

# Weak-value thermostat with 0.2 mK precision

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A new laser-based thermostat sensitive to 0.2 mK at room temperature is reported. The method utilizes a fluid-filled prism and interferometric weak-value amplification to sense nanoradian deviations of a laser beam: due to the high thermo-optic coefficient of the fluid (colorless fluorocarbon), the deviation angle through the prism is sensitive to temperature. We estimate the daily stability of our device to be 0.2 mK, which is limited by drifts in the apparatus, and the narrow 20 mK capture range is the price paid for the weak measurement.

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Resistive thermometry is the workhorse of room-temperature measurement, but its submillikelvin precision is subject to drifts, resistive heating, and a considerable investment (resistance bridge). Optical techniques based on interferometry offer a promising alternative for higher-precision thermometry. For example, displacement interferometry measuring the expansion of a liquid in a capillary has demonstrated precision at the 2  $\mu$ K-level [1], while the difference in frequency between transverse modes of a birefringent crystal microresonator has shown submicrokelvin sensitivity [2]. In addition to these techniques, it is easy to imagine ultrahigh-precision room-temperature thermometers based the thermo-optic property ( $dn/dT$ ) of a material. For example, an ethanol cell in one arm of a heterodyne interferometer in the visible, or a laser locked to a solid silicon Fabry–Perot cavity in the near-infrared. The problem with these optical techniques is that they are not as easy as plugging in a thermistor.

A simpler alternative, perhaps, is the deviation angle  $\delta$  of a laser beam passing through a prism, which depends on the refractive index  $n$  and the prism apex angle  $A$ , and in the case of a minimum-deviation setup is expressed by

$$\frac{d\delta}{dn} = \frac{2 \sin(\frac{A}{2})}{\sqrt{1 - n^2 \sin^2(\frac{A}{2})}}. \quad (1)$$

Liquids such as hydrocarbons (for example, ethanol) or fluorocarbons (for example, index-matching fluid) typically have thermo-optic coefficients of  $(dn/dT) \approx 3 \times 10^{-4}/\text{K}$ , and with  $A = 60^\circ$ , a change of 1 mK would change the deviation angle through a fluid-filled prism by less than 1  $\mu$ rad. Detecting submicroradian deviations is not trivial—an angular deviation of  $\delta = 1 \mu$ rad when measured at a distance of  $L = 0.1$  m corresponds to only a  $\langle x \rangle = 100$  nm standard deflection of the beam. A split photodetector can give nanometer sensitivity in beam-position sensing (using ac-coupled techniques), but for a dc-type measurement such as temperature monitoring, technical noise sources are the main obstacle; for example, a precision beam-steering mount typically drifts by 1  $\mu$ rad/K. So unless something can be done to amplify the deflection of the beam (or to suppress technical noise), the resolution of a fluid-filled prism in detecting changes in temperature would be on the order of several millikelvin. (Other approaches to increasing the

resolution, such as increasing the apex angle and/or cascading several prisms together, could practically give a factor of four improvement.)

Amplification in a weak-value measurement [3,4] occurs when the preselected and postselected states of the measurement system are nearly orthogonal. When orthogonality is achieved with a Sagnac interferometer, the setup becomes highly sensitive to transverse deviations of the beam: instead of a small angular deviation causing a standard deflection of  $\langle x \rangle = \delta \cdot L$  at the detector, a weak-value measurement gives an amplified deflection of  $\langle x \rangle_W = 4\delta k_0 \sigma^2 / \phi$ , where  $k_0$  is the wavenumber,  $\sigma$  is the beam radius, and  $\phi$  is the phase difference between the two paths of the interferometer. Using interferometric weak-value amplification, a system sensitive to subpicoradian deviations of a target mirror was reported where the amplification factor  $\langle x \rangle_W / \langle x \rangle$  exceeded 100 [5]. A modified approach utilized a prism as a dispersive element to detect 10 Hz dithering of the laser frequency, with a reported sensitivity of  $4 \times 10^{-10}$  [6]: a prism in the interferometer meant that changes in the frequency of the laser resulted in beam deflection. In this Letter, we build on the ideas of [6] and report on a weak-value temperature sensor, where a fluid-filled prism is integrated with interferometric weak-value amplification to measure changes in temperature. We investigate the stability and sensitivity of the device, and find that its limited measurement range makes the device best suited as a thermostat.

Our optical setup is shown in Fig. 1: about 1 mW of polarized HeNe laser power in a 1.5 mm diameter beam entered a Sagnac interferometer, configured with three main elements: Jamin beamsplitter, back-to-back fluid-filled prism, and folding prism. The Jamin beamsplitter is essentially a block of glass with half its front surface coated with 50% reflectivity and the opposite half of its back surface coated with 99% reflectivity; we are not aware of a commercially available Jamin beamsplitter offering adequate separation between the reflected beams, and our optic is custom-made (the most expensive component in the setup). The separation between the reflected beams was 2 cm; the distance between the Jamin beamsplitter and the folding prism was 30 cm, and a quadrant photodetector measured the  $x$ -position of the beam exiting the interferometer at the “dark port.” The amplification factor was set by a small tilt of the Jamin beamsplitter (out-of-plane phase difference  $\phi$  between

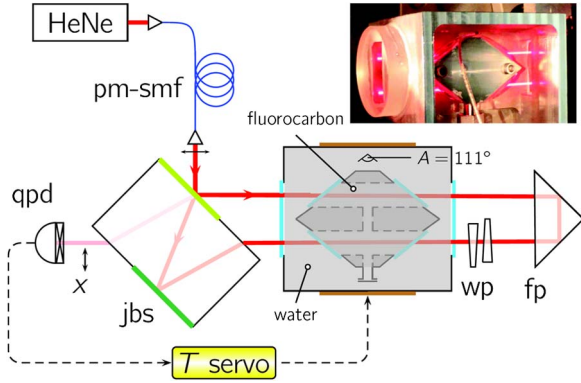


Fig. 1. (Color online) Weak-value thermostat setup: Jamin beamsplitter (jbs), wedge prisms (wp), folding prism (fp), quadrant photodetector (qpd), polarization-maintaining fiber (pm-smf). The ring effectively passes through two fluid-filled prisms.

the interferometer paths), and the weak-value was realized by rotating two wedge prisms inside the interferometer to optimize beam overlap (dark port irradiance minimized). The fluid-filled prism was machined in a single piece of stainless steel with Brewster windows glued onto all four transmission ports. In order to obtain a straight pass through the prism, we submerged it in an aluminum bath of distilled water and chose a water index-matching fluid (colorless fluorocarbon,  $n = 1.33$ ) to fill its interior; the thermo-optic coefficients of distilled water and the index-matching fluid are  $(dn/dT) = 1 \times 10^{-4}/K$  and  $(dn/dT) = 3.2 \times 10^{-4}/K$ , respectively. The salient feature of this design is that only three types of angular deviations are amplified in the interferometer: (a) changes in the deviation angle through the prism, which is the measurand; (b) pitch pivots of the folding prism, which are out-of-plane; or (c) rotation of the wedge prisms, which would be very small, since the wedge is only  $30'$  and the rotation is locked with a setscrew. Other forms of technical noise such as laser beam pointing, tip/tilt of the Jamin beamsplitter, positional instability of the quadrant photodetector, or rotation of the prism are not amplified.

The entire setup was constructed on an optical bench using off-the-shelf optical mounts, and so stability of the design was our first point of interest. We investigated this stability by removing the water-bath and fluid-filled prism from the interferometer and monitoring the position of the beam on the quadrant photodetector over one week. This approach gives a fair estimate of stability without having to go to the lengths of minimizing temperature and refractive-index fluctuations (as would be necessary if the fluid-filled prism were present). Stability data are shown in Fig. 2(a) for 10 min averaging. The daily standard deviation on the weak-value data is  $1.2 \mu\text{m}$ , but hourly fluctuations in beam position can be as high as  $3 \mu\text{m}$ . The stability of the  $y$ -position (out-of-plane) was very similar. As a point of reference, we also plot the drift in beam position when the Jamin beamsplitter was replaced by a mirror in a low-drift mount (a standard deflection configuration). In this case, there was no weak-value amplification: a single beam was reflected from the mirror, traveled through the wedge prisms and folding prism, and was then received at the quadrant

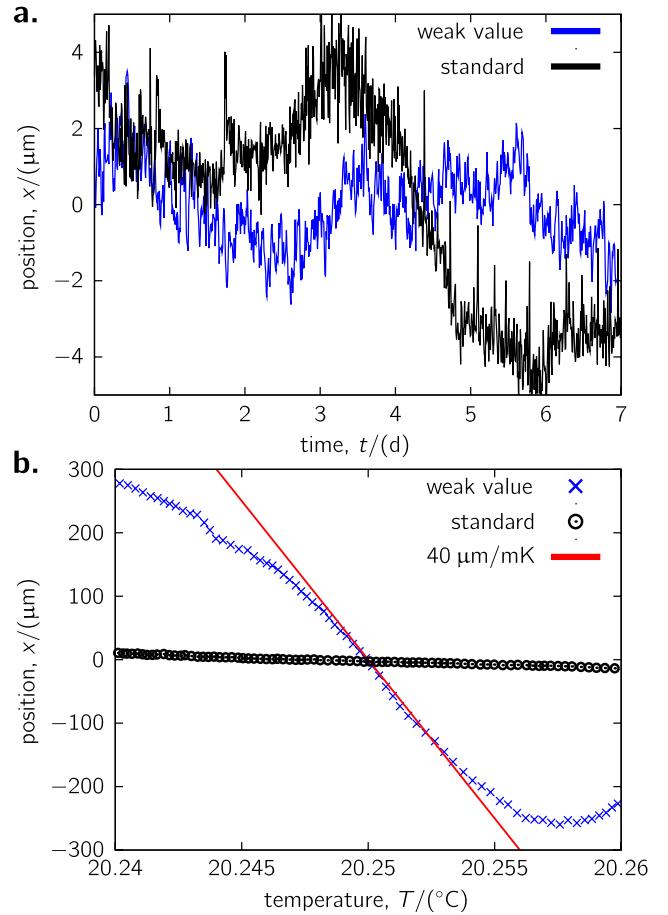


Fig. 2. (Color online) (a) Stability and (b) sensitivity for weak-value and standard deflection configurations.

photodetector. Note that the weak-value and standard deflection setups have a similar technical noise floor, but the weak measurement has an amplification in sensitivity (reported below) without amplification of technical noise. We also briefly note that we have tried to cancel out beam fluctuation by rotating a half-wave plate at the input to the interferometer: the idea was to subtract beam fluctuations at low weak-value amplification factors from those at high. However, using manual rotation, the repeatability of this approach was only about  $10 \mu\text{m}$ , and a motorized rotation stage is something we will consider in future work.

The second point of interest is the sensitivity of the device to changes in temperature. To evaluate sensitivity we slowly varied the temperature of the water-bath and fluid-filled prism and measured the corresponding changes in beam position at the quadrant photodetector. Sensitivity data are shown in Fig. 2(b). Again, we replaced the Jamin beamsplitter with a mirror to get an experimental standard-deflection sensitivity of  $1 \mu\text{m}/\text{mK}$ , and this value is consistent with the  $1.1 \mu\text{m}/\text{mK}$  calculated from theory. The slope of the weak-value deflection data is  $40 \mu\text{m}/\text{mK}$ , and the ratio of the sensitivities gives an estimate of the weak-value amplification factor  $\langle x \rangle_W / \langle x \rangle \approx 40$ . Others have reported amplification factors more than twice as large [5–7], and we have observed similarly high amplification factors when only a hollow prism (with its internal gas pressure varied) was placed

in the interferometer; however, our experience with the thermostat setup of Fig. 1 indicates that higher amplification factors are difficult to achieve when the beams must travel through much liquid and glass. That being said, when the  $40 \mu\text{m}/\text{mK}$  sensitivity of the weak-value measurement is compared with its  $2 \mu\text{m}$  type daily drift, it suggests a practical resolution on the order of  $50 \mu\text{K}$ . On a less positive note, however, the amplification in sensitivity comes at the cost of a very small measurement range: as can be seen in Fig. 2(b), the weak-value thermostat has a high-sensitivity range of less than  $5 \text{ mK}$ , though its capture range is about four times larger. This is expected since it is a general characteristic of weak-value measurements that high amplification is achieved only over a narrow measurement range. Note that if utilized as a thermostat, the setpoint (the center of the capture range) of the device is not fixed since the weak-value could be nulled at the desired temperature by rotating the wedge prisms.

We performed a proof-of-principle experiment using the device as a thermostat: a foil heater on the outside of the water-bath was servo-controlled to hold the weak-value beam position at a constant, and during this servo-control the temperature of the water was measured with four thermistors. Before this experiment the thermistors were placed in a block of copper that sat in an envelope thermally controlled to  $1 \text{ mK}$ , and we found that the largest drift of any thermistor relative to another was  $<70 \mu\text{K}$  over one week. Based on this test we believe the thermistor readings are reliable to well within  $0.1 \text{ mK}$  per day. The performance of the weak-value thermostat at room temperature is shown in Fig. 3, where the beam position on the quadrant photodetector is plotted alongside the temperature read by one thermistor for one day. The two-sigma on the temperature data is  $0.15 \text{ mK}$ , which is only three times larger than the resolution inferred from the stability and sensitivity data of Fig. 2.

To conclude, we have investigated the performance of a weak-value thermostat based on a fluid-filled prism, and the device has shown a daily setpoint precision at the  $0.2 \text{ mK}$  level when compared to several thermistors at room temperature. The device does not offer the microkelvin sensitivity of other optical techniques, but it is not as complex either. A temperature sensor based on an optical cavity, for example, requires difficult matching of the beam into the cavity, whereas the Sagnac interferometer is easy to set up (and provides good stability);

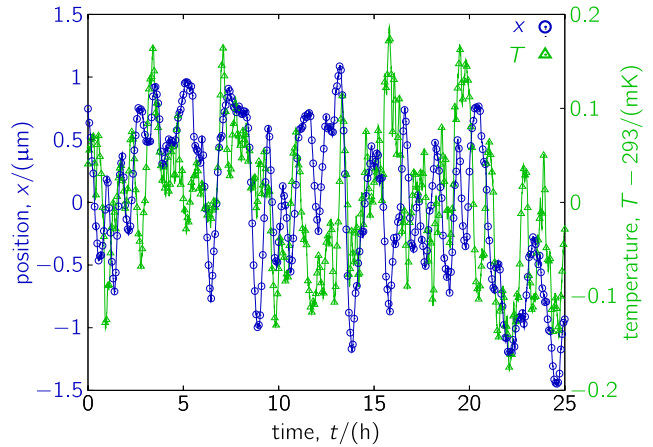


Fig. 3. (Color online) Performance of the weak-value thermostat when heaters were servo-controlled to keep beam position constant.

further, the cavity approach requires modulation of the laser source and demodulation of a locking signal, along with a fast servo to maintain lock. By contrast, the unmodulated HeNe laser with a quadrant detector we employ is a great simplification. However, the submillikelvin sensitivity we achieve does come at the cost of a narrow capture range ( $20 \text{ mK}$ ). The design we have presented is a working proof-of-principle, but future work should avoid materials that would contaminate the fluids (that is, hermetic fused-quartz cells would be preferable to metal-glass-glue), and a more monolithic design should also be considered to minimize beam drift.

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