Diagnosis of Pulsed Squeezing in Multiple Temporal Modes

S. Glancy, E. Knill, T. Gerrits, T. Clement, M. Stevens, S. W. Nam, and R. Mirin

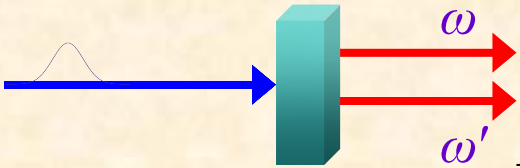
National Institute of Standards and Technology

Boulder, Colorado, USA

Topics

- Multimode squeezing problem
  - temporal/spectral modes (not transverse spatial modes)
- Photon subtraction experiment
- Multimode Gaussian tomography

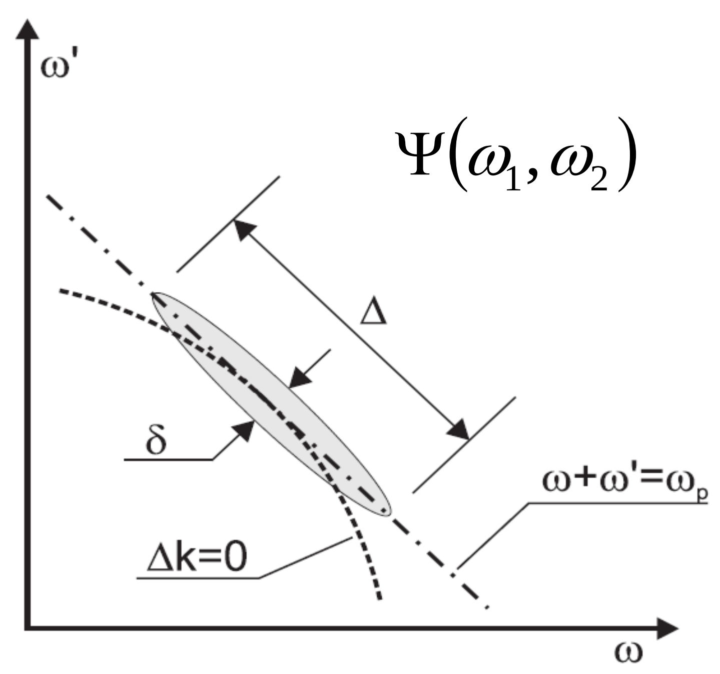
Pulsed squeezing



$$\Psi(\omega,\omega')$$

$$|0\rangle \rightarrow \sqrt{1-\eta}\,|0\rangle + \sqrt{\eta}\iint \mathrm{d}\omega\,\mathrm{d}\omega'\,\Psi(\omega,\omega')\,\hat{a}^{\dagger}(\omega)\hat{a}^{\dagger}(\omega')|0\rangle + \cdots$$

$\Psi(\omega,\omega')$ is the joint wavefunction, determined by broadband energy conservation / crystal phase matching.



from Wasilewski, Lvovsky, Banaszek, and Radzewicz quant-ph/0512215

- If the squeezing is degenerate, $\Psi(\omega,\omega')$ is symmetric, and we use orthonormal decomposition into characteristic modes $\psi_n(\omega)$:

$$\Psi(\omega,\omega') = \sum_{n=1}^{\infty} \zeta_n \psi_n^*(\omega)\psi_n^*(\omega') \quad \text{Now, let} \quad \hat{b}_n = \int \mathrm{d}\omega\, \psi_n(\omega)\hat{a}(\omega)$$

$$|0\rangle \to \sqrt{1-\eta}\,|0\rangle + \sqrt{\eta}\sum_{n=1}^{\infty} \zeta_n \left(\hat{b}_n^{\dagger}\right)^2 |0\rangle + \cdots$$

- Each mode $\psi_n(\omega)$ is squeezed independently by ζ_n.
- For weak squeezing $\psi_n(\omega)$ are approximately Gaussian-Hermite polynomials,

but for strong squeezing they are not.

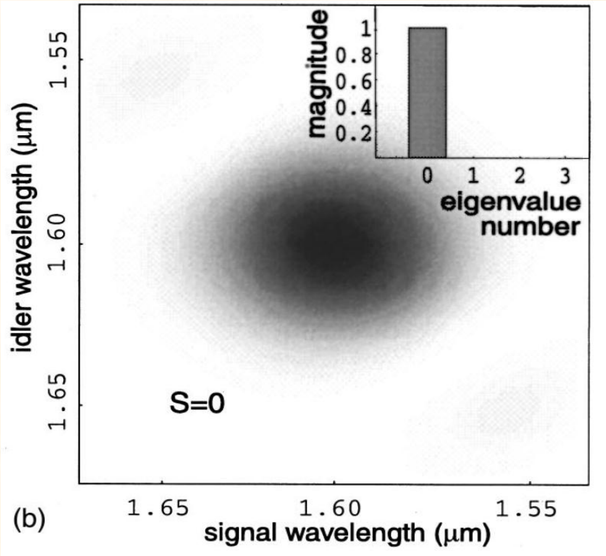
Single Mode Squeezing

- To create single-mode squeezing we need

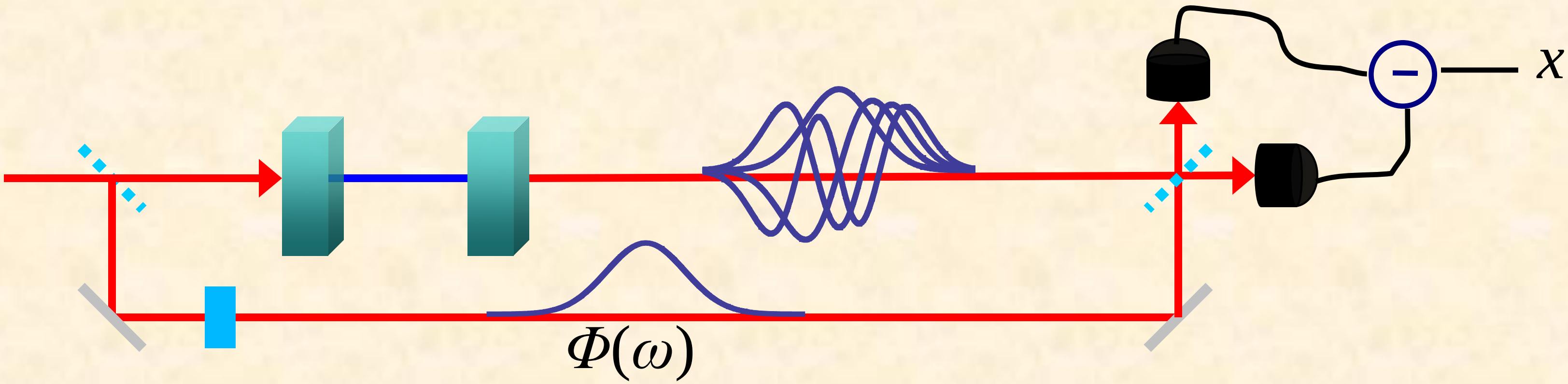
$$\Psi(\omega,\omega') = \sum_{n=0}^{\infty} \zeta_n \psi_n^*(\omega)\psi_n^*(\omega') \rightarrow \psi^*(\omega)\psi^*(\omega')$$

- May be possible by engineering crystal dispersion and phase-matching properties.

- Grice, U'Ren, and Walmsley recommend degenerate, type-II, down-conversion in BBO with an 800 nm pump. [PRA **64**, 063815]



Homodyne Detection



- Local oscillator (LO) is in mode $\Phi(\omega)$.
- The mode overlap is

$$a_n = \int \mathrm{d}\omega \Phi(\omega) \psi_n^{\,*}(\omega) = |a_n| e^{i\alpha_n}$$

- The homodyne signal and its variance are

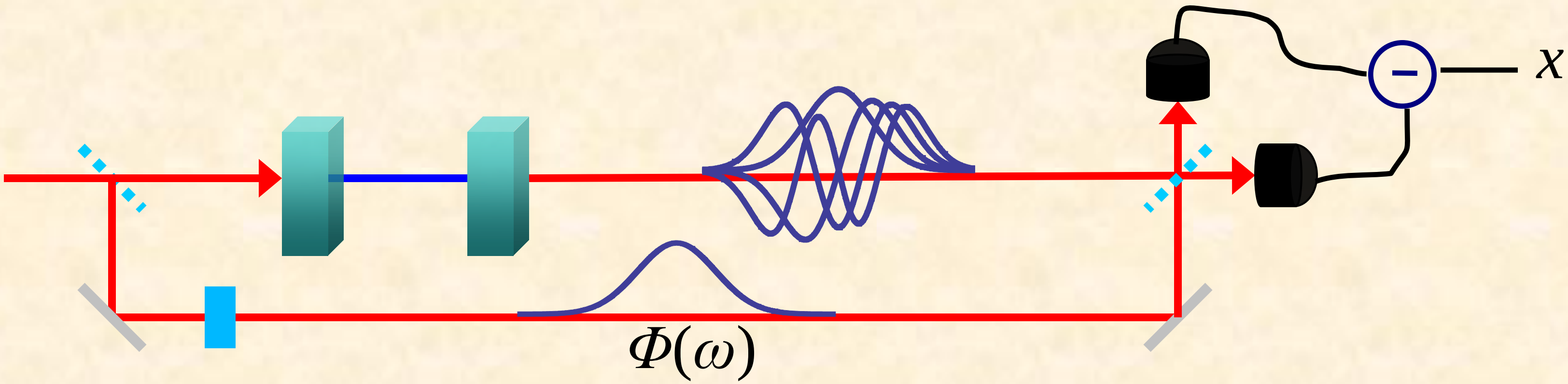
$$x = \sum_{n=1}^{\infty} |a_n| \left(\cos\alpha_n x_n + \sin\alpha_n p_n\right) \qquad \left\langle x^2 \right\rangle = \sum_{n=1}^{\infty} |a_n|^2 \left(\cos^2\alpha_n \left\langle x_n^{\,2} \right\rangle + \sin^2\alpha_n \left\langle p_n^{\,2} \right\rangle\right)$$

$$x = \vec{A}^T \vec{x}$$

$$\left\langle x^2 \right\rangle = \vec{A} V \vec{A}^{\mathrm{T}}$$

where $\vec{q} = \begin{pmatrix} x_1 \\ p_1 \\ x_2 \\ p_2 \\ \vdots \end{pmatrix}$, $\vec{A} = \begin{pmatrix} \mathrm{Re}[a_1] \\ \mathrm{Im}[a_1] \\ \mathrm{Re}[a_2] \\ \mathrm{Im}[a_2] \\ \vdots \end{pmatrix}$, and $V = \begin{pmatrix} \left\langle x_1^{\,2} \right\rangle & & & & \\ & \left\langle p_1^{\,2} \right\rangle & & & \\ & & \left\langle x_2^{\,2} \right\rangle & & \\ & & & \left\langle p_2^{\,2} \right\rangle & \\ & & & & \ddots \end{pmatrix}$

Homodyne Detection



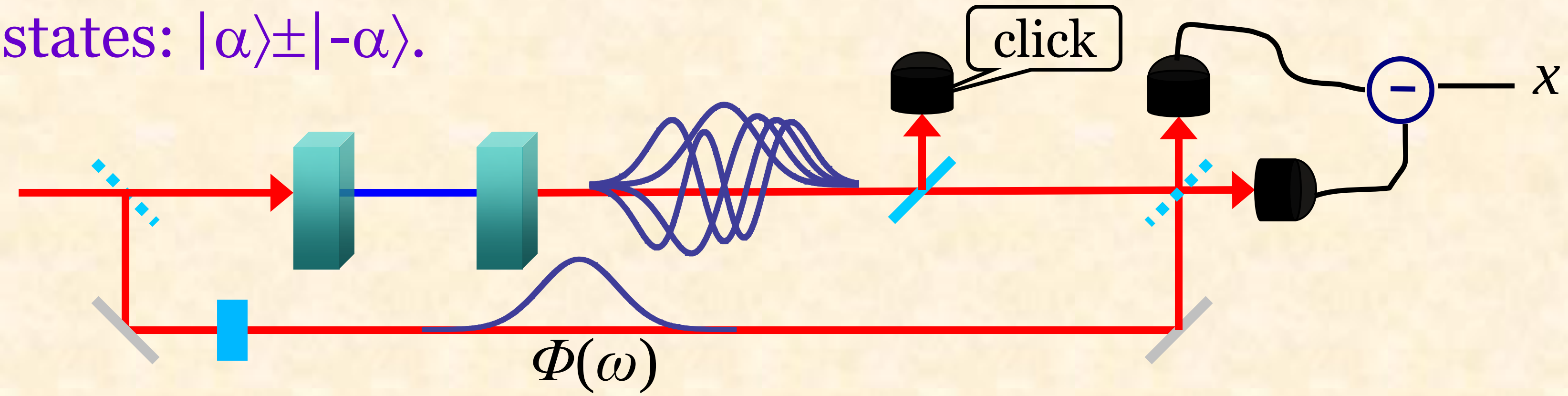
- Because each mode has different levels of squeezing, the state observed by homodyne detection cannot be a pure state of minimum uncertainty, unless the LO shape matches one mode.
- Try to shape LO to match one of the squeezed modes.
  - Shape is not necessarily Gaussian

Multimode Problems for QIP

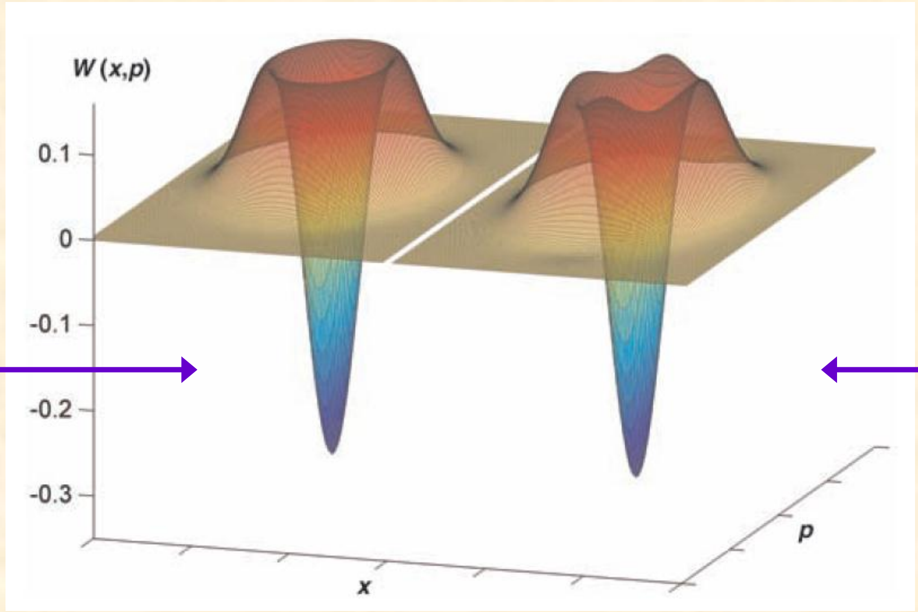
- We have unwanted photons in extra modes.
- They cause no problems for linear optics and homodyne detection.
- They will interact with nonlinearities such as Kerr effect or atoms.
- They are observable by eavesdroppers.
- Extra photons make photon detectors click.

Photon Subtraction

- A method to make superpositions of coherent ("cat") states: $|\alpha\rangle \pm |-\alpha\rangle$.



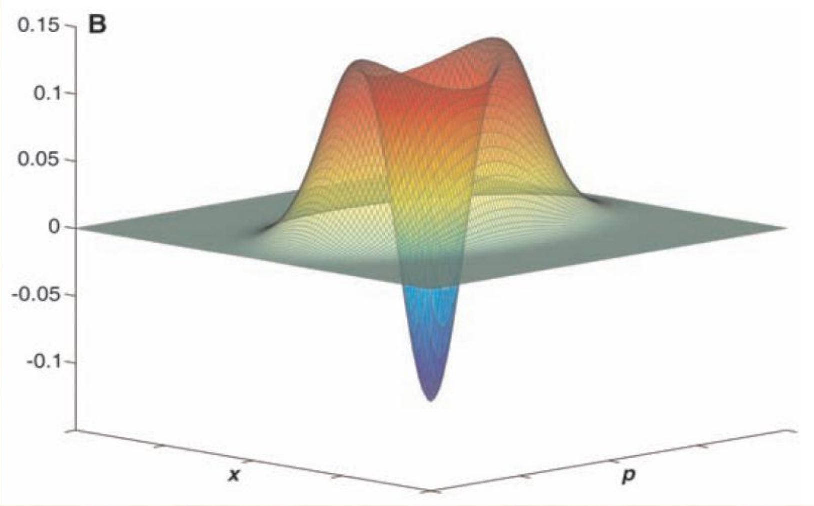
ideal photon subtracted state

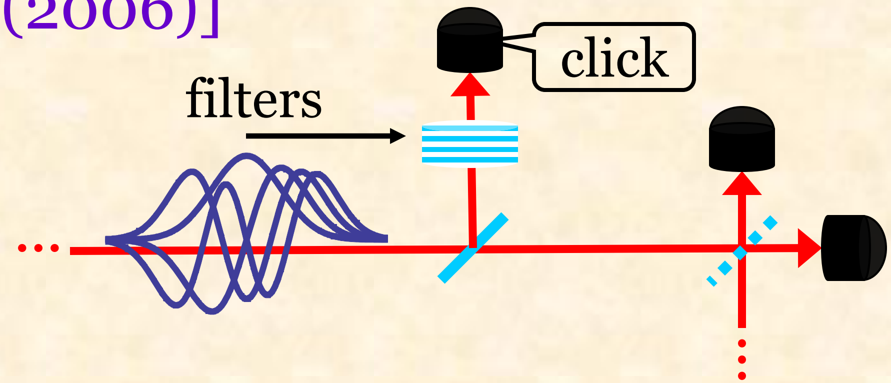


perfect "cat" state $|\alpha|^2=0.8$, $\langle n\rangle=1.2$

Photon Subtraction

- Demonstrated by Ourjoumtsev, Tualle-Brouri, Laurat, Grangier [Science **312**, 83 (2006)]



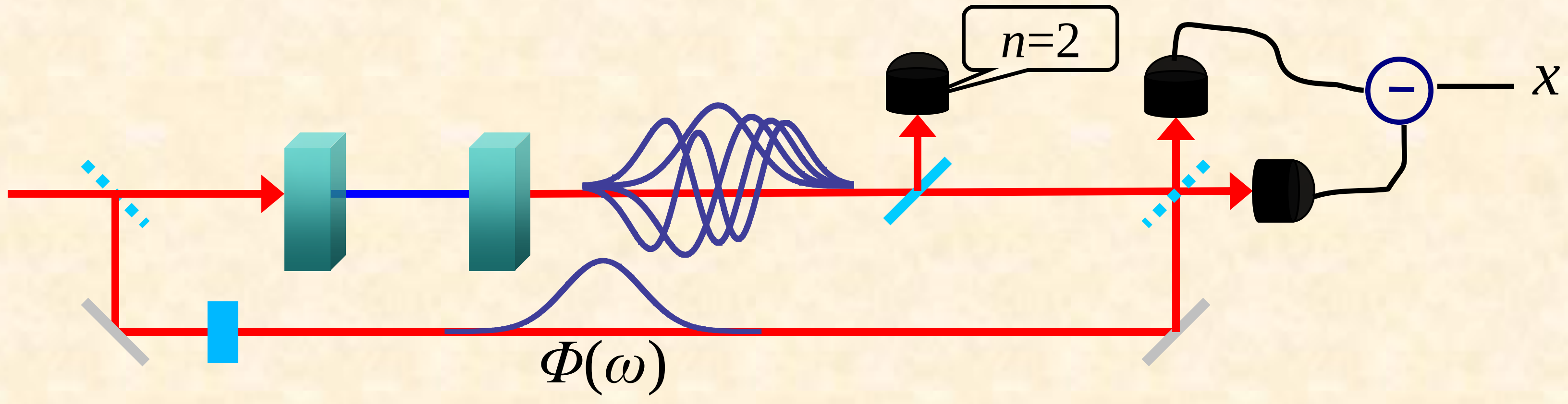


Fidelity=70%
$|\alpha|^2=0.79$
$\langle n\rangle=1.2$

"modal purity" = probability that a click was caused by a photon from the mode matching the local oscillator = 0.82

Our Photon Subtraction

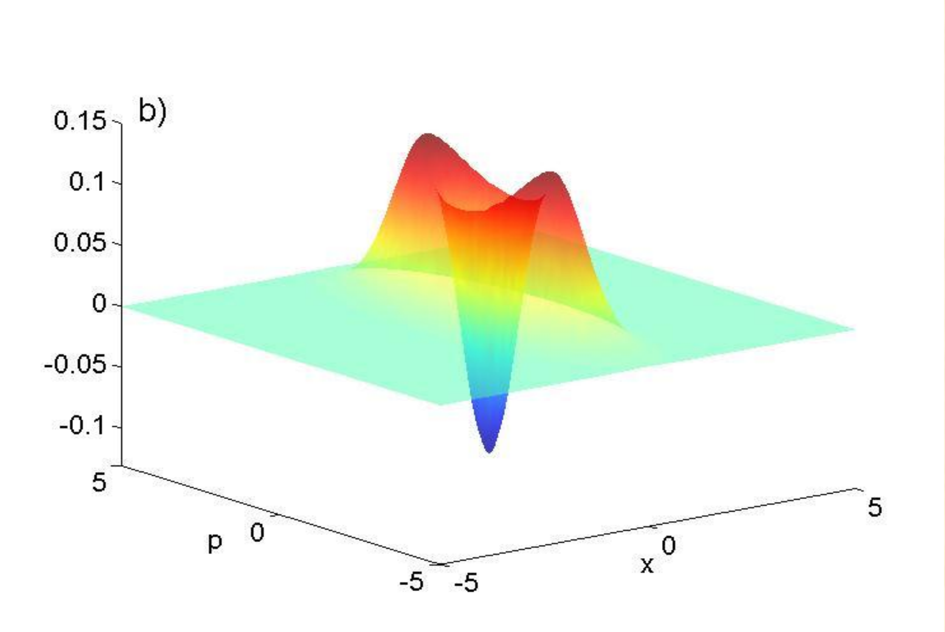
- Subtract two or more photons



- Using superconducting transition edge sensitive photon number resolving detectors.
  - efficiency ~ 90%
  - dark counts limited by black-body radiation
- Subtracting more photons makes a higher fidelity, larger cat, using less squeezing.

Preliminary Results

Single photon subtracted Wigner function



- Fidelity is low (~60±40%) because
  - purity of our squeezed state is too low
  - too many photons that are not matched to the LO.
    - verified by comparison of homodyne signal and photon counting rate
- We want to measure the contents and shapes of the extra modes produced in the squeezing.

Multimode Gaussian Tomography

- We want a method to measure the characteristic mode shapes $\psi_n(\omega)$ and the squeezing ζ_n for (n = 1 to N)
- Full quantum state tomography for ~50 harmonic oscillators is impractical.
- We will limit to Gaussian states.

$$W(\vec{q}) = \frac{1}{(2\pi)^N |\Sigma|^{1/2}} \exp\left[-\frac{1}{2}(\vec{q})^T \Sigma^{-1} \vec{q}\right]$$

where $\vec{q} = \begin{pmatrix} x_1 \\ p_1 \\ x_2 \\ p_2 \\ \vdots \end{pmatrix}$, and Σ is a covariance matrix.

Covariance Matrix Properties

- Real
- Symmetric
- Positive-definite $\Rightarrow$ positive eigenvalues
- Obey uncertainty principle:

$\Sigma + \frac{i}{2} Q$ is positive semidefinite, where $Q = \begin{pmatrix} 0 & 1 & & & \\ -1 & 0 & & & \\ & & 0 & 1 & \\ & & -1 & 0 & \\ & & & & \ddots \end{pmatrix}$.

- All Gaussian state transformation makes $\mathrm{Sp}(2N,\mathbb{R})$.
- Passive linear optical transformations are $\mathrm{SO}(2N) \cap \mathrm{Sp}(2N,\mathbb{R})$.
- Diagonalization of Σ requires SO(2N).

Simon, Mukunda, and Dutta. PRA **49**, 1567

- We choose a set of modes $\beta_n(\omega)$.
- These overlap with the characteristic modes

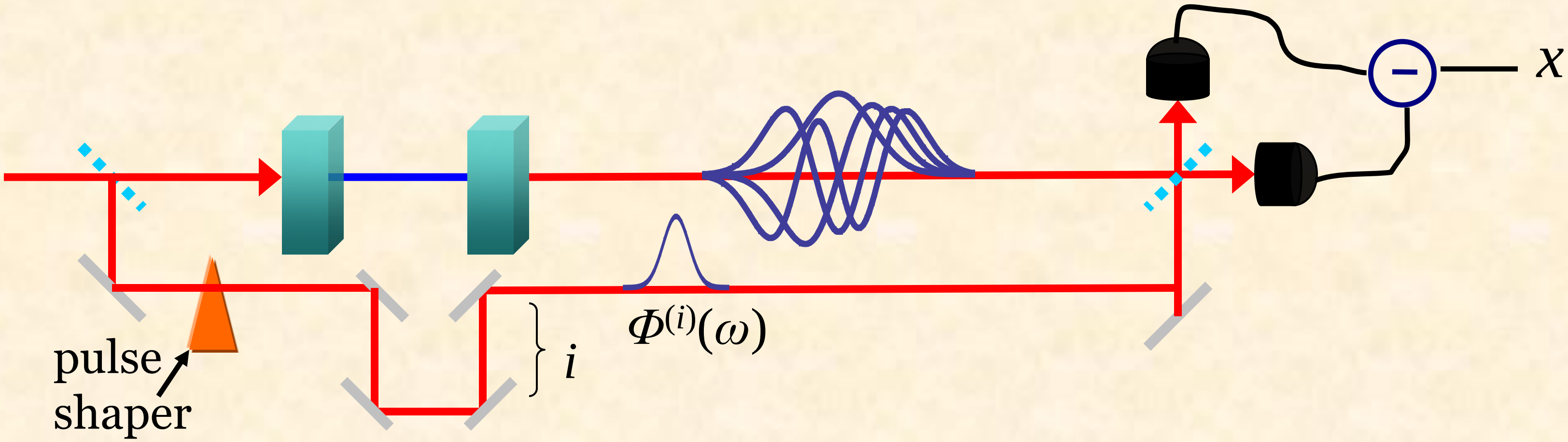
$$b_{ij} = \int \mathrm{d}\omega\, \beta_i(\omega) \psi_j^*(\omega)$$

- The covariance matrices are related by

$$\Sigma = BVB^{\mathrm{T}} \quad \text{where} \quad B = \begin{pmatrix} \mathrm{Re}[b_{11}] & \mathrm{Im}[b_{11}] & \mathrm{Re}[b_{12}] & \mathrm{Im}[b_{12}] & \cdots \\ -\mathrm{Im}[b_{11}] & \mathrm{Re}[b_{11}] & -\mathrm{Im}[b_{12}] & \mathrm{Re}[b_{12}] & \\ \mathrm{Re}[b_{21}] & \mathrm{Im}[b_{21}] & \mathrm{Re}[b_{22}] & \mathrm{Im}[b_{22}] & \\ -\mathrm{Im}[b_{21}] & \mathrm{Re}[b_{21}] & -\mathrm{Im}[b_{22}] & \mathrm{Re}[b_{22}] & \\ \vdots & & & & \ddots \end{pmatrix}$$

- First find Σ using $\beta_n(\omega)$. Then diagonalize Σ to find characteristic modes.

Measurement Scheme



- Shorten LO pulse
- Add large adjustable delay. At each delay measure $x^{(i)}$.
- The overlap between each LO and our chosen modes is

$$c_n^{(i)} = \int \mathrm{d}\omega \Phi^{(i)}(\omega) \beta_n^*(\omega)$$

- For each i, $x^{(i)}$ is a Gaussian random variable with variance

$$v^{(i)} = \vec{C}^{(i)T} \Sigma \vec{C}^{(i)}$$

reminder: $v^{(i)} = \vec{C}^{(i)T} \Sigma \vec{C}^{(i)}$

- Probability to measure data

$$P(x) = \prod_i \frac{1}{\sqrt{2\pi v^{(i)}}} \mathrm{Exp}\left[\frac{-\left(x^{(i)}\right)^2}{2v^{(i)}} \right],$$

- which is like the single variable normal distribution, except the variance changes.
- This gives Log-Likelihood function

$$L(\Sigma) = -\frac{1}{2} \sum_i \left(\mathrm{Log}\left[v^{(i)}\right] + \frac{\left(x^{(i)}\right)^2}{v^{(i)}} \right)$$

- Maybe to maximize this to estimate Σ? How?
- Maybe use some other method? What?

- Given an estimate of Σ, we want to find the set of characteristic modes.
- The characteristic modes have a diagonal covariance matrix *V*.
- We need the similarity transform

$$B\Sigma B^T = V,$$

where *B* can be done with linear optics.
- With *B*, we can transform our modes to characteristic modes.

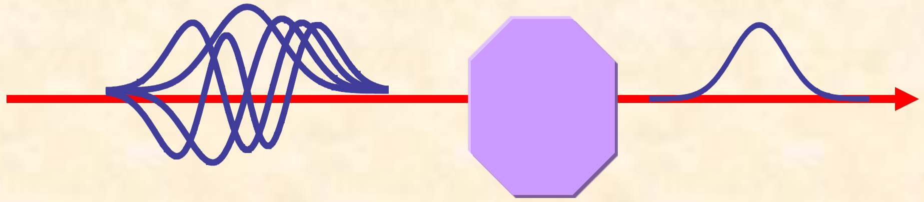
$$\psi_i(\omega) = \sum_j b_{ij}\beta_j(\omega)$$

- How to find *B*?

Concluding Remarks

- Pulsed squeezing makes many temporal modes.
- Extra modes are troublesome for photon subtraction and other QIP applications.
- We want to use homodyne system for multimode Gaussian tomography.

★ Extra credit → design temporal mode filter.



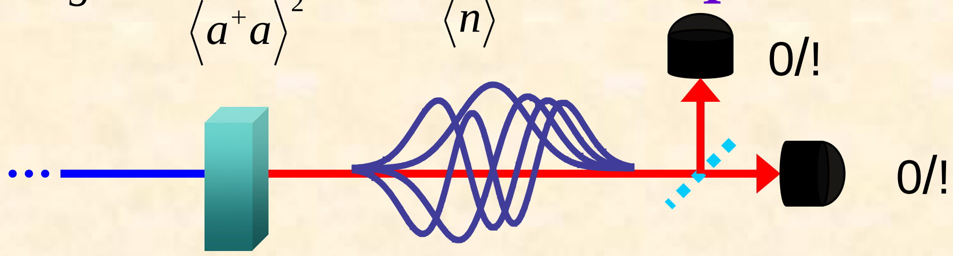
The following slides contain supplementary information not covered in the live talk.

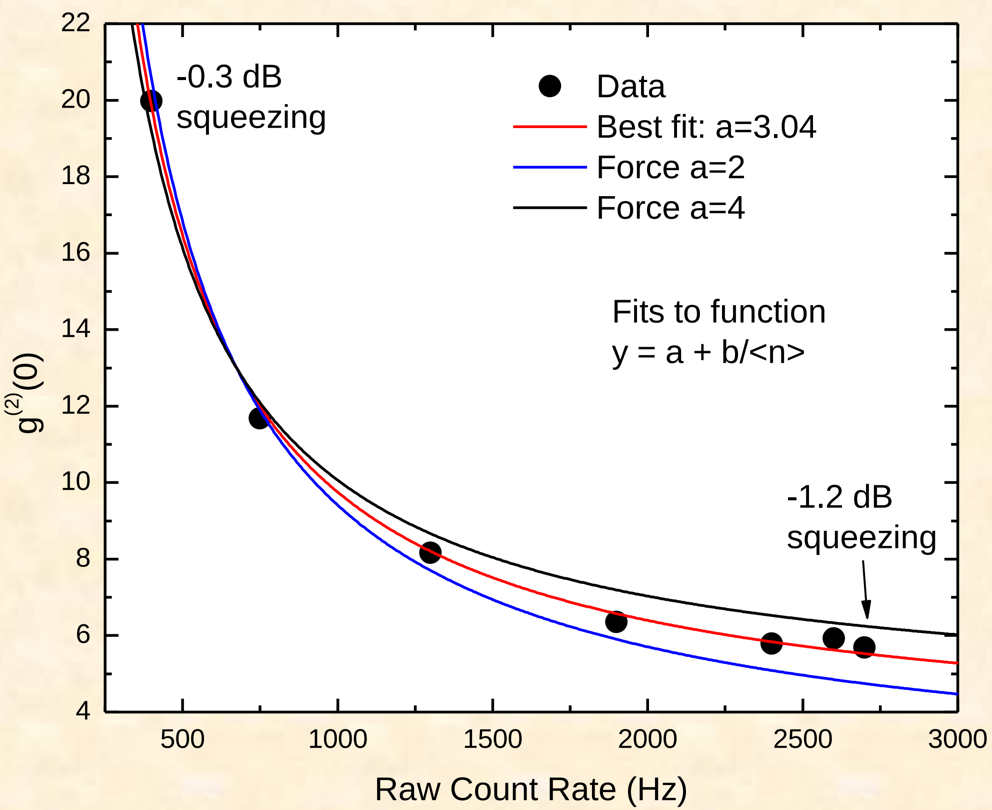
Experiment set-up details

- Ti: Sapphire laser 150 fs pulses
- 860 nm
- 150μm thick $KNbO_3$ crystal

Correlation Measurement

$$g2=\frac{\langle a^{+}a^{+}aa\rangle}{\langle a^{+}a\rangle^{2}}=3+\frac{1}{\langle n\rangle}$$ for squeezed light





When we fit $a+\frac{b}{\langle n\rangle}$ we find a=3.04 but b≈10, which can be explained by photons in extra modes.

Multimode Gaussian Tomography

- We want a method to measure the characteristic mode shapes $\psi_n(\omega)$ and the squeezing ζ_n for n=1 to N.
- Full quantum state tomography for ~50 harmonic oscillators is impractical.
- We will limit to Gaussian states.

$$W(\vec{x}) = \frac{1}{(2\pi)^N |\Sigma|^{1/2}} \exp\left[-\frac{1}{2} (\vec{x} - \vec{\mu})^T \Sigma^{-1} (\vec{x} - \vec{\mu}) \right],$$

where $\vec{x} = \begin{pmatrix} x_1 \\ p_1 \\ x_2 \\ p_2 \\ \vdots \end{pmatrix}$, μ contains means, and Σ is a covariance matrix.

- We choose a set of modes $\beta_n(\omega)$.
- These overlap with the characteristic modes

$$b_{ij} = \int \mathrm{d}\omega\, \beta_i(\omega) \psi_j^*(\omega)$$

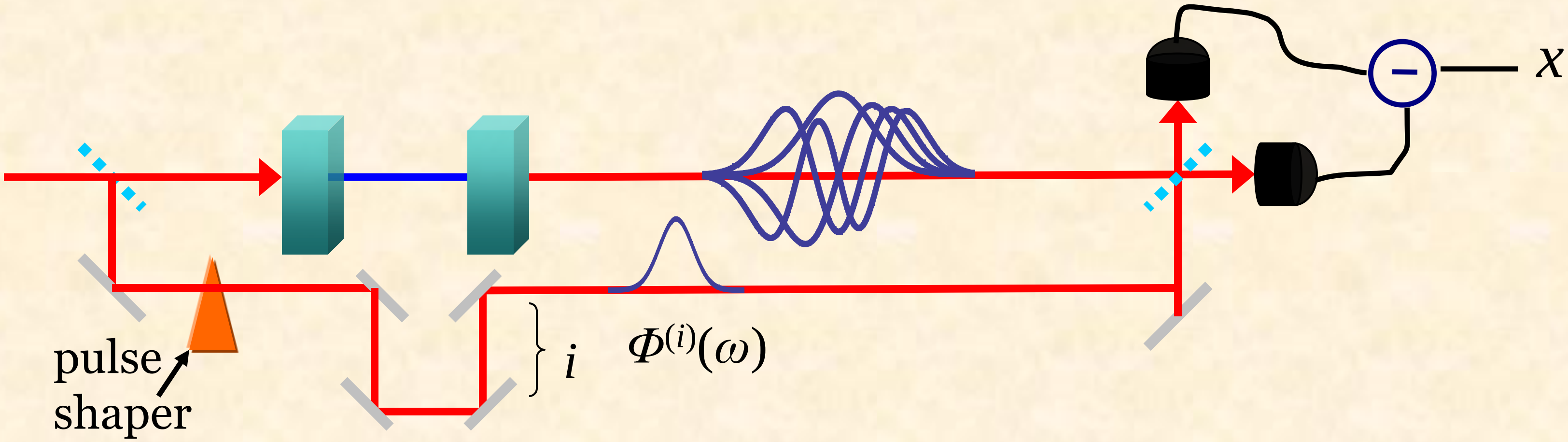
- The covariance matrices and means are related by

$$\Sigma = BVB^{\mathrm{T}} \quad \text{and} \quad \vec{\mu} = B\vec{v}$$

$$\text{where} \quad B = \begin{pmatrix} \mathrm{Re}[b_{11}] & \mathrm{Im}[b_{11}] & \mathrm{Re}[b_{12}] & \mathrm{Im}[b_{12}] & \cdots \\ -\mathrm{Im}[b_{11}] & \mathrm{Re}[b_{11}] & -\mathrm{Im}[b_{12}] & \mathrm{Re}[b_{12}] & \\ \mathrm{Re}[b_{21}] & \mathrm{Im}[b_{21}] & \mathrm{Re}[b_{22}] & \mathrm{Im}[b_{22}] & \\ -\mathrm{Im}[b_{21}] & \mathrm{Re}[b_{21}] & -\mathrm{Im}[b_{22}] & \mathrm{Re}[b_{22}] & \\ \vdots & & & & \ddots \end{pmatrix}$$

- First find Σ using $\beta_n(\omega)$. Then diagonalize Σ to find characteristic modes.

Measurement Scheme



- Shorten LO pulse
- Add large adjustable delay. At each delay measure $x^{(i)}$.
- The overlap is between each LO and our chosen modes is

$$c_n^{(i)} = \int \mathrm{d}\omega \Phi^{(i)}(\omega) \beta_n^*(\omega)$$

- For each i, x(i) is a Gaussian random variable with variance

$$\Sigma^{(i)} = \vec{C}^{(i)T} \Sigma \vec{C}^T$$

- and mean $\mu^{(i)} = \vec{C}^{(i)T} \vec{\mu}$

- Probability to measure data

$$P(x) = \prod_i \frac{1}{\sqrt{2\pi\Sigma^{(i)}}} \mathrm{Exp}\left[\frac{-\left(x^{(i)} - \mu^{(i)}\right)^2}{2\Sigma^{(i)}}\right]$$

$$\Sigma^{(i)} = \vec{C}^{(i)T} \Sigma \vec{C}^{T}$$

$$\mu^{(i)} = \vec{C}^{(i)T} \vec{\mu}$$

- This gives Log-Likelihood function

$$L(\Sigma, \mu) = -\frac{1}{2} \sum_i \left(\mathrm{Log}\left[\Sigma^{(i)}\right] + \frac{\left(x^{(i)} - \mu^{(i)}\right)^2}{\Sigma^{(i)}} \right)$$

- Maybe to maximize this to estimate Σ? How?
- Maybe use some other method? What?

- Given an estimate of Σ, we want to find the set of characteristic modes.
- The characteristic modes have a diagonal covariance matrix V.
- We need the similarity transform

$$B\Sigma B^T = V,$$

where B can be done with linear optics.

- With B, we can transform our modes to characteristic modes.

$$\psi_i(\omega) = \sum_j b_{ij}\beta_j(\omega)$$

- How to find B?