Use of a higher order Heydemann-Welch model to characterize a controlled clearance piston gauge

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Abstract

We present a new method for characterizing a controlled-clearance piston gauge as a primary pressure standard. This method requires operating the piston gauge to jacket pressures of over 80 % of the system pressure. We present measurements on a hydraulic piston gauge with a 290 MPa maximum pressure and a nominal piston diameter of 3.27 mm. Measurements showed that the cylinder becomes stiffer as the jacket pressure increases, and that non-linear models of the Heydemann-Welch parameters improve the determination of the effective area. The relative standard uncertainty in the effective area of the piston gauge ranges from 16.0×10^{-6} to 17.6×10^{-6}, and the agreement to the present NIST pressure scale is within the standard uncertainty.

1. Introduction

Many National Metrology Institutes, including the National Institute of Standards and Technology (NIST), use controlled-clearance piston gauges as primary standards for their hydraulic pressure scales [1-3]. In a controlled-clearance piston gauge, a “jacket” pressure is applied to the outer diameter of the cylinder, which controls the gap between the piston and cylinder [4]. The Heydemann-Welch (HW) method is often used to characterize controlled clearance piston gauges as primary pressure standards. This method requires determining two experimental parameters in the characterization: p_z, which is the jacket pressure that reduces the gap to zero, and d, which is the distortion coefficient due to changing the jacket pressure. In the traditional application of the HW method, both of these parameters are determined assuming linearized ideal behavior of the cylinder component of the piston gauge. The characterization is completed with dimensional measurement of the piston diameter and estimation of piston distortion from linear elasticity theory.

The concept of the HW method is that the effective area of the piston gauge is determined by calculating the piston area at the operating pressure, p, and measuring the area contributed by the piston-cylinder gap as the cylinder deforms from being collapsed onto the piston to its shape at the operating conditions. During characterization, establishing experimental jacket pressures close to the condition of cylinder collapse onto the piston will improve the determination of effective area. Practical considerations, such as deformation in the piston gauge housing and interference between an imperfectly shaped piston and cylinder, limit the jacket pressure that can be applied. In [4], p_j was limited to 40 % of p. One of NIST's controlled-clearance piston gauges (designated as CCPG-2481, manufactured by Fluke Calibration[1]) has been designed to operate at p_j that can in some cases exceed 80 % of p. Operating at high p_j with CCPG-2481 has allowed improving the HW method through higher-order (non-linear) fits of the characterization parameters.

2. The HW method

The pressure generated by a piston gauge is the vertical forces divided by the effective area, A_e, of the piston gauge. The vertical forces are the buoyancy-corrected forces due to mass plus the surface tension of the oil acting on the piston. The effective area is the "calibration coefficient" of the piston gauge, or thought another way, the parameter which when divided into

[1] Certain commercial equipment, instruments, or materials are identified in this paper to foster understanding. Such indentification does not imply recommendation or endorsement by NIST, nor does it imply that the materials or equipment identified are necessarily the best available for the purpose.

the forces gives the pressure. The HW method begins with the piston area, A_p and determines the contribution of the gap to the effective area through measured area changes:

$$A_\mathrm{e} = A_\mathrm{p} + \left(A_{\mathrm{e},p_\mathrm{j}} - A_{\mathrm{e},p_\mathrm{z}}\right) = A_\mathrm{p}\left(1 + \frac{1}{A_\mathrm{p}}\int_{p_\mathrm{z}}^{p_\mathrm{j}} \frac{dA_\mathrm{e}}{dp_\mathrm{j}}\right) \qquad (1)$$

Here, we have assumed a temperature equal to the reference value (23 °C for NIST). A_p is the piston area at pressure p. $A_{\mathrm{e},p_\mathrm{j}}$ and $A_{\mathrm{e},p_\mathrm{z}}$ are the effective areas at the operating jacket pressure and at the jacket pressure of gap closure, p_z. Only the difference in area is needed; the relative difference in areas is typically 5x10^{-4} or less, which can be easily measured with a "reference" piston gauge having a relative uncertainty of 0.1 %. The reference piston gauge must have ppm resolution and repeatability. In the traditional HW method, A_e is assumed to vary linearly with p_j. Measurements on CCPG-2481 show that quadratic or higher terms are needed on A_e vs. p_j. In the terminology of the HW method, $d = -\frac{1}{A_\mathrm{p}}\frac{dA_\mathrm{e}}{dp_\mathrm{j}} = d_0 + d_1 p_\mathrm{j}$

$$A_\mathrm{e} = A_{\mathrm{p}0}\left(1 + bp\right)\left[1 + d_0\left(p_\mathrm{z} - p_\mathrm{j}\right) + \frac{d_1}{2}\left(p_\mathrm{z}^2 - p_\mathrm{j}^2\right)\right] \qquad (2)$$

Crossfloat measurements of CCPG-2481 against the reference gauge are used to measure the effective area as a function of p_j for 10 equally spaced operating pressures from 42 MPa to 290 MPa. For each p, these are done from p_j = 0 to the highest jacket pressure for which there is no operational degradation of the piston gauge. That limit on p_j is always less than p_z, and is indicated by difficulty in establishing pressure equilibrium with the reference gauge. If d_1 is zero, then eq. (2) collapses to the traditional HW method.

The gap closure pressure, p_z, is determined by measuring the fall rate (v) of the piston with the gauge isolated from the hydraulic system. For each p, v is measured from p_j = 0 to a value approaching p_z. In the traditional HW method, p_j is fit to a linear function of $v^{1/3}$, since the gap would be proportional to p_j in that case. The intercept of the function at $v = 0$ is defined as p_z. Measurements on CCPG-2481 show that higher order terms are needed to fit the p_j vs. $v^{1/3}$ data. As the jacket pressure increases, the gap does not close linearly with p_j. This means the cylinder becomes stiffer at high p_j. We have fit the fall rate data to quadratic and cubic terms of $v^{1/3}$ to reduce the error in p_z.

3. Description CCPG-2481 and measurements

The cylinder and piston of CCPG-2481 are made of tungsten carbide (piston: 6 % cobalt binder; cylinder: 10 % cobalt binder). The Young's modulus and Poisson's ratio of a billet of the same casting as the piston were measured using resonant ultrasound. The piston is nominally 3.27 mm in diameter and 56.6 mm in length. At the reference position in the cylinder, 9.0 mm of the piston extends above the upper horizontal surface of the cylinder, and the lower 47.6 mm of the piston forms the gap. The cylinder is 71.0 mm in length. The upper 58.3 mm has a diameter of 25.4 mm and the lower 12.7 mm has a diameter of 35.5 mm. The jacket pressure is applied from 7 mm below the upper surface to 49 mm below the upper surface. The system pressure is sealed with a 4.5 mm ID O-ring on the lower surface of the cylinder. The upward forces produced by the pressure are balanced by a retaining nut on the upper surface of the cylinder.

The jacket pressure was supplied by a secondary standard piston gauge. Equilibrium pressure was measured with a reference piston gauge traceable to the NIST hydraulic pressure scale. Pressure equilibrium was established using the fall rate method. The hydraulic fluid was di(2-ethylhexyl) sebacate. Proximity sensors were used to measure piston height. All signals were sampled by a computer. The piston was rotated at 20 rpm to 30 rpm, and the position (z) was held to within +/- 1 mm from the reference position. For the fall rate measurements, the piston position was recorded symmetrically from above to below the reference position as a function of time. A linear fit of the *z* vs. time data gave *v*.

4. Results and Conclusions

The results of the fall rate measurements for selected pressures are shown in Fig. 1. Intercepts on the y-axis (p_z) are the extrapolations of the fitted functions. The curvature of the data is clearly seen for p = 124 MPa and above. Linear fits always produce lower values of p_z; the linear fit at p = 290 MPa is plotted and p_z is 31.2 MPa less than for the higher order fit. A linear fit with a restricted range of p_j would produce an even lower p_z. At this pressure, a 31 MPa error in p_z would cause a relative error of 32x10^{-6} in effective area.

The results of the crossfloat measurements for selected pressures are shown if Fig. 2. The data is fit to quadratic functions in p_j and the fits are extrapolated to $p_j = p_z$. The curvature at high p_j is clearly evident. A linear fit at p = 290 MPa produces a relative standard error of the residuals of 14.5x10^{-6}, which would translate to an equivalent error in effective area. The uncertainty of the HW model was reduced by operating to high p_j (over 80 % of p_z for most pressures), thus reducing the region of extrapolation.

When the measurements described above are combined with dimensional characterization of the piston diameter and piston distortion calculations, the effective area using the higher order HW model agrees with the existing NIST pressure scale to within -6.6x10^{-6} to 16.3x10^{-6} on a relative basis. Using the linear functions of the traditional HW model, the relative agreement to the NIST pressure scale ranges from -30x10^{-6} to 39x10^{-6}, and the dependence of A_e with p_j is inconsistent with the measured data.

The relative standard uncertainties of the various components contributing to the combined standard uncertainty are shown in Fig. 3. The uncertainty from the dimensional characterization (A_{p0}) is the largest component at about 14x10^{-6}. The uncertainty due to estimating the piston distortion (b) is pressure dependent. The components related to the higher order HW model (p_z and dA_e/dp_j) are 7x10^{-6} or less. The uncertainty of these parameters is independent of the jacket pressure. The relative combined standard uncertainty in the effective area of CCPG-2481 ranges from 16.0x10^{-6} to 17.6x10^{-6}, hence the agreement to the present NIST pressure scale is within the standard uncertainty.

Controlled clearance piston gauges that are characterized using the HW model should be designed and tested to jacket pressures as close as possible to the jacket closure pressure. This allows mapping out the possible non-linear behavior of the gauge, and reducing the amount of extrapolation required to estimate the model parameters. CCPG-2481 becomes stiffer as p_j is increased, which requires including quadratic (or higher) terms in the HW model.

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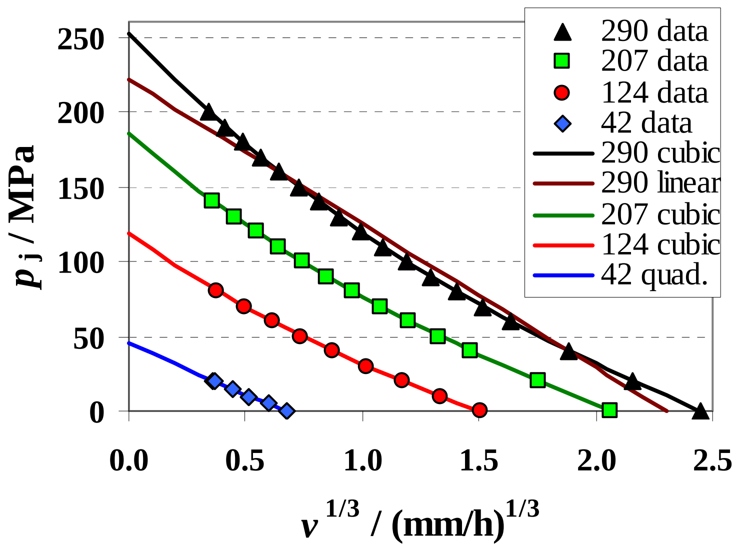

Fig. 1. Fall rate of CCPG-2481 plotted as p_j vs $v^{1/3}$. Fits of data extrapolated to $v = 0$ give p_z.

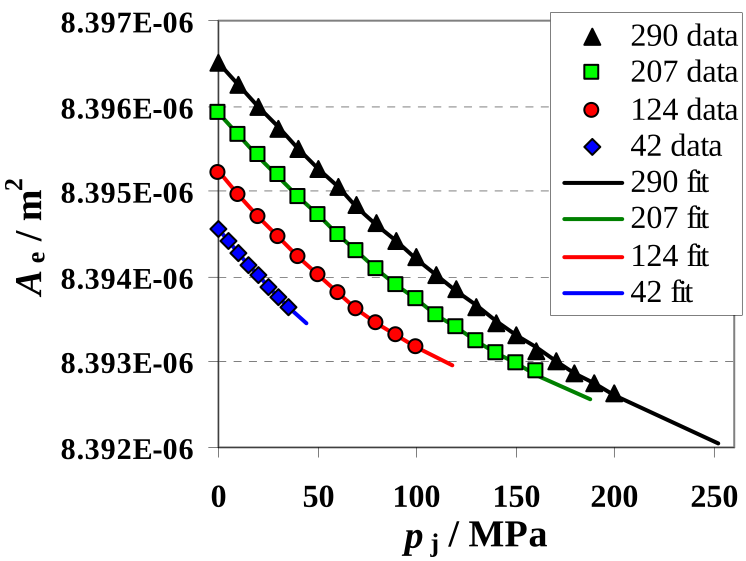

Fig. 2. Effective area of CCPG-2481 from crossfloat to NIST piston gauges. Fits are extrapolated to $p_j = p_z$.

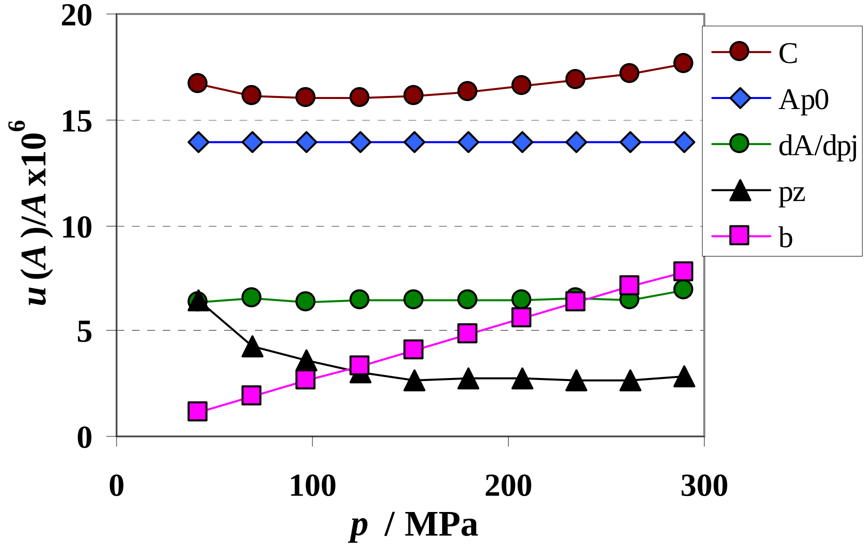

Fig. 3. Relative standard uncertainty components of CCPG-2481. C: combined; A_{p0}: piston at zero gauge pressure; dA/dp_j: cylinder distortion from jacket pressure; p_z: jacket closure pressure; b: piston distortion.