j^{\prime}}} \mathcal{C}_{i} \cap \bigcap_{i: A_{i} \in \mathcal{G}_{j}^{\mathrm{C}} \cap \mathcal{G}_{j^{\prime}}^{\mathrm{C}}} \mathcal{C}_{i}^{\mathrm{C}} \\
& \cap \bigcap_{i: A_{i} \notin \mathcal{G}_{j} \cap \mathcal{G}_{j^{\prime}}} \mathcal{C}_{i} \cap \bigcap_{i: A_{i} \notin \mathcal{G}_{j}^{\mathrm{C}} \cap \mathcal{G}_{j^{\prime}}^{\mathrm{C}}} \mathcal{C}_{\text {C }}^{\mathrm{C}} .
\end{align*}
$$

Since $j^{\prime} \neq j$, assuming $j, j^{\prime}>0, \exists A_{\ell} \in \mathcal{A}$ such that $A_{\ell} \in \mathcal{G}_{j}$ but $A_{\ell} \notin \mathcal{G}_{j^{\prime}}$. Then $A_{\ell} \notin \mathcal{G}_{j} \cap \mathcal{G}_{j^{\prime}}$ and $A_{\ell} \notin \mathcal{G}_{j}^{\mathrm{C}} \cap \mathcal{G}_{j^{\prime}}^{\mathrm{C}}$. Thus the series of intersections contains both $\mathcal{C}_{\ell}$ and $\mathcal{C}_{\ell}^{\mathrm{C}}$, and the intersection of these two sets of points is empty. Since $\emptyset$ is in the series of intersections, $C\left(\mathcal{G}_{j}\right) \cap C\left(\mathcal{G}_{j^{\prime}}\right)=\emptyset$ and we have a contradiction. Thus any two sets in $\left\{\mathcal{U}_{j}\right\}_{j=0}^{N_{\mathcal{P}}-1}$ are disjoint. Since $\bigcup_{j=1}^{N_{\mathcal{P}}-1} \mathcal{U}_{j}$ is the set of all displaced users that can connect to at least one AP, then $\bigcup_{j=0}^{N_{\mathcal{P}}-1} \mathcal{U}_{j}=\mathcal{U}$, and $\left\{\mathcal{U}_{j}\right\}_{j=0}^{N_{\mathcal{P}}-1}$ is a partition of $\mathcal{U}$.

In the absence of CAC at the APs, we have the condition that the number of displaced users that collectively migrate to a group of APs that compose the set $\mathcal{G}_{j} \subseteq \mathcal{A}$ must not exceed the total number of displaced users that lie in the coverage area of $\mathcal{G}_{j}$. For example, in the case of the set of APs from Fig. 4, we have the set of conditions

$$
\begin{aligned}
x_{1} & \leq\left|\mathcal{U}_{4}\right|+\left|\mathcal{U}_{5}\right|+\left|\mathcal{U}_{6}\right|+\left|\mathcal{U}_{7}\right| \\
x_{2} & \leq\left|\mathcal{U}_{2}\right|+\left|\mathcal{U}_{3}\right|+\left|\mathcal{U}_{6}\right|+\left|\mathcal{U}_{7}\right| \\
x_{3} & \leq\left|\mathcal{U}_{1}\right|+\left|\mathcal{U}_{3}\right|+\left|\mathcal{U}_{5}\right|+\left|\mathcal{U}_{7}\right| \\
x_{1}+x_{2} & \leq\left|\mathcal{U}_{2}\right|+\left|\mathcal{U}_{3}\right|+\left|\mathcal{U}_{4}\right|+\left|\mathcal{U}_{5}\right|+\left|\mathcal{U}_{6}\right|+\left|\mathcal{U}_{7}\right| \\
x_{1}+x_{3} & \leq\left|\mathcal{U}_{1}\right|+\left|\mathcal{U}_{3}\right|+\left|\mathcal{U}_{4}\right|+\left|\mathcal{U}_{5}\right|+\left|\mathcal{U}_{6}\right|+\left|\mathcal{U}_{7}\right| \\
x_{2}+x_{3} & \leq\left|\mathcal{U}_{1}\right|+\left|\mathcal{U}_{2}\right|+\left|\mathcal{U}_{3}\right|+\left|\mathcal{U}_{5}\right|+\left|\mathcal{U}_{6}\right|+\left|\mathcal{U}_{7}\right| \\
x_{1}+x_{2}+x_{3} & \leq\left|\mathcal{U}_{1}\right|+\left|\mathcal{U}_{2}\right|+\cdots+\left|\mathcal{U}_{7}\right| .
\end{aligned}
$$

We can state the set of conditions formally as follows. The number of displaced users that migrate to the APs in a set $\mathcal{G}_{j}$ is $\sum_{k: A_{k} \in \mathcal{G}_{j}} x_{k}=\sum_{k: i_{k}\left(\mathcal{G}_{j}\right)=1} x_{k}$. The subset $\mathcal{U}_{\ell}$ of the set of displaced users $\mathcal{U}$ lies within the coverage area of the APs in $\mathcal{G}_{j}$ if there exists $k \in\left\{1,2, \ldots, N_{a}\right\}$ such that $A_{k} \in \mathcal{G}_{j}$ (i.e. $i_{k}\left(\mathcal{G}_{j}\right)=1$ ) and $\mathcal{U}_{\ell} \subset \mathcal{C}_{k}$ (i.e. $f_{k}\left(\mathcal{U}_{\ell}\right)=1$ ). Conversely, the subset $\mathcal{U}_{\ell}$ is not in the coverage area of $\mathcal{G}_{j}$ if $f_{k}\left(\mathcal{U}_{\ell}\right)=0$ for every $A_{k} \in \mathcal{G}_{j}$ or $\sum_{k: i_{k}\left(\mathcal{G}_{j}\right)=1} f_{k}\left(\mathcal{U}_{\ell}\right)=S(j, \ell)=0$. Thus the total number of displaced users that lie in the union of the coverage areas of the APs in $\mathcal{G}_{j}$ is $\sum_{\ell: S(j, \ell) \neq 0}\left|\mathcal{U}_{\ell}\right|$. Because the number of displaced users assigned to the APs that compose $\mathcal{G}_{j}$ cannot exceed the number of displaced users in the coverage area, we have the set of constraints

$$
\begin{equation*}
\sum_{k: i_{k}\left(\mathcal{G}_{j}\right)=1} x_{k} \leq \sum_{\ell: S(j, \ell) \neq 0}\left|\mathcal{U}_{\ell}\right|, \quad \forall \mathcal{G}_{j} \subseteq \mathcal{P}(\mathcal{A}) \backslash \emptyset \tag{11}
\end{equation*}
$$

We must modify the above set of constraints to account for the limited capacity at the target APs. Recalling the definition from Section II-A, $m_{i}$ is the available capacity at the $i$ th AP,
measured in the number of displaced users that can join the AP without being rejected by the CAC algorithm. The number of displaced users that migrate to a set of APs $\mathcal{G}_{j}$ cannot exceed either the number of displaced users that were in the aggregate coverage area of the APs that compose $\mathcal{G}_{j}$ or the total available capacity of the APs that compose $\mathcal{G}_{j}$. The modified constraint is then

$$
\begin{equation*}
\sum_{k: i_{k}\left(\mathcal{G}_{j}\right)=1} x_{k} \leq \min \left(\sum_{\ell: S(j, \ell) \neq 0}\left|\mathcal{U}_{\ell}\right|, \sum_{k: i_{k}\left(\mathcal{G}_{j}\right)=1} m_{k}\right), \forall \mathcal{G}_{j} \subseteq \mathcal{P}(\mathcal{A}) \backslash \emptyset \tag{12}
\end{equation*}
$$

Recalling that the cost function is the number of displaced users that successfully connect to the available APs, we have an ILP where we must maximize

$$
\begin{equation*}
J\left(x_{1}, x_{2}, \ldots, x_{N_{a}}\right)=\sum_{i=1}^{N_{a}} x_{i} \tag{13}
\end{equation*}
$$

such that the set of constraints given by Eq. (12) are satisfied.

## III. Simulations

In this section, we present simulation results that illustrate the performance of the probabilistic assignment scheme that accounts for the loading at the target APs versus the performance of a simple scheme in which each displaced user chooses a target AP based only on the RSS. The simulations also use the ILP described in Section II-B to provide a lower bound for the algorithms' performance. The performance metric we use is $\mathrm{E}\left\{N_{R}\right\}=N_{u}-\mu_{S}$, the average number of users that are not able to connect to a target AP on the first attempt.

The simulations were performed in Matlab. For each value of $N_{u}$ and $N_{a}$ that we considered, we performed 1000 individual runs. The set of parameters that we used are listed in Table III. Each run examined $N_{u}$ displaced users that were uniformly distributed within a unit circle centered at the origin of the $x-y$ plane that represented the coverage area of the failed AP. Overlapping the unit circle were $N_{a}$ circles that represented the coverage areas of available APs. The coverage radii of the available APs, $\left\{R_{i}\right\}, i=1,2, \ldots, N_{a}$, were uniformly distributed over the range [1,4] (i.e. 1-4 times the radius of the failed AP). Given a radius $R_{i}$ for a target AP, its location was uniformly distributed in a circle of radius $R_{i}+1$ centered at the origin, which guaranteed overlap of the target AP and failed AP coverage areas. We also computed a transmit power level for each target AP, assuming that the power was proportional to the coverage area. We normalized the power levels so that the transmit power of an AP with unit radius was 1 , and $P_{i}$, the power of an AP with radius $R_{i}$, was $P_{i}=R_{i}^{2}$. Capacities for the target APs were generated as follows. We computed the total capacity of the $i$ th AP, $C_{i}$, to be proportional to the coverage area of the AP; we chose a proportion of $10 / \pi$. We computed the available capacity to be a uniform random variate over the range $\left[0,0.1 C_{i}\right]$.

Once the network topology and AP capacities were computed for a given run, the probabilistic algorithm examined each displaced user and determined the set of visible target APs. Based on the capacities of the APs in the set, the user

TABLE I
Parameter Values Used in Simulations

| Parameter | Value |
| :---: | :---: |
| $N_{u}$ | 5 to 50 |
| $N_{a}$ | 4,6 |
| $R_{\text {failed AP }}$ | 1 |
| $R_{i}$ | uniform between 1 to 4 |
| $m_{i}$ | uniform over $0-10 \%$ of $i$ th AP capacity |



Fig. 5. Plot of $\mathrm{E}\left\{N_{R}\right\}$ for handover migration schemes based on maximum RSS, random choice of a AP based on available capacities, and solving constrained ILP for (a) 4 APs and (b) 6 APs, versus the number of displaced users.
was assigned to one of them using the probabilities given by Eq. (5). The number of users rejected by the target APs due to CAC was computed by taking the sum of the differences of the number of users assigned to each AP and the free capacity of the AP. The RSS algorithm assumed that the RSS for each AP was proportional to the square of the distance from the AP. Using this value, each displaced user was assigned to the target AP with the highest RSS from the group of APs in whose coverage areas the user was located. The number of rejected users was computed in the same fashion as for the probabilistic algorithm. Finally, the ILP maximized the cost function in Eq. (13) subject to the set of constraints in Eq. (12). The number of rejected users in this case was the difference of total number of users $N_{u}$ and the number of assigned users $\sum_{i=1}^{N_{a}} x_{i}$.

We plot our results in Fig. 5(a) and Fig. 5(b) for $N_{a}=4$ and $N_{a}=6$ respectively. The data points in the figure are the mean number of rejected users, averaged over 1000 trials for each point. The error bars were computed by taking the sample variance of the number of rejected users for each scheme and setting the length of each bar to be one standard deviation. The results show that the probabilistic approach is superior to using the RSS in all cases, which we expected. The probabilistic algorithm's performance is clearly suboptimal for small $N_{u}$ but approaches the ILP's performance as $N_{u}$ becomes large. Furthermore, the difference in performance between the RSSbased algorithm and the probabilistic algorithm is larger for larger $N_{a}$; this is also the case when we consider the difference between the probabilistic algorithm and the ILP. The probabilistic algorithm and ILP are also more sensitive to $N_{a}$ than the RSS-based algorithm, showing greater improvement as $N_{a}$ increases. The variance of the cost of each approach increased steeply for $5 \leq N_{u} \leq 20$, and then plateaued for


Fig. 6. Plot of $\mathrm{E}\left\{N_{R}\right\}$ for three handover migration schemes versus the number of displaced users, where maximum AP load is $10 \%$.
$N_{u}>20$. The ILP and probabilistic cost variances, which were nearly equal, were greater than the variance of the RSS cost. Increasing $N_{a}$ caused an increase in the variance for each scheme, although the increase was greatest for the ILP and probabilistic approaches.

We also considered the case where the $i$ th target AP's load was uniform over $\left[0,0.1 C_{i}\right]$ (available capacity was uniform over $\left[0.9 C_{i}, C_{i}\right]$ ); we plotted the results in Fig. 6. For this case, the probabilistic algorithm did not perform as well as the RSS-based approach, although the expected value of $N_{R}$ in all cases was very small and should not prevent using the probabilistic approach over the full range of target AP load levels.

## IV. Conclusions

In this paper, we derived expressions for the probability that a randomly chosen user would be successfully connected to a new AP and for the expected number of displaced users that would be successfully connected on their first attempt and not require migration to a secondary AP. Our analysis showed that good performance can be achieved by using a simple heuristic by which displaced users that can see multiple APs randomly choose one according to the fraction of the total available resources associated with each AP, and that this approach is nearly optimal, especially when the number of displaced users is large relative to the available free capacity of the target APs and the target APs are heavily loaded.

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