CODATA Recommended Values of the Fundamental Physical Constants: 2014*

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This paper gives the 2014 self-consistent set of values of the constants and conversion factors of physics and chemistry recommended by the Committee on Data for Science and Technology (CODATA). These values are based on a least-squares adjustment that takes into account all data available up to 31 December 2014. Details of the data selection and methodology of the adjustment are described. The recommended values may also be found at http://physics.nist.gov/ constants. © 2016 AIP Publishing LLC for the National Institute of Standards and Technology. [http://dx.doi.org/10.1063/1.4954402]

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I. Introduction

This report describes work carried out under the auspices of the Task Group on Fundamental Constants, one of several task groups of the Committee on Data for Science and Technology (CODATA) founded in 1966 as an interdisciplinary committee of the International Council for Science (ICSU). It gives a detailed account of the 2014 CODATA multivariate least-squares adjustment of the values of the constants as well as the resulting 2014 set of over 300 self-consistent recommended values. The cutoff date for new data to be considered for possible inclusion in the 2014 adjustment was at the close of 31 December 2014, and the new set of values first became available on 25 June 2015 at http://physics.nist.gov/ constants, part of the website of the Fundamental Constants Data Center of the National Institute of Standards and Technology (NIST), Gaithersburg, Maryland, USA.

A. Background

The compilation of a carefully reviewed set of values of the fundamental constants of physics and chemistry arguably began over 85 years ago with the paper of Birge (1929). In 1969, 40 years after the publication of Birge’s paper, the CODATA Task Group on Fundamental Constants was established for the following purpose: to periodically provide the scientific and technological communities with a self-consistent set of internationally recommended values of the basic constants and conversion factors of physics and chemistry based on all the data available at a given point in time. The Task Group first met this responsibility with its 1973 multivariate least-squares adjustment of the values of the constants (Cohen and Taylor, 1973), which was followed 13 years later by the 1986 adjustment (Cohen and Taylor, 1987). Starting with its third adjustment in 1998 the Task Group has carried them out every 4 years; if the 2014 adjustment is assumed. This includes, as a relative standard uncertainty, denoted $u_r$, As an aid to the reader, included near the end of this report is a comprehensive list of symbols and abbreviations.

Because of its importance, we do once again state that, as a working principle, the validity of the physical theory underlying the 2014 adjustment is assumed. This includes, as in previous adjustments, special and general relativity, quantum mechanics, quantum electrodynamics (QED), the standard model of particle physics, including CPT invariance, and for all practical purposes the exactness of the relations $K_f = 2e/h$ and $K_K = h/e^2$, where $K_f$ and $K_K$ are the Josephson and von Klitzing constants, respectively, and $e$ is the elementary charge and $h$ is the Planck constant.

There continues to be no observed time variation of the values of the constants relevant to the data used in adjustments carried out in our current era. Indeed, a recent summary based on frequency ratio measurements of various transitions in different atomic systems carried out over a number of years in several different laboratories gives $-0.7(2.1) \times 10^{-17}$ per year as the constraint on the fine-structure constant $\alpha$ and $-0.2(1.1) \times 10^{-16}$ per year for the proton-to-electron mass ratio $m_p/m_e$ (Godun et al., 2014).

In general, a result considered for possible inclusion in a CODATA adjustment is identified by the institution where the work was primarily carried out and by the last two digits of the year in which it was published in an archival journal. Even if a result is labeled with a “15” identifier, it can be safely assumed that it was available by the 31 December 2014 closing date for new data. A new result was considered to have met this date if published, or if the Task Group received a preprint describing the work by that date and it had already been, or was


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about to be, submitted for publication. However, this closing
date does not apply to clarifying information requested from
authors. The name of an institution is always given in full
together with its abbreviation when first used, but for the
convenience of the reader the abbreviations and full institutional
names are also included in the aforementioned comprehensive
list of symbols and abbreviations near the end of this report.

B. Highlights of the CODATA 2014 adjustment

We summarize here the most significant advances made, or
lack thereof, in our knowledge of the values of the fundamental
constants in the past 4 years and, where appropriate, their
impact. The multivariate least-squares methodology employed
in the four previous adjustments is employed in the 2014
adjustment but in this case with \( N = 141 \) items of input data,
\( M = 74 \) variables or unknowns, and \( v = N - M = 67 \) degrees
of freedom. The chi square statistic is \( \chi^2 = 50.4 \) with probability
\( p(50.4/67) = 0.93 \) and the Birge ratio is \( \chi_B = \sqrt{50.4/67} = 0.87 \).
This adjustment includes data for the Newtonian constant of
gravitation \( G \), although it is independent of the other constants.

1. Planck constant \( h \), elementary charge \( e \),
Boltzmann constant \( k \), Avogadro constant \( N_A \),
and the redefinition of the SI

It is planned that at its meeting in the fall of 2018, the 26th
General Conference on Weights and Measures (CGPM) will
adopt a resolution to revise the International System of Units
(SI). This new SI, as it is sometimes called, will be defined by
assigning exact values to the following seven defining constants:
the ground-state hyperfine-splitting frequency of the \( ^{133}\text{Cs} \) atom
\( \Delta \nu_{\text{Cs}} \), the speed of light in vacuum \( c \), the Planck constant \( h \),
the elementary charge \( e \), the Boltzmann constant \( k \), the Avogadro
constant \( N_A \), and the luminous efficacy of monochromatic
radiation of frequency 540 THz, \( \text{K}_{\text{cd}} \). As a result of the
significant advances made since CODATA-10 in watt-balance
measurements of \( h \), x-ray-crystal-density (XRCD) measure-
ments of \( N_A \) using silicon spheres composed of highly enriched
silicon, and acoustic-gas-thermometry (AGT) measurements
of the molar gas constant, the relative standard uncertainties of
the four constants \( h, e, k, \) and \( N_A \) have been reduced (respectively, in
parts in \( 10^9 \)) from 4.4, 2.2, 91, and 4.4 in CODATA-10 to 1.2,
0.61, 57, and 1.2, in CODATA-14. (The defining constants \( \Delta \nu_{\text{Cs}}, c, \) and \( \text{K}_{\text{cd}} \) will retain their present values.)

This is a truly major development, because these uncer-
tainties are now sufficiently small that the adoption of the new
SI by the 26th CGPM is expected. It has been made possible to
a large extent by the resolution of the disagreement between
different watt-balance measurements of \( h \) and the disagree-
ment of the value of \( h \) inferred from the XRCD value of \( N_A \)
with one of the watt-balance values. These disagreements led
the Task Group to increase the initial assigned uncertainties of
the 2010 data that contributed to the determination of \( h \) by
a factor of 2. Further, the reduction in the relative uncertainty
of \( k \) from \( 9.1 \times 10^{-7} \) to \( 5.7 \times 10^{-7} \) is in large part a consequence
of three new AGT determinations of the molar gas constant
with relative uncertainties (in parts in \( 10^9 \)) of 0.9, 1.0, and 1.4.
The significant reductions in the uncertainties of \( h, e, k, \) and \( N_A \)
have also led to the reduction of the uncertainties of many other
constants and conversion factors.

The values of the constants to be adopted by the CGPM for
the redefinition will be based on a special least-squares adjustment
carried out by the Task Group during the summer of 2017. Data
for this adjustment must be described in a paper that has been
published or accepted for publication by 1 July 2017.

2. Relative atomic mass of the electron \( A_e(e) \)

The relative standard uncertainty of the 2014 recommended
value of \( A_e(e) \) is \( 2.9 \times 10^{-11} \), nearly 14 times smaller than that
of the 2010 recommended value. It is based on extremely accurate
measurements, using a specially designed triple Penning trap, of
the ratio of the electron spin-precession (or spin-flip) frequency
in hydrogenic carbon and silicon ions to the cyclotron frequency
of the ions, together with the theory of the electron bound-state
\( g \)-factor in the ions. The uncertainties of the measurements
are so small that the data used to obtain the CODATA-10 value of
\( A_e(e) \) are no longer competitive and are excluded from the 2014
adjustment. Thus, there is no discussion of antiprotonic helium
in this report. The new value of \( A_e(e) \) will eliminate a potentially
significant source of uncertainty in obtaining the fine-structure
constant from anticipated high-accuracy atom-recoil measure-
ments of \( h/m \) for an atom of mass \( m \).

3. Proton magnetic moment in units of the nuclear
magneton \( \mu_p/\mu_N \)

The CODATA-10 recommended value of the magnetic
moment of the proton in nuclear magnetons \( \mu_p/\mu_N \), where
\( \mu_N = e \hbar/2m_p \) and \( m_p \) is the proton mass, has a relative standard
uncertainty of \( 8.2 \times 10^{-9} \) and is calculated from other measured
constants including the electron to proton mass ratio. However,
because of the development of a unique double Penning trap
similar to the triple Penning trap mentioned in the previous
section, for the first time a value of \( \mu_p/\mu_N \) from direct
measurements of the spin-flip and cyclotron frequencies of
a single proton with an uncertainty of \( 3.3 \times 10^{-9} \) has become
available. As a consequence, the uncertainty of the 2014
recommended value is \( 3.0 \times 10^{-9} \), which is 2.7 times smaller
than that of the 2010 value, and similar reductions in the
uncertainties of other constants that depend on the \( \mu_p/\mu_N \) result.

4. Fine-structure constant \( \alpha \)

Improved numerical calculations of the 8th- and 10th-order
mass-independent coefficients of the theoretical expression
for the electron magnetic-moment anomaly \( a_e \) have allowed
full advantage to be taken of the \( 2.4 \times 10^{-10} \) relative standard
uncertainty of the experimental value of \( a_e \) for the determination
of the fine-structure constant; the relative uncertainty of the 2014
recommended value of \( \alpha \) is \( 2.3 \times 10^{-10} \) compared with
\( 3.2 \times 10^{-10} \) for the CODATA-10 value. However, because of
the somewhat unexpected large size of the 10th-order co-
efficient, the 2014 recommended value of \( \alpha \) is fractionally
smaller than the CODATA-10 value by 4.7 parts in \( 10^{10} \).
5. Relative atomic masses

A new atomic mass evaluation, called AME2012, was completed and published by the Atomic Mass Data Center (now transferred from France to China), and its recommended values are generally used for the various relative atomic masses required for the 2014 adjustment, including that for the neutron. Because AME2012 is a self-consistent evaluation based on data included in CODATA-10, those data are neither discussed nor included in CODATA-14. However, two new, highly precise pairs of cyclotron frequency ratios relevant to the determination of the masses of the deuteron, triton, and helium (nucleus of the $^3$He atom) were reported after the completion of AME2012 and are included in this adjustment. Yet, because the values of the relative atomic mass of $^3$He implied by the relevant ratio in each pair disagree, the initial uncertainty of each of these ratios is multiplied by 2.8 to reduce the inconsistency to an acceptable level.

6. Newtonian constant of gravitation $G$

Three new values of $G$ obtained by different methods have become available for CODATA-14 with relative standard uncertainties of $1.9 \times 10^{-5}$, $2.4 \times 10^{-5}$, and $15 \times 10^{-5}$, respectively, but have not resolved the considerable disagreements that have existed among the measurements of $G$ for the past 20 years. These inconsistencies led the Task Group to apply an expansion factor of 14 to the initial uncertainty of each of the 11 values available for the 2010 adjustment and to adopt their weighted mean with its relative uncertainty of $12 \times 10^{-5}$ as the 2010 recommended value. The expansion factor 14 was chosen so that the smallest and largest values would differ from the recommended value by about twice its uncertainty. For the 2014 adjustment the Task Group has decided that its usual practice in such cases, which is to choose an expansion factor that reduces the normalized residual of each datum to less than 2, should be followed instead. Thus an expansion factor of 6.3 is chosen and the weighted mean of the 14 values with its relative uncertainty $4.7 \times 10^{-5}$ is adopted as the 2014 recommended value. Because of the three new values of $G$, the 2014 recommended value is larger than the 2010 value by 3.6 parts in $10^5$.

7. Proton radius $r_p$ and theory of the muon magnetic-moment anomaly $a_\mu$

The very precise value of the root-mean-square charge radius of the proton $r_p$ obtained from spectroscopic measurements of a Lamb-shift transition frequency in the muonic hydrogen atom $\mu$p was omitted from CODATA-10 because of its significant disagreement with the value from electron-proton elastic scattering and from spectroscopic measurements of hydrogen and deuterium. Although the originally measured Lamb-shift frequency has been reevaluated, the result from a second frequency that gives a value of $r_p$ consistent with the first has been reported, and improvements were made to the theory required to extract $r_p$ from the Lamb-shift frequencies, the disagreement persists. The Task group has, therefore, decided to omit the muonic hydrogen result for $r_p$ from the 2014 adjustment.

Similarly, because the value of the muon magnetic-moment anomaly $a_\mu$(th) predicted by the theoretical expression for the anomaly significantly disagreed with the value obtained from a seminal experiment at Brookhaven National Laboratory, USA, the theory was omitted from CODATA-10. Even though much effort has been devoted in the past 4 years to improving the theory, the disagreement and concerns about the theory remain. Thus the Task Group has also decided not to employ the theory of $a_\mu$ in the 2014 adjustment.

C. Outline of the paper

Some constants that have exact values in the International System of Units (SI) (BIPM, 2006), which is the unit system used in all CODATA adjustments, are recalled in Sec. II. Sections III through XII discuss the input data with an emphasis on the new results that have become available during the past 4 years. As discussed in Appendix E of CODATA-98, in a least-squares analysis of the values of the constants, the numerical data, both experimental and theoretical, also called observational data or input data, are expressed as functions of a set of independent variables or unknowns called adjusted constants. The functions themselves are called observational equations, and the least-squares methodology yields best estimates of the adjusted constants in the least-squares sense. Basically, the methodology provides the best estimate of each adjusted constant by automatically taking into account all possible ways its value can be determined from the input data. The best values of other constants are calculated from the best values of the adjusted constants.

The analysis of the input data is discussed in Sec. XIII. It is carried out by directly comparing measured values of the same quantity, by comparing measured values of different quantities through inferred values of $\alpha$, $h$, and $k$, and by carrying out least-squares calculations. These investigations are the basis for the selection of the final input data used to determine the adjusted constants, and hence the entire 2014 CODATA set of recommended values.

Section XIV provides, in several tables, the set of over 300 CODATA-14 recommended values of the basic constants and conversion factors of physics and chemistry, including the covariance matrix of a selected group of constants. The report concludes with Sec. XV, which includes a comparison of a representative subset of 2014 recommended values with their 2010 counterparts, comments on some of the implications of CODATA-14 for metrology and physics, and some suggestions for future work, both experimental and theoretical, that could advance our knowledge of the values of the fundamental constants.

II. Special Quantities and Units

Table I gives the values of a number of exactly known constants of interest. The speed of light in vacuum $c$ is exact as a consequence of the definition of the meter in the SI and the magnetic constant (vacuum permeability) $\mu_0$ is exact because of the SI definition of the amper (BIPM, 2006). Thus the electric constant (vacuum permittivity) $\varepsilon_0 = 1/\mu_0 c^2$ is also exact. The molar mass of carbon 12, $M(^{12}C)$, is exact as a consequence of the SI definition of the mole, as is the molar
mass constant \( M_e = M(^{12}\text{C})/12 \). By definition, the relative atomic mass of the carbon 12 atom \( A(^{12}\text{C})=12 \) is exact. The quantities \( K_{J-90} \) and \( R_{K-90} \) are the exact, conventional values of the Josephson and von Klitzing constants adopted by the International Committee for Weights and Measures (CIPM) in 1989 for worldwide use starting 1 January 1990 for measurements of electrical quantities using the Josephson and quantum-Hall effects (BIPM, 2006). Quantities measured in terms of these conventional values are labeled with a subscript 90.

### III. Relative Atomic Masses

The relative atomic masses of some particles and ions are used in the least-squares adjustment. These values are extracted from measured atom and ion masses by calculating the effect of the bound-electron masses and the binding energies, as discussed in the following sections.

#### A. Relative atomic masses of atoms

Results from the periodic atomic mass evaluations (AMEs) carried out by the Atomic Mass Data Center (AMDC), Centre de Spectrométrie Nucléaire et de Spectrométrie de Masse (CSNSM), Orsay, France, have long been used as input data in CODATA adjustments. Indeed, results from AME2003, the most recent evaluation at the time, were employed in the 2006 and 2010 CODATA adjustments. In 2008 a memorandum between the Institute of Modern Physics, Chinese Academy of Sciences (IMP), in Lanzhou, PRC, and CSNSM was signed that initiated the transfer of the AMDC from CSNSM to IMP. The transfer was concluded in 2013 after the completion of AME2012, which superseded its immediate predecessor, AME2003. The results of the 2012 evaluation, which was a collaborative effort between IMP and CSNSM, are published (Audi et al., 2012; Wang et al., 2012) and are also available on the AMDC website at http://amdc.impanes.ac.cn/evaluation/data2012/ame.html.

The AME2012 relative atomic mass values of interest for the 2014 adjustment are given in Table II; additional digits were supplied in 2014 to the Task Group by M. Wang of the AMDC to reduce rounding errors. However, the AME2012 values for \( A(^{3}\text{He}) \) and \( A(^{3}\text{He}) \) from which the relative atomic masses of the deuteron \( \text{d} \) and helium \( \text{h} \) (nucleus of the \(^{3}\text{He} \)) atom can be obtained are not included. This is because the AME2012 value for \( A(^{3}\text{He}) \) is based to a large extent on preliminary data from the group of R. Van Dyck at the University of Washington (UWash), Seattle, Washington, USA, that have been superseded by recently reported final data (Zafonte and Van Dyck, 2015). Further, the AME2012 value for \( A(^{3}\text{He}) \) is partially based on very old UWash data that have been superseded by newer and much more accurate data given in the paper that reports the final \( A(^{3}\text{He}) \)-related data. These new UWash results are discussed below in Sec. IIIC together with new measurements related to the triton and helium from the group of E. Myers at Florida State University (FSU), Tallahassee, Florida, USA.

The covariances among the AME2012 values in Table II are taken from the file covariance.covar available at the AMDC website indicated above and are used as appropriate in our calculations. They are given in the form of correlation coefficients in Table XIX, Sec. XIII.

In the four previous CODATA adjustments, the recommended value of the relative atomic mass of the neutron \( A(n) \) was based on the wavelength of the 2.2 MeV \( \gamma \) ray emitted in the reaction \( n + p \rightarrow d + \gamma \) as measured in the 1990s. In the current adjustment the AME2012 value in Table II is taken as an input datum and \( A(n) \) as an adjusted constant, because the 2012 AME is an internally consistent evaluation that uses all available data relevant to the determination of \( A(n) \).

#### B. Relative atomic masses of ions and nuclei

The mass of an atom or ion is the sum of the nuclear mass and the masses of the electrons minus the mass equivalent of the binding energy of the electrons. To produce an ion \( X^{+} \) with net charge \( ne \), the energy needed to remove \( n \) electrons from the neutral atom is the sum of the electron ionization energies \( E_i(X^{+}) \):

\[
\Delta E_0(X^{+}) = \sum_{i=0}^{n-1} E_i(X^{+}) \tag{1}
\]

\( \Delta E_0(X^{+}) \) is the average of the electron ionization energies of the ion.

---

**TABLE I.** Some exact quantities relevant to the 2014 adjustment

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed of light in vacuum</td>
<td>( c )</td>
<td>299 792 458 m s(^{-1} )</td>
</tr>
<tr>
<td>Magnetic constant</td>
<td>( \mu_0 )</td>
<td>( 4\pi \times 10^{-7} \text{ N A}^{-2} = 12.566 370 614 \ldots \times 10^{-7} \text{ N A}^{-2} )</td>
</tr>
<tr>
<td>Electric constant</td>
<td>( \epsilon_0 )</td>
<td>( (\mu_0 c^2)^{-1} = 8.854 187 817 \ldots \times 10^{-12} \text{ F m}^{-1} )</td>
</tr>
<tr>
<td>Molar mass of ( ^{12}\text{C} )</td>
<td>( M(^{12}\text{C}) )</td>
<td>( 12 \times 10^{-3} \text{ kg mol}^{-1} )</td>
</tr>
<tr>
<td>Molar mass constant</td>
<td>( M_0 )</td>
<td>( M(^{12}\text{C})/12 = 10^{-3} \text{ kg mol}^{-1} )</td>
</tr>
<tr>
<td>Relative atomic mass of ( ^{12}\text{C} )</td>
<td>( A(^{12}\text{C}) )</td>
<td>12</td>
</tr>
<tr>
<td>Conventional value of Josephson constant</td>
<td>( K_{J-90} )</td>
<td>483 597.9 GHz V(^{-1} )</td>
</tr>
<tr>
<td>Conventional value of von Klitzing constant</td>
<td>( R_{K-90} )</td>
<td>25 812.807 ( \Omega )</td>
</tr>
</tbody>
</table>

---

**TABLE II.** Relative atomic masses used in the least-squares adjustment as given in the 2012 atomic mass evaluation and the defined value for \(^{13}\text{C} \)

<table>
<thead>
<tr>
<th>Atom</th>
<th>Relative atomic mass ( A_i(X) )</th>
<th>Relative standard uncertainty ( u_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{n} )</td>
<td>1.008 664 915 85(49)</td>
<td>4.9 \times 10^{-10}</td>
</tr>
<tr>
<td>(^1\text{H} )</td>
<td>1.007 825 032 231(93)</td>
<td>9.3 \times 10^{-11}</td>
</tr>
<tr>
<td>(^2\text{H} )</td>
<td>3.016 049 277(24)</td>
<td>7.9 \times 10^{-10}</td>
</tr>
<tr>
<td>(^4\text{He} )</td>
<td>4.002 603 254 130(63)</td>
<td>1.6 \times 10^{-10}</td>
</tr>
<tr>
<td>(^{12}\text{C} )</td>
<td>12</td>
<td>(exact)</td>
</tr>
<tr>
<td>( ^{32}\text{Si} )</td>
<td>27.976 926 534 65(44)</td>
<td>1.6 \times 10^{-11}</td>
</tr>
<tr>
<td>( ^{36}\text{Ar} )</td>
<td>35.967 545 105(29)</td>
<td>8.1 \times 10^{-10}</td>
</tr>
<tr>
<td>( ^{38}\text{Ar} )</td>
<td>37.962 732 112(21)</td>
<td>5.5 \times 10^{-9}</td>
</tr>
<tr>
<td>( ^{40}\text{Ar} )</td>
<td>39.962 383 1237(24)</td>
<td>6.0 \times 10^{-11}</td>
</tr>
<tr>
<td>( ^{85}\text{Rb} )</td>
<td>86.909 180 5319(65)</td>
<td>7.5 \times 10^{-11}</td>
</tr>
<tr>
<td>( ^{107}\text{Ag} )</td>
<td>106.905 0916(26)</td>
<td>2.4 \times 10^{-4}</td>
</tr>
<tr>
<td>( ^{109}\text{Ag} )</td>
<td>108.904 7553(14)</td>
<td>1.3 \times 10^{-8}</td>
</tr>
<tr>
<td>( ^{133}\text{Cs} )</td>
<td>132.905 451 9615(86)</td>
<td>6.5 \times 10^{-11}</td>
</tr>
</tbody>
</table>
For a neutral atom we have \( n = 0 \) and \( \Delta E_B(X^0) = 0 \); for a bare nucleus \( n = Z \). In the 2014 least-squares adjustment, we use the removal energies expressed in terms of wave numbers given by

\[
\begin{align*}
\Delta E_B(\text{H}^+) / \hbar c &= 1.0967877174307(10) \times 10^7 \text{ m}^{-1}, \\
\Delta E_B(\text{He}^+) / \hbar c &= 1.0971854390(13) \times 10^7 \text{ m}^{-1}, \\
\Delta E_B(\text{He}^{2+}) / \hbar c &= 6.3721954487(28) \times 10^7 \text{ m}^{-1}, \\
\Delta E_B(\text{C}^{6+}) / \hbar c &= 83.083962(72) \times 10^7 \text{ m}^{-1}, \\
\Delta E_B(\text{Si}^{13+}) / \hbar c &= 420.608(19) \times 10^7 \text{ m}^{-1}, \\
\end{align*}
\]

which follow from the data tabulated in Table III. In that table, the value for \( ^1\text{H} \) is from Jentschura et al. (2005), and the rest are from the NIST online Atomic Spectra Database (ASD, 2015), in which the value for \( ^3\text{H} \) is based on a calculation by Kotochigova (2006). In general, because of the relatively small size of the uncertainties of the ionization energies given in Table III, any correlations that might exist among them or with other data used in the CODATA-14 are unimportant. However, there is a significant covariance between the two carbon binding-energy values, because a large part of the uncertainty is due to common uncertainties in the lower ionization stages; this yields the correlation coefficient

\[
\sigma[E_B(\text{C}^{5+}) / \hbar c, E_B(\text{C}^{6+}) / \hbar c] = 0.999976. \tag{2}
\]

The relative atomic mass of an atom, its ions, the relative atomic mass of the electron, and the relative atomic mass equivalent of the binding energy of the removed electrons are related according to

\[
A_i(X) = A_i(X^{n+}) + nA_i(e) - \frac{\Delta E_B(X^{n+})}{m_a c^2}, \tag{3}
\]

where \( m_a = m(\text{C}^{12})/12 \) is the unified atomic mass constant. Equation (3) is the form of the observational equation for \( A_i(X) \) used in previous adjustments with \( A_i(X^{n+}) \) and \( A_i(e) \) taken as adjusted constants with the binding-energy term taken to be exact. However, because for \( ^{28}\text{Si} \) the binding-energy uncertainty is not negligible compared with the uncertainty of \( A_i(^{28}\text{Si}) \), we adopt the following new approach for treating binding energies in all calculations in which they are required. Since the binding energies are known most accurately in terms of their wave number equivalents, and since \( R_{\omega_a} = \alpha^2 m_a c^2 / 2 \hbar \) and \( m_a = A_i(e)m_a \), one can write

\[
\frac{\Delta E_B(X^{n+})}{m_a c^2} = \frac{\alpha^2 A_i(e)}{2R_{\omega_a}} \frac{\Delta E_B(X^{n+})}{\hbar c}. \tag{4}
\]

Thus, in the 2014 adjustment we replace the binding-energy term in Eq. (3) by Eq. (4) and take the binding energy \( \Delta E_B(X^{n+}) / \hbar c \) as both an input datum and an adjusted constant, thereby obtaining a new form of observational equation for \( A_i(X) \) expressed solely in terms of adjusted constants. Although this requires taking binding energies as input data rather than exactly known quantities, it allows all binding-energy uncertainties and covariances to be properly taken into account. This new form of observational equation is used for the AME2012 values of \( A_i(^{2}\text{H}) \), \( A_i(^{3}\text{H}) \), \( A_i(^{4}\text{He}) \), and \( A_i(^{28}\text{Si}) \), and the new way of treating binding energies is used in the observational equations for a number of frequency ratios; see Table XXIV, Sec. XIII.

### C. Relative atomic mass of the deuteron, triton, and helion

We consider here the recent data of the University of Washington and Florida State University groups mentioned above relevant to the determination of the relative atomic masses of the nuclei of the \( ^{2}\text{H} \) (deuteron D), \( ^{3}\text{H} \) (tritium T), and \( ^{3}\text{He} \) atoms, or deuteron d, triton t, and helion h, respectively. The data are cyclotron frequency ratios obtained in a Penning trap and it is these ratios that are used as input data in the adjustment to determine \( A_i(d) \), \( A_i(t) \), and \( A_i(h) \), which are taken as adjusted constants. These new results became available shortly before the 31 December 2014 closing date of the adjustment and were published in 2015.

The UWash group reports as the final values of the cyclotron frequency ratios \( d \) and \( h \) to \( ^{12}\text{C}^{6+} \) (Zafonte and Van Dyck, 2015)

\[
\frac{\omega_d(d)}{\omega_k(12^{6+})} = 0.992996654743(20) \quad [2.0 \times 10^{-11}], \tag{5}
\]

\[
\frac{\omega_h(h)}{\omega_k(12^{6+})} = 1.32636586219(19) \quad [1.4 \times 10^{-11}]. \tag{6}
\]

These ratios are correlated because of the image charge correction applied to each; based on the published uncertainty budgets and additional information provided by Van Dyck (2015), their correlation coefficient is
r[ωc(h)/ωc(12C6+), ωc(h)/ωc(3He+)] = 0.306. \hspace{1cm} (7)

The relative atomic masses follow from the relations

\[ \frac{ωc(d)}{ωc(12C6+)} = \frac{A_1(12C6^+)}{6A_1(d)}, \hspace{1cm} (8) \]

\[ \frac{ωc(h)}{ωc(12C6+)} = \frac{A_1(12C6^+)}{3A_1(h)}, \hspace{1cm} (9) \]

where

\[ A_1(12C6^+) = 12 - 6A_1(c) + \frac{ΔE_B(12C6^+)}{m_ec^2}, \hspace{1cm} (10) \]

which takes into account the definition \( A_1(12C) = 12 \).

An overview of the University of Washington Penning trap mass spectrometer (UW-PTMS), which was developed over several decades, is given by Zafonte and Van Dyck (2015); a discussion of the various experimental effects that can influence UW-PTMS cyclotron frequency measurements is given by Van Dyck, et al. (2006). The later paper also reports a preliminary value of \( A_1(2^H) \) based on the analysis of \( ωc(d)/ωc(12C6+) \) data obtained in three early data runs. The final result of the UW deuterium measurements given in Eq. (5) is based on 10 data runs, each of which yields one frequency ratio and lasted more than a month when the time required to check all experimental effects is included. Corrections for six significant experimental effects are applied to each of the 10 ratios before their weighted mean is calculated. The largest of these by far is that for image charge; its fractional magnitude is \(-245 \times 10^{-12}\) for each ratio. Each correction has an uncertainty, but since the \( 9.9 \times 10^{-12} \) relative standard uncertainty \( u_c \) of the image charge correction is the same for each ratio, it is omitted from the individual ratio uncertainties. Rather, Zafonte and Van Dyck (2015) take it into account by combining it with the uncertainty \( u_c = 17.4 \times 10^{-12} \) of the weighted mean calculated without the image charge uncertainty, thereby obtaining the 20 parts in 1012 final uncertainty.

Although there were seven successful helium runs to determine \( ωc(h)/ωc(13C6+) \), Zafonte and Van Dyck (2015) decided to exclude runs three and four from their final analysis because they were found to contain two \( 12C6^+ \) ions instead of one. To avoid the problem of isolating a single \( 12C6^+ \) ion, they used a single \( 13C6^+ \) ion in the three other runs and scaled the results using the well-known values of \( A_1(12C6^+) \) and \( A_1(13C6^+) \) without adding any significant uncertainty to what they would have obtained if a \( 12C6^+ \) ion had been used. Zafonte and Van Dyck (2015) treat the five individual \( ωc(h)/ωc(12C6^+) \) frequency ratios as they did the 10 \( ωc(d)/ωc(12C6^+) \) ratios; the fractional image charge correction is \(-515 \times 10^{-12} \) with \( u_c = 8.9 \times 10^{-12} \), for the weighted mean of the five ratios \( u_c = 11.2 \times 10^{-12} \), and for the final value \( u_c = 14 \times 10^{-12} \).

The cyclotron frequency ratios of HD\(^+\) to \(^3\)He\(^+\) and to \( t \) reported by the FSU group are (Myers et al., 2015)

\[ \frac{ωc(HD\(^+\))}{ωc(^3He\(^+\))} = 0.998 048 885 153(48) \hspace{1cm} [4.8 \times 10^{-11}], \hspace{1cm} (11) \]

\[ \frac{ωc(HD\(^+\))}{ωc(t)} = 0.998 054 687 288(48) \hspace{1cm} [4.8 \times 10^{-11}]. \hspace{1cm} (12) \]

As for the two UWash ratios, these ratios are correlated, but in this case because of the correction to account for imbalance between the cyclotron radii of the two ions. Based on the published uncertainty budgets and additional information provided by Myers (2015), their correlation coefficient is

\[ r[ωc(HD\(^+\))/ωc(^3He\(^+\)), ωc(HD\(^+\))/ωc(t)] = 0.875. \hspace{1cm} (13) \]

The relevant equations for these data are

\[ \frac{ωc(HD\(^+\))}{ωc(^3He\(^+\))} = \frac{A_1(^3He\(^+\))}{A_1(HD\(^+\))}, \hspace{1cm} (14) \]

\[ \frac{ωc(HD\(^+\))}{ωc(t)} = \frac{A_1(t)}{A_1(HD\(^+\))}, \hspace{1cm} (15) \]

where

\[ A_1(^3He\(^+\)) = A_1(h) + A_1(e) - \frac{E_I(^3He\(^+\))}{m_e c^2} \hspace{1cm} (16) \]

\[ E_I(^3He\(^+\))/hc = 43 888 919.36(3) \hspace{1cm} m^{-1}, \hspace{1cm} (17) \]

\[ A_1(HD\(^+\)) = A_1(p) + A_1(d) + A_1(e) - \frac{E_I(HD\(^+\))}{m_e c^2} \hspace{1cm} (18) \]

\[ E_I(HD\(^+\))/hc = 13 122 468.415(6) \hspace{1cm} m^{-1}. \hspace{1cm} (19) \]

The ionization wave number in Eq. (17) is from Table III, and the value in Eq. (19) is from Liu et al. (2010) and Sprecher et al. (2010).

In the FSU experiment pairs of individual ions, either HD\(^+\) and \(^3\)H\(^+\) or HD\(^+\) and \(^3\)He\(^+\), are confined at the same time in a Penning trap at 4.2 K with an applied magnetic flux density of 8.5 T. The cyclotron frequency of one ion centered in the trap in an orbit with a radius of about 45 \( \mu \)m is determined while the other ion is kept in an outer orbit with a radius of about 1.1 mm to reduce perturbations on the inner ion due to Coulomb interactions. The two ions are then interleaved. In a typical run lasting up to 10 h about 20 cyclotron frequency measurements are made on each ion. The temporal variation of the magnet flux density is accounted for by simultaneously fitting a fourth-order polynomial to the individual cyclotron frequencies as a function of time. In total 34 HD\(^+\)/\(^3\)He\(^+\) and HD\(^+\)/\(^3\)H\(^+\) runs were carried out over a 5 month period. For each frequency ratio the standard uncertainty of the mean of the individual values before correction for two systematic effects is \( 17 \times 10^{-12} \). The correction for cyclotron radius imbalance for each is \( 22 \times 10^{-12} \) and for the polarizability of the HD\(^+\) ion, \( 94 \times 10^{-12} \) with negligible uncertainty. These two uncertainty components lead to the final uncertainty for each of \( 48 \times 10^{-12} \).

Since the cyclotron frequencies in Eqs. (11) and (12) are both measured with reference to the same molecular ion HD\(^+\) and there is a sizable correlation coefficient between the frequency ratio measurements, Myers et al. (2015) obtain a value for the ratio \( ωc(^3H\(^+\))/ωc(^3He\(^+\)) \) with only one-half the \( 4.8 \times 10^{-11} \) uncertainty of that for either of the ratios determined with HD\(^+\). They are thus able to deduce for the mass difference between the tritium and helium-3 atoms, \( m(^3H) - m(^3He) \)
m^{3}(\text{He}) = 1.995\ 934\ (7) \times 10^{-5} \text{ eV}/\text{c}^2, \text{ which has a significantly smaller uncertainty than any other value.}\n
The value of A_{\text{e}}^{3}(\text{He}) deduced by Myers et al. (2015) from their data, 3.016\ 029\ 322\ 43(19), exceeds the value deduced by Zafonte and Van Dyck (2015) from their data, 3.016\ 029\ 321\ 675(43), by 3.9 times the standard uncertainty of their difference \delta_{\text{data}} or 3.9\sigma. (Throughout the paper, \sigma as used here is the standard uncertainty \delta_{\text{data}} of the difference between two values.) How this disagreement is treated in the 2014 adjustment is discussed in Sec. XIII. The THe-Trap experiment currently underway at the Max-Planck-Institut für Kernphysik (MPIK), Heidelberg, Germany, the aim of which is to determine the ratio A_{\text{e}}^{3}(\text{H})/A_{\text{e}}^{3}(\text{He}) in order to determine the Q-value of tritium, may clarify the cause of this discrepancy; see Diehl et al. (2011) and Streubel et al. (2014).

IV. Atomic Transition Frequencies

Comparison of theory and experiment for transition frequencies in hydrogen, deuterium, and muonic hydrogen provides information on the Rydberg constant, and on the charge radii of the proton and deuteron. Hyperfine splittings in hydrogen and fine-structure splittings in helium are also briefly considered.

A. Hydrogen and deuterium transition frequencies, the Rydberg constant \( R_{\infty} \), and the proton and deuteron charge radii \( r_{p}, r_{d} \)

The transition frequency between states \( i \) and \( i' \) with energy levels \( E_{i} \) and \( E_{i'} \) in hydrogen or deuterium is given by

\[
\hbar \nu_{i'i} = E_{i'} - E_{i}. \tag{20}
\]

The energy levels are given by

\[
E_{i} = -\frac{\alpha^{2} m_{e} c^{2}}{2 n_{i}^{2}} (1 + \delta_{i}) = -\frac{R_{\infty} \hbar c}{n_{i}^{2}} (1 + \delta_{i}), \tag{21}
\]

where \( R_{\infty} \) is the Rydberg constant, \( n_{i} \) is the principal quantum number of state \( i \), and \( \delta_{i} \), where \( |\delta_{i}| \ll 1 \), contains the details of the theory of the energy level.

1. Theory of hydrogen and deuterium energy levels

References to the original works are generally omitted; these may be found in earlier detailed CODATA reports, in Eides, Grotch, and Shelyuto (2001, 2007), and in Sapirstein and Yennie (1990). Uncertainties we assign to the individual theoretical contributions are categorized as either correlated or uncorrelated. Correlations we consider arise in two forms. One case is where the uncertainties are mainly of the form \( C/n_{i}^{2} \), where \( C \) is the same for all states with the same \( L \) and \( j \). Such uncertainties are denoted by \( u_{0} \), while the uncorrelated uncertainties are denoted by \( u_{i} \). The other correlations we consider are those between corrections for the same state in different isotopes, where the correction only depends on the mass of the isotope. Calculations of the uncertainties of the energy levels and the corresponding correlation coefficients are described in Sec. IV.A.1.1.

\[a. \text{ Dirac eigenvalue.} \text{ The Dirac eigenvalue for an electron bound to a stationary point nucleus is} \]

\[ E_{D} = f(n,j)m_{e}c^{2}, \tag{22} \]

where

\[ f(n,j) = \left[ 1 + \frac{\left( Z\alpha \right)^{2}}{(n - \delta)^{2}} \right]^{-1/2}, \tag{23} \]

and \( n \) and \( j \) are the principal and total angular-momentum quantum numbers of the bound state,

\[ \delta = j + \frac{1}{2} \left[ (j + \frac{1}{2})^{2} - (Z\alpha)^{2} \right]^{1/2}, \tag{24} \]

and \( Z \) is the charge number of the nucleus.

For a nucleus with a finite mass \( m_{N} \), we have

\[ E_{M}(\text{H}) = mc^{2} + \frac{1}{2} \left[ m_{e}c^{2} - f(n,j) - 1 \right]^{2}m^{2}_{N}c^{2} \]

\[ + \frac{1 - \delta_{0}}{\kappa(2 \ell + 1)} \left( Z\alpha \right)^{2}m^{2}_{N}c^{2} + \ldots \tag{25} \]

for hydrogen or

\[ E_{M}(\text{D}) = mc^{2} + \frac{1}{2} \left[ m_{e}c^{2} - f(n,j) - 1 \right]^{2}m^{2}_{N}c^{2} \]

\[ + \frac{1}{\kappa(2 \ell + 1)} \left( Z\alpha \right)^{2}m^{2}_{N}c^{2} + \ldots \tag{26} \]

for deuterium, where \( \ell \) is the nonrelativistic orbital angular-momentum number, \( \delta_{0} \) is the Kronecker delta, \( \kappa = (-1)^{\ell - 1/2} (j + \frac{1}{2}) \) is the angular-momentum-parity quantum number, \( M = m_{e} + m_{N} \), and \( m_{\ell} = m_{e}m_{N}/(m_{e} + m_{N}) \) is the reduced mass.

\[b. \text{ Relativistic recoil.} \text{ The leading relativistic-recoil correction, to lowest order in } Z\alpha \text{ and all orders in } m_{e}/m_{N}, \text{ is} \]

\[ E_{R} = \frac{m^{3}_{e}}{m^{3}_{N}} \left( Z\alpha \right)^{6} m_{e}c^{2} \]

\[ \times \left\{ \frac{1}{3} \delta_{0} \ln(Z\alpha)^{2} - \frac{8}{3} \ln(k_{0}(n, \ell)) - \frac{1 - \delta_{0}}{9} \right\} \frac{1}{\ell(\ell + 1)(2\ell + 1)} \tag{27} \]

where

\[ a_{n} = -2 \left[ \ln \left( \frac{2}{n} \right) + \sum_{i=1}^{n} \frac{1}{i + 1} - \frac{1}{2n} \right] \delta_{0} + \frac{1}{\ell(\ell + 1)(2\ell + 1)}. \tag{28} \]

Values we use for the Bethe logarithms \( \ln(k_{0}(n, \ell)) \) in Eqs. (27), (38), and (65) are given in Table IV.

Additional contributions to lowest order in the mass ratio and of higher order in \( Z\alpha \) are

\[ E_{R} = \frac{m_{e}}{m_{N}} \left( Z\alpha \right)^{6} m_{e}c^{2} \left[ D_{60} + D_{72} Z\alpha \ln^{2}(Z\alpha)^{2} + \ldots \right], \tag{29} \]

where
Table IV. Relevant values of the Bethe logarithms \( \ln k_0(\ell, \alpha) \) 

<table>
<thead>
<tr>
<th>( n )</th>
<th>S</th>
<th>P</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.984 128 556</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2.811 769 893</td>
<td>-0.030 016 709</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2.767 663 612</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2.749 811 840</td>
<td>-0.041 954 895</td>
<td>-0.006 740 939</td>
</tr>
<tr>
<td>6</td>
<td>2.735 664 207</td>
<td>-0.008 147 204</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>2.730 267 261</td>
<td>-0.008 785 043</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>-0.009 342 954</td>
<td></td>
</tr>
</tbody>
</table>

\[
D_{60} = \left( 4 \ln 2 - \frac{7}{2} \right) \delta_{10} + \left[ 3 - \frac{\ell(\ell + 1)}{n^2} \right] \frac{2(1 - \delta_{10})}{4\ell^2(1 + \delta_{10})}, \tag{30}
\]

\[
D_{72} = \frac{11}{60\pi} \delta_{10}. \tag{31}
\]

The uncertainty in the relativistic recoil correction is taken to be

\[
[0.1\delta_{10} + 0.01(1 - \delta_{10})] E_R. \tag{32}
\]

Covariances follow from the \( (m_e/m_N)/n^3 \) scaling of the uncertainty.

c. Nuclear polarizability. For the nuclear polarizability in hydrogen, we use

\[
E_p(H) = -0.070(13)\hbar \frac{\delta_{10}}{n^3} \text{ kHz}, \tag{33}
\]

and for deuterium

\[
E_p(D) = -21.37(8)\hbar \frac{\delta_{10}}{n^3} \text{ kHz}. \tag{34}
\]

Presumably the polarizability effect is negligible for states of higher \( \ell \) in either hydrogen or deuterium.

d. Self energy. The one-photon self energy of an electron bound to a stationary point nucleus is

\[
E_{SE}^{(2)} = \frac{\alpha}{\pi} \frac{(Z\alpha)^4}{n^3} F(Z\alpha) m_e c^2, \tag{35}
\]

where

\[
F(Z\alpha) = A_{41} \ln(Z\alpha)^2 + A_{40} + A_{50}(Z\alpha) + A_{62}(Z\alpha)^2 \ln(Z\alpha)^2 + A_{61}(Z\alpha)^2 \ln(Z\alpha)^2 + G_{SE}(Z\alpha)(Z\alpha)^2, \tag{36}
\]

with

\[
A_{41} = \frac{4}{3} \delta_{10}, \tag{37}
\]

\[
A_{40} = -\frac{4}{5} \ln k_0(n, \ell) + \frac{10}{9} \delta_{10} - \frac{1}{2k(2\ell + 1)} (1 - \delta_{10}), \tag{38}
\]

\[
A_{50} = \left( \frac{139}{32} - 2\ln 2 \right) \pi \delta_{10}, \tag{39}
\]

\[
A_{62} = -\delta_{10}, \tag{40}
\]

\[
A_{61} = \left[ 4 \left( 1 + \frac{1}{2} + \cdots + \frac{1}{n} \right) + \frac{28}{3} \ln 2 - 2 \ln n \right. \\
\left. - \frac{601}{180} \frac{77}{45n^2} \right] \delta_{10} + \frac{n^2 - 1}{n^2} \left[ \frac{2}{15} + \frac{\delta_{10}}{3} \right] \delta_{11}, \tag{41}
\]

Values for \( G_{SE}(\alpha) \) in Eq. (36) are listed in Table V. See CODATA-10 for details. The uncertainty of the self-energy contribution to a given level is due to the uncertainty of \( G_{SE}(\alpha) \) listed in that table and is taken to be \( u_n \).

Following convention, \( F(Z\alpha) \) is multiplied by the reduced-mass factor \( (m_e/m_N)^2 \), except the magnetic-moment term \(-1/[2k(2\ell + 1)]\) in \( A_{50} \) which is instead multiplied by the factor \( (m_e/m_N)^2 \), and the argument \( (Z\alpha)^2 \) of the logarithms is replaced by \( (m_e/m_N)(Z\alpha)^2 \).

e. Vacuum polarization. The stationary point nucleus second-order vacuum-polarization level shift is

\[
E_{VP}^{(2)} = \frac{\alpha}{\pi} \frac{(Z\alpha)^4}{n^3} H(Z\alpha) m_e c^2, \tag{42}
\]

where

\[
H(Z\alpha) = H^{1(1)}(Z\alpha) + H^{R}(Z\alpha), \tag{43}
\]

\[
H^{1(1)}(Z\alpha) = V_{40} + V_{50}(Z\alpha) + V_{61}(Z\alpha)^3 \ln(Z\alpha)^2 + G_{VP}^{(1)}(Z\alpha)(Z\alpha)^2, \tag{44}
\]

with

\[
V_{40} = -\frac{4}{15} \delta_{10}, \tag{45}
\]

\[
V_{50} = \frac{5}{48} \pi \delta_{10}, \tag{46}
\]

\[
V_{61} = -\frac{2}{15} \delta_{10}. \tag{47}
\]

Values of \( G_{VP}^{(1)}(Z\alpha) \) are given in Table VI, and

Table V. Values of the function \( G_{SE}(\alpha) \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>( S_{1/2} )</th>
<th>( P_{1/2} )</th>
<th>( P_{3/2} )</th>
<th>( D_{3/2} )</th>
<th>( D_{5/2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-30.290 240(20)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-31.185 150(90)</td>
<td>-0.973 50(20)</td>
<td>-0.486 50(20)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-31.047 70(90)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-30.912 040(40)</td>
<td>-1.164 00(20)</td>
<td>-0.609 00(20)</td>
<td></td>
<td>0.031 63(22)</td>
</tr>
<tr>
<td>6</td>
<td>-30.711 (47)</td>
<td></td>
<td></td>
<td></td>
<td>0.034 17(26)</td>
</tr>
<tr>
<td>8</td>
<td>-30.606 (47)</td>
<td></td>
<td></td>
<td></td>
<td>0.035 84(22)</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.035 12(22)</td>
</tr>
</tbody>
</table>
TABLE VI. Values of the function $G^{(1)}_{\nu \nu}(\alpha)$

<table>
<thead>
<tr>
<th>$n$</th>
<th>$S_{1/2}$</th>
<th>$P_{1/2}$</th>
<th>$P_{3/2}$</th>
<th>$D_{1/2}$</th>
<th>$D_{3/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.618724</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-0.808872</td>
<td>-0.064006</td>
<td>-0.01432</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-0.814530</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-0.806597</td>
<td>-0.080007</td>
<td>-0.017666</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-0.791450</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>-0.781197</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$G^{(R)}_{\nu \nu}(Z \alpha) = \left( \frac{19}{45} \frac{\pi^2}{27} \right) \delta_{10} + \left( \frac{1}{16} \frac{31 \pi^2}{2880} \right) \pi(Z \alpha) \delta_{10} + \cdots$.  

(46)

Higher-order terms are negligible. We multiply Eq. (42) by $G_{\nu \nu}$ and include a factor of $\pi(Z \alpha) \delta_{10}$ in the argument of the logarithm in Eq. (43).

Vacuum polarization from $\mu^+\mu^-$ pairs is

$$E^{(2)}_{\nu \nu \nu} = \frac{\alpha}{\pi} \frac{(Z \alpha)^4}{n^3} \frac{m_e}{m_{\mu}} \left( \frac{m_{\mu}}{m_e} \right)^2 \frac{3}{2} m_e c^2,$$

(47)

and hadronic vacuum polarization is given by

$$E^{(2)}_{\nu \nu \nu} = 0.671(15) E^{(2)}_{\nu \nu \nu}.$$  

(48)

Uncertainties are of type $u_0$. The muonic and hadronic vacuum-polarization contributions are negligible for higher-$\ell$ states.

f. Two-photon corrections. The two-photon correction is

$$E^{(4)} = \frac{(2 \pi^2)}{3} (Z \alpha)^4 n^3 m_e c^2 F^{(4)}(Z \alpha),$$

(49)

where

$$F^{(4)}(Z \alpha) = B_{40} + B_{50}(Z \alpha) + B_{63}(Z \alpha)^2 \ln^3(Z \alpha)^2 - \ln^2(Z \alpha) - 2 + B_{61}(Z \alpha) \ln(Z \alpha)^2 - 2 + B_{60}(Z \alpha) + B_{72}(Z \alpha)^3 \ln^2(Z \alpha)^2 - 2 + B_{71}(Z \alpha)^3 \ln(Z \alpha)^2 - 2 + B_{70}(Z \alpha)^3 + \cdots,$$

(50)

with

$$B_{40} = \left[ \frac{3 \pi^2}{2} \ln 2 - 10 \frac{\pi^2}{27} - \frac{2179}{648} \frac{9}{4} \pi(3) \delta_{10} + \left[ \frac{\pi^2}{2} - \frac{\pi^2}{12} - \frac{197}{144} - \frac{3 \pi(3)}{4} \right] \frac{1}{\kappa(2 \ell + 1)} \delta_{10} \right],$$

(51)

$$B_{50} = -21.55447(13) \delta_{10},$$

(52)

Values for $B_{60}$ used in the adjustment are listed in Table VII. In CODATA-10, the entries for states with $\ell = 1$ in the corresponding Table IX are incorrect, which had negligible effect on the results. Corrected values are listed here in Table VII. The values of $N(nL)$, which appear in Eq. (54), are listed in Table VIII. The uncertainties are negligible.

Values used in the adjustment for $B_{60}$ and $B_{61}$ are listed in Table IX. For the $S$-state values, the first number in parentheses is the state-dependent uncertainty $u_0(B_{60})$, and the second number in parentheses is the state-independent uncertainty $u_0(B_{60})$ that is common to all $S$-state values of $B_{60}$. For higher-$\ell$ states, the notation $B_{60}$ indicates that the number listed in the table is the value of the line center shift for the level, in contrast to the total real part of the two-photon correction. See CODATA-10 for a complete explanation. For $S$ states, the difference between $B_{60}$ and $B_{61}$ is negligible compared to the uncertainty of the value of $B_{60}$. The uncertainties of $B_{60}$ for higher-$\ell$ states are taken to be independent.

For $S$ states, the next term $B_{72}$ is state independent, but its value is not known. However, the state dependence of the following term is

$$\Delta B_{71}(nS) = B_{71}(nS) - B_{71}(1S) \pi \left( \frac{247}{36} \frac{16}{3} \ln^2 \right) \times \left[ \frac{3}{4} - \frac{1}{n} + \frac{1}{4n^2} + \gamma + \psi(n) - \ln n \right].$$

(56)

TABLE VII. Values of $B_{60}$ used in the 2014 adjustment

<table>
<thead>
<tr>
<th>$n$</th>
<th>$B_{60}(nS_{1/2})$</th>
<th>$B_{60}(nP_{1/2})$</th>
<th>$B_{60}(nP_{3/2})$</th>
<th>$B_{60}(nD_{3/2})$</th>
<th>$B_{60}(nD_{3/2})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>48.958 590 24(1)</td>
<td>-0.092 224 453(1)</td>
<td>-0.121 307 400(1)</td>
<td>-0.000</td>
<td>-0.000</td>
</tr>
<tr>
<td>2</td>
<td>41.062 164 31(1)</td>
<td>0.157 775 547(1)</td>
<td>-0.121 307 400(1)</td>
<td>-0.000</td>
<td>-0.000</td>
</tr>
<tr>
<td>3</td>
<td>38.904 222(1)</td>
<td>0.191 192 600(1)</td>
<td>-0.121 307 400(1)</td>
<td>-0.000</td>
<td>-0.000</td>
</tr>
<tr>
<td>4</td>
<td>37.909 514(1)</td>
<td>0.191 192 600(1)</td>
<td>-0.121 307 400(1)</td>
<td>-0.000</td>
<td>-0.000</td>
</tr>
<tr>
<td>5</td>
<td>36.963 391(1)</td>
<td>0.191 192 600(1)</td>
<td>-0.121 307 400(1)</td>
<td>-0.000</td>
<td>-0.000</td>
</tr>
<tr>
<td>8</td>
<td>36.504 940(1)</td>
<td>0.00(0)</td>
<td>0.00(0)</td>
<td>0.00(0)</td>
<td>0.00(0)</td>
</tr>
<tr>
<td>12</td>
<td>0.00(0)</td>
<td>0.00(0)</td>
<td>0.00(0)</td>
<td>0.00(0)</td>
<td>0.00(0)</td>
</tr>
</tbody>
</table>
with a relative uncertainty of 50%. We include this difference, which is listed in Table IX, along with an estimated uncertainty of \(\delta B_{171} = \Delta B_{171}/2\).

The three-photon contribution in powers of \(Z\alpha\) is

\[
E^{(6)} = \left(\frac{\alpha}{\pi}\right)^3 \frac{(Z\alpha)^4}{n^3} m_e c^2 \left[ C_{40} + C_{50}(Z\alpha) + \cdots \right].
\] (57)

The leading term \(C_{40}\) is

\[
C_{40} = -\left(\frac{568a_4 + 85\zeta(5)}{9} \right) \frac{\pi^2}{72} \frac{\zeta(3)}{2304} \frac{211n^2}{27} - \left(\frac{100a_4 + 215\zeta(5)}{3} \right) \frac{\pi^2}{72} \frac{\zeta(3)}{25n^2} + \left(\frac{-239\pi^2n^2}{135} \right) \frac{2160}{9} \frac{2898\pi^2n^2}{18} + \left(\frac{-239\pi^2n^2}{1710n^2} \right) \frac{28259}{810} \frac{5184}{28.259} \frac{1}{\kappa(2\ell + 1)}.
\] (58)

As with the one-photon correction, the two-photon correction is multiplied by the reduced-mass factor \((m_i/m_e)^3\), except the magnetic-moment term proportional to \(1/\kappa(2\ell + 1)\) in \(B_{40}\) which is multiplied by the factor \((m_i/m_e)^2\), and the argument \((Z\alpha)^2\) of the logarithms is replaced by \((m_i/m_e)(Z\alpha)^2\).

### h. Finite nuclear size

For \(S\) states the leading and next-order correction to the level shift due to the finite size of the nucleus is given by

\[
E_{NS} = E_{NS} \left\{ -C_{40} \frac{m_i r_N}{m_e \hbar^2} Z\alpha \ln \left( \frac{m_i r_N}{m_e \hbar^2} \frac{Z\alpha}{n} \right) + \psi(n) \right. + \left. \gamma \frac{(5n + 9)(n - 1)}{4n^2} \right\} \left(\frac{Z\alpha}{n}\right)^2, \] (59)

where

\[
E_{NS} = \frac{2}{3} \left(\frac{m_i}{m_e}\right)^3 \frac{(Z\alpha)^2}{n^3} m_e c^2 \left(\frac{Zar_N}{\hbar c}\right)^2.
\] (60)

\(r_N\) is the bound-state root-mean-square (rms) charge radius of the nucleus, \(\hbar c\) is the Compton wavelength of the electron divided by \(2\pi\), \(C_{40}\) and \(C_{50}\) are constants that depend on the charge distribution in the nucleus with values \(C_{40} = 0.47(4)\) for hydrogen or \(C_{40} = 2.0(1)\) and \(C_{50} = 0.38(4)\) for deuterium.

For the \(P_{1/2}\) states in hydrogen the leading term is

\[
E_{NS} = E_{NS} \left(\frac{Z\alpha}{n}\right)^2 \frac{(n^2 - 1)}{4n^2}.
\] (61)

For \(P_{3/2}\) states and higher-\(\ell\) states the nuclear-size contribution is negligible.

### i. Nuclear-size correction to self energy and vacuum polarization

For the lowest-order self energy and vacuum polarization the correction due to the finite size of the nucleus is

<table>
<thead>
<tr>
<th>(n)</th>
<th>(B_{40}(nS_{1/2}))</th>
<th>(B_{40}(nP_{1/2}))</th>
<th>(B_{40}(nP_{3/2}))</th>
<th>(B_{40}(nD_{3/2}))</th>
<th>(B_{40}(nD_{5/2}))</th>
<th>(\Delta B_{171}(nS_{1/2}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-81.3(0.3)(19.7)</td>
<td>-1.6(3)</td>
<td>-1.7(3)</td>
<td>16(8)</td>
<td>22(11)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-66.2(0.3)(19.7)</td>
<td>0.015(5)</td>
<td>-0.009(5)</td>
<td>29(15)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-63.0(0.6)(19.7)</td>
<td>0.014(7)</td>
<td>-0.010(7)</td>
<td>28(14)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-61.3(0.8)(19.7)</td>
<td>-2.0(3)</td>
<td>-2.2(3)</td>
<td>25(12)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-59.3(0.8)(19.7)</td>
<td>-0.008(4)</td>
<td>-0.005(2)</td>
<td>16(8)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-58.3(2.0)(19.7)</td>
<td>-0.010(7)</td>
<td>-0.009(5)</td>
<td>29(15)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.015(5)</td>
<td>-0.009(5)</td>
<td>29(15)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.014(7)</td>
<td>-0.010(7)</td>
<td>28(14)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table IX.** Values of \(B_{40}\), \(B_{50}\), or \(\Delta B_{171}\) used in the 2014 adjustment. The uncertainties of \(B_{40}\) are explained in the text.
\[ \begin{align*}
E_{\text{NSE}} &= \left(4\ln 2 - \frac{23}{4}\right)\alpha(Z\alpha)\mathcal{E}_{\text{NS}}\delta_{\text{m}}, \\
E_{\text{NVP}} &= \frac{3}{4}\alpha(Z\alpha)\mathcal{E}_{\text{NS}}\delta_{\text{m}},
\end{align*} \]
respectively.

\textit{j. Radiative-recoil corrections.} Corrections for radiative-recoil effects are
\[ E_{\text{RR}} = \frac{m_i^3}{m_e^2 m_N} \left( \frac{\alpha(Z\alpha)^2}{\pi^2} - m_e c^2 \delta_{\text{m}} \right) \]
\[ \times \left[ 6\zeta(3) - 2\pi^2 \ln 2 + \frac{35\pi^2}{36} \frac{448}{27} \right] \]
\[ \times \frac{2}{3} \frac{\alpha(Z\alpha)^2}{\pi^2} \ln^2(X\alpha)^2 \right) + \cdots \right]. \tag{64} \]

The uncertainty is \((Z\alpha)^2\ln(Z\alpha)^2\) relative to the square brackets with a factor of 10 for \( \delta_{\text{m}} \) and 1 for \( u_n \). Corrections for higher-\( \ell \) states are negligible.

\textit{k. Nucleus self energy.} The nucleus self energy correction for \( S \) states is
\[ E_{\text{SEN}} = \frac{4Z^2 \alpha(Z\alpha)^2}{3\pi^2} \left( \frac{m_i^3}{m_N} + c^2 \right) \left[ \ln \left( \frac{m_N}{m_i(Z\alpha)} \right) \right] \delta_{\text{m}} - \ln \kappa_i(n, \ell), \tag{65} \]
with an uncertainty \( u_0 \) given by Eq. (65) with the factor in the square brackets replaced by 0.5. For higher-\( \ell \) states, the correction is negligible.

\textit{l. Total energy and uncertainty.} The energy \( E_X(nL_j) \) of a level (where \( L = S, P, \ldots \) and \( X = H, D \)) is the sum of the various contributions listed above. Uncertainties in the fundamental constants that enter the theoretical expressions are taken into account through the formalism of the least-squares adjustment. To take uncertainties in the theory into account, a correction \( \delta_X(nL_j) \) that is zero with an uncertainty that is the rms sum of the uncertainties of the individual contributions
\[ u^2[\delta_X(nL_j)] = \sum_i \left[ u_{0i}(XnL_j) + u_{ni}(XnL_j) \right], \tag{66} \]
where \( u_{0i}(XnL_j) \) and \( u_{ni}(XnL_j) \) are the components of uncertainty \( u_0 \) and \( u_n \) of contribution \( i \), is added to the level. The corrections \( \delta_X(nL_j) \), which includes their uncertainties, are taken as input data in the least-squares adjustment. Covariances are taken into account by calculating all the covariances and including them in the input data for the adjustment.

Covariances of the \( \delta_X \) for a given isotope are
\[ u[\delta_X(n_1L_j), \delta_X(n_2L_j)] = \sum_i u_{0i}(Xn_2L_j) u_{0i}(Xn_1L_j). \tag{67} \]
Covariances between \( \delta_X \) for hydrogen and deuterium for states of the same \( n \) are

\[ u[\delta(n_1L_j), \delta(n_2L_j)] = \sum_{i=\{\}} u_{0i}(HnL_j) u_{0i}(DnL_j) + u_{ni}(HnL_j) u_{ni}(DnL_j), \tag{68} \]
and for \( n_1 \neq n_2 \)

\[ u[\delta(n_1L_j), \delta(n_2L_j)] = \sum_{i=\{\}} u_{0i}(Hn_1L_j) u_{0i}(Dn_2L_j), \tag{69} \]
where the summation is over the uncertainties common to hydrogen and deuterium.

Values of \( u[\delta_X(nL_j)] \) are given in Table XVI of Sec. XIII, and the non-negligible covariances of the \( \delta_X \) are given as correlation coefficients in Table XVII.

\textit{m. Transition frequencies between levels with } \( n = 2 \) \textit{and the fine-structure constant } \( \alpha \). QED predictions for hydrogen transition frequencies between levels with \( n = 2 \) are obtained from a least-squares adjustment that does not include an experimental value for the transitions being calculated (items A39, A40.1, or A40.2 in Table XVI), where the constants \( A_{\text{e}}(e) \), \( A_{\text{e}}(p) \), \( A_{\text{e}}(d) \), and \( \alpha \) are assigned the 2014 values. The results are
\[ \nu_H(2P_{1/2} - 2S_{1/2}) = 1.057843.7(2.1) \text{ kHz} [2.0 \times 10^{-9}], \]
\[ \nu_H(2S_{1/2} - 2P_{3/2}) = 9.911197.8(2.1) \text{ kHz} [2.1 \times 10^{-7}], \]
\[ \nu_H(2P_{1/2} - 2P_{3/2}) = 10.969041.530(41) \text{ kHz} [3.7 \times 10^{-9}], \tag{70} \]
which are consistent with the experimental results given in Table XVI.

Data for the hydrogen and deuterium transitions yield a value for the fine-structure constant \( \alpha \), which follows from a least-squares adjustment that includes all the transition frequency data in Table XVI, the 2014 adjusted values of \( A_{\text{e}}(e) \), \( A_{\text{e}}(p) \), and \( A_{\text{e}}(d) \), but no other input data for \( \alpha \). The result is
\[ \alpha^{-1} = 137.035992(55) [4.0 \times 10^{-7}], \tag{71} \]
and is also given in Table XX.

\section*{2. Experiments on hydrogen and deuterium}
Table X gives the hydrogen and deuterium transition frequencies used to determine the Rydberg constant \( R_{\infty} \), items A26 to A48 in Table XVI, Sec. XIII. The only difference between this table and the corresponding Table XI in CODATA-10 is that the value for the \( 1S_{1/2} - 2S_{1/2} \) hydrogen transition frequency obtained by the group at the Max-Planck-Institut für Quantenoptik (MPQ), Garching, Germany, used in the 2010 adjustment is superseded by two new values obtained by the same group but with significantly smaller uncertainties (first two entries of Table X):
\[ \nu_H(1S_{1/2} - 2S_{1/2}) = 246606143187.035(10) \]
\[ [4.2 \times 10^{-15}], \tag{72} \]
\[ \nu_H(1S_{1/2} - 2S_{1/2}) = 246606143187.018(11) \]
\[ [4.4 \times 10^{-15}], \tag{73} \]
The result reported by Parthey et al. (2011) has a relative standard uncertainty \( u_r = 4.2 \times 10^{-15} \), about one-third that of the value used in the 2010 adjustment. The reduction was achieved using a more stable spectroscopy laser and by reducing the uncertainties from the principal systematic effects, namely, the second-order Doppler shift and ac and dc Stark shifts.

The result reported by Matveev et al. (2013) has a relative standard uncertainty of \( 4.4 \times 10^{-15} \). In contrast to the 2011 measurement, which used an on-site transportable cesium fountain clock as the frequency reference, the 2013 measurement used the nonmovable cesium fountain clock at the Physikalisch-Technische Bundesanstalt (PTB) in Braunschweig, Germany, by connecting the MPQ experiment in Garching to the PTB clock in Braunschweig via a 920 km fiber link. Another difference is the use of an improved detector of the Lyman-\( \alpha \) photons emitted when the excited hydrogen atom beam is deexcited from the 2S state. The new results are consistent with each other and with that used in 2010. However, the 2011 and 2013 values are correlated, and based on the published uncertainty budgets and information provided by the researchers (Udem, 2014) the correlation coefficient is estimated to be 0.707. This correlation coefficient is included in Table XVII, Sec. XIII, together with the correlation coefficients of the other correlated frequencies in Table X as discussed in CODATA-98.

3. Nuclear radii

It follows from Eqs. (59) and (60) in Sec. IV.A.1.h that transition frequencies in hydrogen and deuterium atoms depend on the bound-state rms charge radius \( r_p \) and \( r_d \) of their respective nuclei, the proton \( p \) and deuteron \( d \). Accurate values for these radii can be obtained if they are taken as adjusted constants in a least-squares adjustment together with experimental H and D transition frequency input data and theory. The values so determined are often referred to as the H-D spectroscopic values of \( r_p \) and \( r_d \).

a. Electron scattering. Values of \( r_p \) and \( r_d \) are also available from electron-proton (e-p) and electron-deuteron (e-d) elastic scattering data. If these values are included as input data in an adjustment together with the H and D spectroscopic data and theory, a combined least-squares adjusted value for \( r_p \) and \( r_d \) can be obtained. In the 2010 adjustment the value of \( r_d \) determined by Sick (2008) from an analysis of the available data on e-d elastic scattering was used as an input datum and it

is again used as an input datum in the 2014 adjustment. That value is $r_d = 2.130(10)$ fm.

The e-p scattering value of $r_p$ is more problematic. Two such values were used in the 2010 adjustment: $r_p = 0.895(18)$ fm and $r_p = 0.8791(79)$ fm. The first is due to Sick (2008) based on his analysis of the data then available. The second is from a seminal experiment, described in detail by Bernauer et al. (2014), carried out at the Johannes Gutenberg Universität Mainz (or simply the University of Mainz), Mainz, Germany, with the Mainz microtron electron-beam accelerator MAMI. Both values are discussed in CODATA-10, as is the significant disagreement of the H-D spectroscopic and e-p scattering values of $r_p$ with the value determined from spectroscopic measurements of the Lamb shift in muonic hydrogen. The cause of the disagreement remains unknown, and we review the current situation in the following section and in Sec. XIII.B.2.

To assist in deciding what scattering value or values of $r_p$ to use in the 2014 adjustment, the Task Group invited a number of researchers active in the field to attend its annual meeting in November 2014. It also helped organize the Workshop on the Determination of the Fundamental Constants held in Eltville, Germany, in February 2015, the proceedings of which are published in J. Phys. Chem. Ref. Data 44(3) (2015). A point of concern at these meetings was how best to extract $r_p$ from the MAMI data and from the combination of the MAMI data with the remaining available data. As a result of the discussions at these meetings two pairs of knowledgeable researchers, J. Arrington and I. Sick, and J. C. Bernauer and M. O. Distler, provided the Task Group with their best estimate of $r_p$ from all the available data. The Arrington-Sick value, $r_p = 0.879(11)$ fm, has been published (Arrington and Sick, 2015) but was initially transmitted to the Task Group as a private communication in 2015; the Bernauer-Distler value, $r_p = 0.880(11)$ fm, was also transmitted to the Task Group in 2015 as a private communication but has not been published. The two values are highly consistent even though different approaches and assumptions were used to obtain them. We therefore adopt as the e-p scattering-data input datum for $r_p$ in the 2014 adjustment

$$r_p = 0.879(11) \quad [1.3 \times 10^{-2}] \text{ fm}, \quad (74)$$

which is the weighted mean of the two values, but the uncertainty is the average of their uncertainties since each value was based on essentially the same data.

The Task Group well recognizes that Eq. (74) is not the last word on the subject and that the topic remains one of active interest; see, for example, the papers by Kraus et al. (2014), Arrington (2015), Lorenz et al. (2015), and Sick (2015). Particularly noteworthy is the paper by Lee, Arrington, and Hill (2015), which did not become available until well after the 31 December closing date of the 2014 adjustment, that provides a new and improved analysis to obtain $r_p = 0.895(20)$ fm from the MAMI data, $r_p = 0.916(24)$ fm from the remaining available data but not including the MAMI data, and $r_p = 0.904(15)$ fm by combining these two values. Three recent papers by Griffioen, Carlson and Maddox (2016), Higinbotham et al. (2016), and Horbatsch and Hessels (2016) obtain results consistent with the smaller muonic hydrogen radius in Eq. (78). However, Bernauer and Distler (2016) point out a number of problems with the analyses in these papers.

b. Isootope shift and the deuteron-proton radius difference. From a comparison of experiment and theory for the hydrogen-deuterium isotope shift, one can extract a value for the difference of the squares of the charge radii for the deuteron and proton, based on the 2014 recommended values, given by

$$r_d^2 - r_p^2 = 3.81948(37) \text{ fm}^2. \quad (75)$$

The corresponding result based on the 2010 recommended values,

$$r_d^2 - r_p^2 = 3.81989(42) \text{ fm}^2, \quad (76)$$

differs from the current value due mainly to the change in the relative atomic mass of the electron used to evaluate the theoretical expression for the isotope shift. The electron mass appears as an overall factor as can be seen from the leading term (Jentschura et al., 2011)

$$\Delta f_{1S-2S,d} - \Delta f_{1S-2S,p} \approx -\frac{3}{4} R_{\alpha \omega c} \left(\frac{m_e}{m_d} - \frac{m_e}{m_p}\right). \quad (77)$$

c. Muonic hydrogen. The first reported value of $r_p$ from spectroscopic measurements of the Lamb shift in the muonic hydrogen atom $\mu$-p obtained by the Charge Radius Experiment with Muonic Atoms (CREMA) collaboration (Pohl et al., 2010) and the problem, now often called the “proton radius puzzle,” that resulted from its significant disagreement with the scattering and spectroscopic values is discussed in CODATA-10. As a result of this inconsistency the Task Group decided that the initial $\mu$-p value of $r_p$ should be omitted from the 2010 adjustment and the 2010 recommended value should be based on only the two available e-p scattering values (see previous section) and the H-D spectroscopic data and theory. Consequently the disagreement of the $\mu$-p value with the 2010 recommended value was 7$\sigma$ and with the H-D spectroscopic value, 4.4$\sigma$.

The $\mu$-p Lamb-shift experiment employs pulsed laser spectroscopy at a special muon beam line at the proton accelerator of the Paul Scherrer Institute (PSI), Villigen, Switzerland. Muonic atoms in the 2S state (lifetime 1 ps) are formed when muons from the beam strike a gaseous H2 target, a 5 ns near-resonance laser pulse (tunable from 50 THz to 55 THz) induces transitions to the 2P state (lifetime 8.5 ps), the atoms decay to the 1S ground state by emitting 1.9 keV $K\alpha$ x rays, and a resonance curve is obtained by counting the number of x rays as a function of laser frequency. (Because of the large electron vacuum-polarization effect in $\mu$-p, the 2S$_{1/2}$ level is far below both the 2P$_{1/2}$ and 2P$_{3/2}$ levels.)

The transition initially measured was 2S$_{1/2}(F = 1) - 2P_{1/2}(F = 2)$ at 50 THz or 6$\mu$m. More recently, the CREMA collaboration reported their result for the transition 2S$_{1/2}(F = 0) - 2P_{3/2}(F = 1)$ at 55 THz or 5.5$\mu$m, as well as their reevaluation of the 50 THz data (Antognini et al., 2013b).
This reevaluation reduced the original frequency by 0.53 GHz and its uncertainty from 0.76 GHz to 0.65 GHz. By comparison, the uncertainty of the 55 THz frequency is 1.05 GHz.

The theory that relates the muonic hydrogen Lamb-shift transition frequencies to \( r_p \) reflects the calculations and critical investigations of many researchers, and a number of issues have been resolved since the closing date of the 2010 adjustment; a detailed review is given by Antognini et al. (2013a). Using the theory in this paper the CREMA collaboration reports, in the same paper in which they give their two measured transition frequencies (Antognini et al., 2013b), the following value as the best estimate of \( r_p \) from muonic hydrogen:

\[
r_p = 0.840\,87(39)\,\text{fm}.
\]

This may be compared with the muonic hydrogen value \( r_p = 0.841\,69(66)\,\text{fm} \) available for consideration in the 2010 adjustment based on the \( 2S_{1/2}(F = 1) - 2P_{3/2}(F = 2) \) transition frequency before reevaluation and the theory as it existed at the time. Because the new result in Eq. (78) is smaller and has a smaller uncertainty than this value and that neither the H-D data nor theory have changed significantly, it is not surprising that the disagreement still persists and that the Task Group has decided to omit the \( \mu \)-p result for \( r_p \) in the 2014 adjustment. Indeed, the disagreement of the value in Eq. (78) with the 2014 CODATA recommended value 0.8751(61) fm is 5.6\( \sigma \) and with the H-D spectroscopic value 0.8759(77) fm is 4.5\( \sigma \).

The negative effect of including the value of \( r_p \) in Eq. (78) as an input datum in the 2014 adjustment and why the Task Group concluded that it should be excluded is discussed further in Sec. XIII.B.2. However, we do point out here the two following facts.

First, if the least-squares adjustment that leads to the value of \( \alpha^{-1} \) given in Eq. (71) is carried out with the value in Eq. (78) included as an input datum, the result is \( \alpha^{-1} = 137.035\,876(35) \) [2.6 \times 10^{-7}] , which differs from the 2014 recommended value by 3.5\( \sigma \).

Second, Karshenboim (2014) has proposed a different approach to the calculation of the next-to-leading higher-order proton-size contribution to the theory of the muonic hydrogen Lamb shift than was used by Antognini et al. (2013b) to obtain the value of \( r_p \) in Eq. (78). The approach changes the value in Eq. (78) to \( r_p = 0.840\,22(56)\,\text{fm} \) (Karshenboim, 2014), for which the disagreement with the 2014 CODATA recommended value is 5.7\( \sigma \) and with the H-D spectroscopic value is 4.6\( \sigma \). [The slightly different value \( r_p = 0.840\,29(55)\,\text{fm} \) was subsequently published after the closing date of the 2014 adjustment by Karshenboim et al. (2015).] Because this value of \( r_p \) is consistent with the value in Eq. (78), because the disagreement for the two values with the 2014 CODATA recommended value and H-D spectroscopic value are very nearly the same, and because the suggested approach has not yet been widely accepted, we have used the value of \( r_p \) in Eq. (78) for our discussion here and analysis in Sec. XIII.B.2.

A recent review of the proton radius puzzle is given by Carlson (2015).

\[ V. \text{Magnetic Moments and } g\text{-factors} \]

The magnetic-moment vector of a charged lepton is

\[
\mu_\ell = g_\ell \frac{e}{2m_\ell} \hat{s},
\]

where \( \ell = e, \mu, \) or \( \tau, g_\ell \) is the \( g\)-factor, with the convention that it has the same sign as the charge of the particle, \( e \) is the (positive) unit charge, \( m_\ell \) is the lepton mass, and \( \hat{s} \) is its spin.

Since the spin has projection eigenvalues of \( s_z = \pm \hbar/2 \), the magnetic moment is defined to be

\[
\mu_\ell = g_\ell \frac{e\hbar}{2m_\ell},
\]

and the Bohr magneton is just

\[
\mu_B = \frac{e\hbar}{2m_e}.
\]

The \( g\)-factor differs from the Dirac value of 2 because of the lepton magnetic-moment anomaly \( a_\ell \) defined by

\[
|g_\ell| = 2(1 + a_\ell).
\]

The theoretical value of the anomaly is given by QED, predominantly electroweak, and predominantly strong-interaction effects denoted by

\[
a_\ell(\text{th}) = a_\ell(\text{QED}) + a_\ell(\text{weak}) + a_\ell(\text{had}),
\]

respectively.

\[ B. \text{Hyperfine structure and fine structure} \]

In principle, together with theory, hyperfine-structure measurements other than in muonium, and fine-structure measurements other than in hydrogen and deuterium, could provide accurate values of some constants, most notably the fine-structure constant \( \alpha \). Indeed, it has long been the hope that a competitive value of \( \alpha \) could be obtained from experimental measurements and theoretical calculations of \( ^4\text{He} \) fine-structure transition frequencies. However, as discussed in CODATA-10, no such data were available for the 2010 adjustment and this is also the case for the 2014 adjustment.

For completeness, we note that there have been significant improvements in both the theory and experimental determination of the hyperfine splitting in positronium [see Adkins et al. (2015), Eides and Shelyuto (2015), Ishida (2015), and Miyazaki et al. (2015) and the references cited therein]. Also, Marsman, Horbatsch, and Hessels (2015a, 2015b) have found that the measured values of helium fine-structure transition frequencies can be significantly influenced by quantum-mechanical interference between neighboring resonances even if separated by thousands of natural linewidths. The shifts they calculated for reported experimental values of the \( 2^3\text{P}_1 - 2^3\text{P}_2 \) fine-structure interval in \(^4\text{He} \) improve their agreement with theory.
A. Electron magnetic-moment anomaly $a_e$ and the fine-structure constant $\alpha$

A value for the fine-structure constant $\alpha$ is obtained by equating theory and experiment for the electron magnetic-moment anomaly.

1. Theory of $a_e$

The QED contribution to the electron anomaly is

$$a_e(\text{QED}) = A_1 + A_2(m_e/m_\mu) + A_3(m_e/m_\tau)$$
$$+ A_5(m_e/m_\mu, m_e/m_\tau),$$  \hspace{1cm} (84)

where the mass-dependent terms arise from vacuum-polarization loops. Each term may be expanded in powers of the fine-structure constant as

$$A_i = \sum_{n=1}^{\infty} A_i^{(2n)} \left( \frac{\alpha}{\pi} \right)^n,$$  \hspace{1cm} (85)

where $A_i^{(2)} = A_i^{(3)} = A_i^{(4)} = 0$.

The mass-independent terms $A_i^{(2n)}$ are known accurately through sixth order:

$$A_1^{(2)} = \frac{1}{2},$$  \hspace{1cm} (86)
$$A_1^{(4)} = -0.328\ 478\ 965\ 579\ 193\ \ldots,$$  \hspace{1cm} (87)
$$A_1^{(6)} = 1.181\ 241\ 456\ \ldots.$$  \hspace{1cm} (88)

Recent numerical evaluations have yielded values for the eighth- and tenth-order coefficients (Aoyama et al., 2014)

$$A_1^{(8)} = -1.912\ 98(84),$$  \hspace{1cm} (89)
$$A_1^{(10)} = 7.795(336).$$  \hspace{1cm} (90)

Higher-order coefficients are assumed to be negligible.

Mass-dependent coefficients for the electron, based on the CODATA-14 values of the mass ratios, are

$$A_2^{(4)}(m_e/m_\mu) = 5.197\ 386\ 76(23) \times 10^{-7} \rightarrow 24.182 \times 10^{-10} a_e,$$  \hspace{1cm} (91)
$$A_2^{(4)}(m_e/m_\tau) = 1.837\ 98(33) \times 10^{-9} \rightarrow 0.086 \times 10^{-10} a_e,$$  \hspace{1cm} (92)
$$A_2^{(6)}(m_e/m_\mu) = -7.373\ 941\ 71(24) \times 10^{-6}$$
$$\rightarrow -0.797 \times 10^{-10} a_e,$$  \hspace{1cm} (93)
$$A_2^{(6)}(m_e/m_\tau) = -6.5830(11) \times 10^{-8} \rightarrow -0.007 \times 10^{-10} a_e,$$  \hspace{1cm} (94)

Additional series expansions in the mass ratios yield (Kurz et al., 2014a)

$$A_2^{(8)}(m_e/m_\mu) = 9.161\ 970\ 83(33) \times 10^{-4} \rightarrow 0.230 \times 10^{-10} a_e,$$  \hspace{1cm} (95)
$$A_2^{(8)}(m_e/m_\tau) = 7.4292(12) \times 10^{-6} \rightarrow 0.002 \times 10^{-10} a_e.$$  \hspace{1cm} (96)

A numerical calculation gives the next term (Aoyama et al., 2012b)

$$A_2^{(10)}(m_e/m_\mu) = -0.003\ 82(39) \rightarrow -0.002 \times 10^{-10} a_e.$$  \hspace{1cm} (97)

Additional terms have been calculated, but are negligible at the current level of accuracy; see, for example, Aoyama et al. (2014) and Kurz et al. (2014a).

All contributing terms of each order in $\alpha$ are combined to yield the total coefficients in the series

$$a_e(\text{QED}) = \sum_{n=1}^{\infty} C_e^{(2n)} \left( \frac{\alpha}{\pi} \right)^n,$$  \hspace{1cm} (98)

with

$$C_e^{(2)} = 0.5,$$  \hspace{1cm} (99)
$$C_e^{(4)} = -0.328\ 478\ 444\ 00 \ldots,$$  \hspace{1cm} (99)
$$C_e^{(6)} = 1.181\ 234\ 017 \ldots,$$  \hspace{1cm} (99)
$$C_e^{(8)} = -1.912\ 06(84),$$  \hspace{1cm} (99)
$$C_e^{(10)} = 7.79(34),$$  \hspace{1cm} (99)

where higher-order coefficients are negligible.

The electroweak contribution, calculated as in CODATA-98 but with present values of $G_F$ and $\sin^2 \theta_W$ (see Sec. XII), is

$$a_e(\text{weak}) = 0.029\ 73(23) \times 10^{-12} = 0.2564(20) \times 10^{-10} a_e.$$  \hspace{1cm} (100)

The hadronic contribution is the sum

$$a_e(\text{had}) = a_e^{\text{LO,VP}}(\text{had}) + a_e^{\text{NLO,VP}}(\text{had}) + a_e^{\text{LL}}(\text{had}) + \ldots,$$  \hspace{1cm} (101)

where $a_e^{\text{LO,VP}}(\text{had})$ and $a_e^{\text{NLO,VP}}(\text{had})$ are the leading order and next-to-leading order (with an additional photon or electron loop) hadronic vacuum-polarization corrections given by Nomura and Teubner (2013)

$$a_e^{\text{LO,VP}}(\text{had}) = 1.866(11) \times 10^{-12},$$  \hspace{1cm} (102)
$$a_e^{\text{NLO,VP}}(\text{had}) = -0.2234(14) \times 10^{-12},$$  \hspace{1cm} (103)

and $a_e^{\text{NNLO,VP}}(\text{had})$ is the next-to-next-to-leading order vacuum-polarization correction (Kurz et al., 2014b)

$$a_e^{\text{NNLO,VP}}(\text{had}) = 0.0280(10) \times 10^{-12},$$  \hspace{1cm} (104)

and where $a_e^{\text{LL}}(\text{had})$ is the hadronic light-by-light scattering term given by Prades, de Rafael, and Vainshtein (2010)

$$a_e^{\text{LL}}(\text{had}) = 0.035(10) \times 10^{-12}.$$  \hspace{1cm} (105)

The total hadronic contribution is

$$a_e(\text{had}) = 1.734(15) \times 10^{-12} = 14.95(13) \times 10^{-10} a_e.$$  \hspace{1cm} (106)
The theoretical value is the sum of all the terms:

\[ a_e(\text{th}) = a_e(\text{QED}) + a_e(\text{weak}) + a_e(\text{had}), \]  

(107)
and has the standard uncertainty

\[ u[a_e(\text{th})] = 0.037 \times 10^{-12} = 0.32 \times 10^{-10} a_e, \]

(108)
where the largest contributions to the uncertainty are from the 8th- and 10th-order QED terms. This uncertainty is taken into account in the least-squares adjustment by writing

\[ a_e(\text{th}) = a_e(\alpha) + \delta_e, \]

(109)
where 0.000(37) \times 10^{-12} is the input datum for \( \delta_e \). It is noteworthy that the uncertainty of \( a_e(\text{th}) \) is significantly smaller than the uncertainty of the most accurate experimental value (see following section), which is \( 2.4 \times 10^{-10} a_e \).

2. Measurements of \( a_e \)

Two experimental values of \( a_e \) obtained using a Penning trap were initially included in CODATA-10. The first is the classic 1987 result from the University of Washington discussed in CODATA-98 with a relative standard uncertainty \( u_t = 3.7 \times 10^{-9} \) and which assumes that CPT invariance holds for the electron-positron system (Van Dyck, Schwinberg, and Dehmelt, 1987). The second is the much more accurate 2008 result from Harvard University discussed in CODATA-10 with \( u_t = 2.4 \times 10^{-10} \) (Hanneke, Fogwell, and Gabrielse, 2008). These results are items B22.1 and B22.2 in Table XVIII, Sec. XIII, with identifications UWash-87 and HarvU-08; the values of \( \alpha \) that can be inferred from them based on the theory of \( a_e \) discussed in the previous section are given in Table XX, Sec. XIII.A. The University of Washington result was not included in the final adjustment on which the CODATA 2010 recommended values are based because of its comparatively low weight and is also omitted from the 2014 final adjustment. It is only considered to help provide an overall picture of the data available for the determination of the 2014 recommended value of \( \alpha \).

B. Muon magnetic-moment anomaly \( a_\mu \)

Only the measured value of the muon magnetic-moment anomaly was included in the 2010 adjustment of the constants due to concerns about some aspects of the theory. The concerns still remain, thus the Task Group decided not to employ the theoretical expression for \( a_\mu \) in the 2014 adjustment. The theory and measurement of \( a_\mu \) and the reasons for the Task Group decision are presented in the following sections.

1. Theory of \( a_\mu \)

The relevant mass-dependent terms are

\[ A_2^{(4)}(m_\mu/m_e) = 1.0942583092(72) \rightarrow 5.06 \times 10^{-3} a_\mu, \]  

(110)

\[ A_2^{(6)}(m_\mu/m_e) = 0.000078079(14) \rightarrow 3.61 \times 10^{-7} a_\mu, \]

(111)
\[ A_2^{(6)}(m_\mu/m_e) = 22.86837998(17) \rightarrow 2.46 \times 10^{-4} a_\mu, \]

(112)
\[ A_2^{(6)}(m_\mu/m_e) = 0.00036063(11) \rightarrow 3.88 \times 10^{-9} a_\mu, \]

(113)
\[ A_2^{(8)}(m_\mu/m_e) = 132.6852(60) \rightarrow 3.31 \times 10^{-6} a_\mu, \]

(114)
\[ A_2^{(8)}(m_\mu/m_e) = 0.042494(53) \rightarrow 1.06 \times 10^{-9} a_\mu, \]

(115)
\[ A_2^{(10)}(m_\mu/m_e) = 742.18(87) \rightarrow 4.30 \times 10^{-8} a_\mu, \]

(116)
\[ A_2^{(10)}(m_\mu/m_e) = -0.068(5) \rightarrow -3.94 \times 10^{-12} a_\mu, \]

(117)
\[ A_3^{(6)}(m_\mu/m_e,m_\mu/m_e) = 0.000527762(94) \rightarrow 5.67 \times 10^{-9} a_\mu, \]

(118)
\[ A_3^{(8)}(m_\mu/m_e,m_\mu/m_e) = 0.06272(4) \rightarrow 1.57 \times 10^{-9} a_\mu, \]

(119)
\[ A_3^{(10)}(m_\mu/m_e,m_\mu/m_e) = 2.011(10) \rightarrow 1.17 \times 10^{-10} a_\mu. \]

(120)

New mass-dependent contributions reported since CODATA-10 are Eqs. (114), (116), (117), (119), and (120) from Aoyama et al. (2012a) and Eq. (115) from Kurz et al. (2014a).

The total for \( a_\mu(\text{QED}) \) is

\[ a_\mu(\text{QED}) = \sum_{n=1}^{\infty} C^{(2n)}_\mu \left( \frac{n}{\pi} \right)^n, \]

(121)
with

\[ C^{(2)}_\mu = 0.5, \]

(122)
\[ C^{(4)}_\mu = 0.765857423(16), \]

(123)
\[ C^{(6)}_\mu = 24.05050982(27), \]

(124)
\[ C^{(8)}_\mu = 130.8774(61), \]

(125)
\[ C^{(10)}_\mu = 751.92(93). \]

(126)

Based on the 2014 recommended value of \( \alpha \), this yields

\[ a_\mu(\text{QED}) = 0.00116584718858(87) \quad [7.4 \times 10^{-10}], \]  

(127)
where an uncertainty of \( 0.8 \times 10^{-12} \) is included to account for uncalculated 12th-order terms (Aoyama et al., 2012a).

As for the electron, there are additional contributions:

\[ a_\mu(\text{th}) = a_\mu(\text{QED}) + a_\mu(\text{weak}) + a_\mu(\text{had}). \]

(128)

The primarily electroweak contribution is (Czarnecki, Marciano, and Vainshtein, 2003; Gnendiger, Stöckinger, and Stöckinger-Kim, 2013)

\[ a_\mu(\text{weak}) = 154(1) \times 10^{-11}. \]

(129)

Separate contributions (lowest-order vacuum polarization, next-to-lowest-order, and light-by-light) to the hadronic term are

\[ a_\mu(\text{had}) = a_\mu^{\text{LO,V}}(\text{had}) + a_\mu^{\text{NLO,V}}(\text{had}) + a_\mu^{\text{LL}}(\text{had}) + \cdots, \]

(130)
where (Hagiwara et al., 2011)

\[
a_{\mu}^{\text{LO,VP}}(\text{had}) = 6949(43) \times 10^{-11},
\]

(127)

\[
a_{\mu}^{\text{NLO,VP}}(\text{had}) = -98.40(72) \times 10^{-11},
\]

(128)

and (Jegerlehner and Nyffeler, 2009)

\[
a_{\mu}^{\text{LL}}(\text{had}) = 116(40) \times 10^{-11}.
\]

(129)

The total hadronic contribution is then

\[
a_{\mu}(\text{had}) = 6967(59) \times 10^{-11}.
\]

(130)

In analogy with the electron we have finally

\[
a_{\mu}(\text{th}) = a_{\mu}(\text{QED}) + 7121(59) \times 10^{-11}.
\]

(131)

with the standard uncertainty

\[
u[a_{\mu}(\text{th})] = 59 \times 10^{-11} = 51 \times 10^{-11} a_{\mu}.
\]

(132)

The largest contributions to the uncertainty are from \(a_{\mu}^{\text{LO,VP}}(\text{had})\) and \(a_{\mu}^{\text{LL}}(\text{had})\); their respective uncertainties of \(43 \times 10^{-11}\) and \(40 \times 10^{-11}\) are nearly the same. By comparison, the \(0.087 \times 10^{-11}\) uncertainty of \(a_{\mu}(\text{QED})\) is negligible. However, the \(63 \times 10^{-11}\) uncertainty of the experimental value of \(a_{\mu}\), which is discussed in the following section, and \(u[a_{\mu}(\text{th})]\) are nearly the same. Based on the 2014 recommended value of \(\alpha\), Eq. (131) yields

\[
a_{\mu}(\text{th}) = 1.165 918 39(59) \times 10^{-3}
\]

(133)

for the theoretically predicted value of \(a_{\mu}\).

2. Measurement of \(a_{\mu}\): Brookhaven

The experimental determination of \(a_{\mu}\) at Brookhaven National Laboratory (BNL), Upton, New York, USA, has been discussed in the past four CODATA reports. The quantity measured is the anomaly difference frequency \(f_{s} = f_{s} - f_{e}\), where \(f_{s} = |g_{s}|(e\hbar/2m_{\mu})B/\hbar\) is the muon spin-flip (or precession) frequency in the applied magnetic flux density \(B\) and \(f_{e} = eB/2\pi m_{\mu}\) is the corresponding muon cyclotron frequency. The flux density is eliminated from these expressions by determining its value using proton nuclear magnetic resonance (NMR) measurements. This means that the muon anomaly is calculated from

\[
a_{\mu}(\text{exp}) = \frac{R}{[\mu_{B}/\mu_{P}] - R}
\]

(134)

where \(R = f_{s}/f_{p}\) and \(f_{p}\) is the free proton NMR frequency corresponding to the average flux density \(B\) seen by the muons in their orbits in the muon storage ring.

The final value of \(R\) obtained in the experiment is, from Table XV of Bennett et al. (2006),

\[
R = 0.003 707 2063(20),
\]

which is used as an input datum in the 2014 adjustment and is the same as used in the 2010 adjustment. It is datum B24 in Table XVIII with identification BNL-06. Based on this value of \(R\), Eq. (134), and the 2014 recommended value of \(\mu_{B}/\mu_{P}\), whose uncertainty is negligible in this context, the experimental value of the muon anomaly is

\[
a_{\mu}(\text{exp}) = 1.165 920 89(63) \times 10^{-3}.
\]

(135)

Further, with the aid of Eq. (225), the equation for \(R\) can be written as

\[
R = -\frac{a_{\mu}}{1 + a_{e} m_{e} \mu_{e}}.
\]

(137)

where use has been made of the relations \(g_{\mu} = -2(1 + a_{e})\), \(g_{e} = -2(1 + a_{p})\), and \(a_{\mu}\) is replaced by the theoretical expression given in Eq. (109) for the observational equation. If the theory of \(a_{\mu}\) were not problematic and used in adjustment calculations, then \(a_{\mu}\) in Eq. (137) would be its theoretical expression, which mainly depends on \(\alpha\). If the theory is omitted, then \(a_{\mu}\) in that equation is simply taken to be an adjusted constant. The following section discusses why the Task Group decided to do the latter.

3. Comparison of theory and experiment for \(a_{\mu}\)

The difference between the experimental value of \(a_{\mu}\) in Eq. (136) and the theoretical value in Eq. (133) is \(250(86) \times 10^{-11}\), which is 2.9 times the standard uncertainty of the difference or 2.9\(\sigma\). The terms \(a_{\mu}^{\text{LO,VP}}(\text{had})\) and \(a_{\mu}^{\text{LL}}(\text{had})\) are the dominant contributors to the uncertainty of \(a_{\mu}(\text{th})\), and a smaller value of either will increase the disagreement while a larger value will decrease it.

The value for \(a_{\mu}^{\text{LO,VP}}(\text{had})\) used to obtain \(a_{\mu}(\text{th})\) is \(6949(43) \times 10^{-11}\), but an equally credible value, 6923(42) \(\times 10^{-11}\), is also available (Davier et al., 2011). If it is used instead, the discrepancy is 276(86) \(\times 10^{-11}\) or 3.2\(\sigma\). Both values are based on very thorough analyses using theory and experimental data from the production of hadrons in \(e^{+}e^{-}\) collisions. Davier et al. (2011) also obtain the value 7015(47) \(\times 10^{-11}\) using both \(e^{+}e^{-}\) annihilation data and data from the decay of the \(\tau\) into hadrons. For this value the discrepancy is reduced to 2.1\(\sigma\). A result due to Jegerlehner and Szafirn (2011) also obtained using \(e^{+}e^{-}\) annihilation data and \(\tau\) decay data but which is only in marginal agreement with this value is 6910(47) \(\times 10^{-11}\). However, it agrees with the first two values, which are based solely on \(e^{+}e^{-}\) data. For this value the discrepancy is 3.3\(\sigma\). In a lengthy paper Benayoun et al. (2013) used the hidden local symmetry model or HLS to obtain values of \(a_{\mu}^{\text{LO,VP}}(\text{had})\) that are significantly smaller than others and that lead to a discrepancy of between 4\(\sigma\) and 5\(\sigma\). However, Davier and Malaescu (2013) give a number of reasons why these values may not be reliable.

The value for \(a_{\mu}^{\text{LL}}(\text{had})\) used to obtain \(a_{\mu}(\text{th})\) is 116(40) \(\times 10^{-11}\), but there are other credible values in the
literature. For example, Prades, de Rafael, and Vainshtein (2010) give 105(26) × 10^{-11} while Melnikov and Vainshtein (2004) find 136(25) × 10^{-11}. A more recent value, due to Narukawa et al. (2014), is 118(20) × 10^{-11}, and the largest is 188(4) × 10^{-11} reported by Goecke, Fischer, and Williams (2013). Others range from 80(40) × 10^{-11} to 107(17) × 10^{-11}. Recent brief overviews of $\alpha^\text{LL}_{\text{had}}$ may be found in Dorokhov, Radzhabov, and Zhevylakov (2014a, 2014b), Nyffeler (2014), and Adikaram et al. (2015), which describe the many obstacles to obtaining a reliable estimate of its value. The conclusion that emerges from these papers is that $\alpha^\text{LL}_{\text{had}}$ is quite model dependent and the reliability of the estimates is questionable. Note that in calculating the value 116(40) × 10^{-11} used to obtain Eq. (133), Nyffeler (2009) adds the uncertainty components linearly and rounds the result up from 39 × 10^{-11} to 40 × 10^{-11}. This result is also thoroughly discussed in the detailed review by Jegerlehner and Nyffeler (2009) (see especially Table 13).

Although much work has been done over the past 4 years to improve the theory of $\alpha_{\text{a}}$, the discrepancy between experiment and theory remains at about the 3σ level. Further, there is a significant spread in the values of $\alpha^\text{LL}_{\text{had}}$ and $\alpha^\text{LL}_{\text{had}}$, the two most problematic contributions to $\alpha_{\text{a}}$, it is not obvious which are the best values. Expanding the uncertainty of $\alpha_{\text{a}}(\text{th})$ to reflect this spread would reduce its contribution to the determination of the 2014 CODATA recommended value of $\alpha_{\text{a}}$, significantly. Expanding the uncertainties of both $\alpha_{\text{a}}(\text{th})$ and $\alpha_{\text{a}}(\text{exp})$ to reduce the discrepancy to an acceptable level and including both would mean that the recommended value would cease to be a useful reference value for future comparisons of theory and experiment; it might tend to cover up an important physics problem rather than emphasizing it. For all these reasons, the Task Group chose not to include $\alpha_{\text{a}}(\text{th})$ in the 2014 adjustment and to base the 2014 recommended value on experiment only as was the case in 2010.

C. Proton magnetic moment in nuclear magnetons $\mu_p / \mu_N$

The 2010 recommended value of the magnetic moment of the proton $\mu_p$ in units of the nuclear magneton $\mu_N = e\hbar / 2m_p$ has a relative standard uncertainty $u_\epsilon = 8.2 \times 10^{-9}$. It was not measured directly but calculated from the relation $\mu_p / \mu_N = \langle \mu_p / \mu_B \rangle / A_{\text{e}}(p) / A_{\text{e}}(e)$, where $A_{\text{e}}(p)$ and $A_{\text{e}}(e)$ are adjusted constants and $\mu_B = e\hbar / 2m_e$ is the Bohr magneton. The proton magnetic moment in units of $\mu_B$ is calculated from $\mu_p / \mu_B = (\mu_p / \mu_B) / (\mu_e / \mu_B)$, where $\mu_e / \mu_B = 1 + a_8$ is extremely well known and $\mu_e / \mu_B$ is an adjusted constant determined mainly by the experimentally measured value of the bound-state magnetic-moment ratio in hydrogen $\mu_e(H) / \mu_B(H)$ taken as an input datum.

Now, however, a directly determined value of $\mu_p / \mu_N$ with $u_\epsilon = 3.3 \times 10^{-9}$ obtained from measurements on a single proton in a double Penning trap at the Institut für Physik, Johannes Gutenberg Universität Mainz (or simply the University of Mainz), Mainz, Germany, is available for inclusion in the 2014 adjustment. The spin-flip transition frequency in a magnetic flux density $B$ is

$$\omega_s = \frac{\Delta E}{\hbar} = \frac{2\mu_p B}{\hbar},$$

and the cyclotron frequency for the proton is

$$\omega_c = \frac{eB}{m_p}.$$

The ratio for the same magnetic flux density $B$ is just

$$\frac{\omega_s}{\omega_c} = \frac{\mu_p}{\mu_N}.$$

The value employed as an input datum in the 2014 adjustment is

$$\frac{\mu_p}{\mu_N} = 2.792 847 3498(93) \ [3.3 \times 10^{-9}],$$

which is the result reported by Mooser et al. (2014) but with an additional digit for both the value and uncertainty provided to the Task Group by coauthor Blaum (2014). This value, which we identify as UMZ-14, is the culmination of an extensive research program carried out over many years. Descriptions of the key advances made in the development of the double Penning trap used in the experiment are given in a number of publications (Ulmer, Blaum et al., 2011; Ulmer, Rodegheri et al., 2011; Mooser et al., 2013; Mooser, Kracke et al., 2013; Ulmar et al., 2013).

The double Penning trap consists of a precision trap of inner diameter 7 mm and an analysis trap of inner diameter 3.6 mm connected by transport electrodes, all mounted in the horizontal bore of a superconducting magnet with $B \approx 1.89$ T. Storage times for a single proton of no less than 1 year are achieved by enclosing the entire apparatus in a sealed vacuum chamber cooled to 4 K where pressures below $10^{-14}$ Pa are reached. The measurement sequence is straightforward but its implementation is complex; the cited papers should be consulted for details. Sputtered atoms created by electrons from a field emission gun hitting a polyethylene target are ionized in the center of the precision trap. A single proton is obtained from the resulting ion cloud, its cyclotron frequency of approximately 29 MHz is determined, and an electromagnetic signal near the proton’s precession frequency of approximately 81 MHz is applied to it. The proton is then transported to the analysis trap where the spin state of the proton is determined and the proton returned to the precision trap. By varying the frequency of the spin-flip signal and repeating the process many times, a resonance curve of the probability $P(\omega_s / \omega_c)$ as a function of $\omega_s$ is obtained. Its maximum is located at $\omega_s / \omega_c = \mu_p / \mu_N$. The statistical relative standard uncertainty of the value given in Eq. (141) is $2.6 \times 10^{-9}$, and the net fractional correction for systematic effects is $-0.64(2.04) \times 10^{-9}$. The largest contributor by far to the uncertainty of the net correction is the $2 \times 10^{-9}$ relative uncertainty assigned by Mooser et al. (2014) for the possible effect of nonlinear drifts of the magnetic field.

The relationships given at the beginning of this section show how the 2010 recommended value of $\mu_p / \mu_N$ is based on the relation

\[ \omega_s = \frac{\Delta E}{\hbar} = \frac{2\mu_p B}{\hbar}, \]
where \( \mu_Z / \mu_N = -(1 + \alpha_z) A_z(p) \mu_p / A_z(e) \mu_e \),

\[(142)\]

which is the source of the observational equation B29 in Table XXIV.

For completeness, we note that a value of \( \mu_p / \mu_N \) with \( \omega_r = 8.9 \times 10^{-6} \) from the double Penning trap in an earlier stage of development has been published by the Mainz group (Rodegheri et al., 2012), as has a value with \( \omega_r = 2.5 \times 10^{-6} \) obtained by a direct measurement at Harvard University with a different type of Penning trap (Lees et al., 2012). Also noteworthy is the use of a similar double Penning trap at CERN to test CPT invariance by comparing the antiproton-to-proton charge-to-mass ratio, demonstrating that they agree within an uncertainty of 69 parts in 10^{12} (DiSciacca et al., 2013).

### D. Atomic g-factors in hydrogenic \(^{12}\)C and \(^{28}\)Si and \( A_z(e) \)

The most accurate values of the relative atomic mass of the electron \( A_z(e) \) are obtained from measurements of the electron g-factor in hydrogenic ions, silicon and carbon in particular, and theoretical expressions for the g-factors. In fact, the uncertainties of the values so obtained are now so small that none of the data previously used to determine \( A_z(e) \) remain of interest.

For a hydrogenic ion \( X \) with a spinless nucleus and atomic number \( Z \), the energy-level shift in an applied magnetic flux density \( B \) in the \( z \) direction is given by

\[
E = -\mu \cdot B = -g(X) \frac{e}{2m_e} J_z B,
\]

\[(143)\]

where \( J_z \) is the electron angular-momentum projection in the \( z \) direction and \( g(X) \) is the atomic g-factor. In the ground \( 1S_{1/2} \) state, \( J_z = \pm h/2 \), so the splitting between the two levels is

\[
\Delta E = |g(X)| \frac{eB}{2m_e},
\]

\[(144)\]

and the spin-flip transition frequency is

\[
\omega_s = \frac{\Delta E}{\hbar} = |g(X)| \frac{eB}{2m_e},
\]

\[(145)\]

### Table XI. Theoretical contributions and total value for the g-factor of hydrogenic carbon 12 based on the 2014 recommended values of the constants

<table>
<thead>
<tr>
<th>Contribution</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dirac ( g_D )</td>
<td>(-1.998 721 354 392 1(6))</td>
<td>Eq. (149)</td>
</tr>
<tr>
<td>( \Delta g_{\text{He}}^{(2s)} )</td>
<td>(-0.002 323 672 435(4))</td>
<td>Eq. (157)</td>
</tr>
<tr>
<td>( \Delta g_{\text{He}}^{(2p)} )</td>
<td>(0.000 000 008 511)</td>
<td>Eq. (160)</td>
</tr>
<tr>
<td>( \Delta g_{\text{He}}^{(3s)} )</td>
<td>(0.000 003 545 677(25))</td>
<td>Eq. (164)</td>
</tr>
<tr>
<td>( \Delta g_{\text{He}}^{(4s)} )</td>
<td>(-0.000 000 029 618)</td>
<td>Eq. (166)</td>
</tr>
<tr>
<td>( \Delta g_{\text{He}}^{(5s)} )</td>
<td>(0.000 000 111)</td>
<td>Eq. (167)</td>
</tr>
<tr>
<td>( \Delta g_{\text{He}}^{(6s)} )</td>
<td>(-0.000 000 000 001)</td>
<td>Eq. (168)</td>
</tr>
<tr>
<td>( \Delta g_{\text{He}}^{(7s)} )</td>
<td>(-0.000 000 087 629)</td>
<td>Eqs. (169), (170)</td>
</tr>
<tr>
<td>( \Delta g_{\text{He}}^{(8s)} )</td>
<td>(-0.000 000 000 408(1))</td>
<td>Eq. (172)</td>
</tr>
<tr>
<td>( g_{(12\text{C}^{1+})} )</td>
<td>(-2.001 041 590 183(26))</td>
<td>Eq. (173)</td>
</tr>
</tbody>
</table>

In the same flux density, the ion’s cyclotron frequency is

\[
\omega_c = \frac{q_X B}{m_X},
\]

\[(146)\]

where \( q_X = (Z - 1)e \) is the net charge of the ion and \( m_X \) is its mass. Thus the frequency ratio is

\[
\frac{\omega_s}{\omega_c} = \frac{|g(X)|}{2(Z-1)} \frac{m_X}{m_e} = \frac{|g(X)|}{2(Z-1)} A_z(X),
\]

\[(147)\]

where \( A_z(X) \) is the relative atomic mass of the ion.

### 1. Theory of the bound-electron g-factor

The bound-electron g-factor is given by

\[
g(X) = g_D + \Delta g_{\text{rad}} + \Delta g_{\text{rec}} + \Delta g_{\text{ns}} + \cdots,
\]

\[(148)\]

where the individual terms on the right-hand side are the Dirac value, radiative corrections, recoil corrections, nuclear-size corrections, and the dots represent possible additional corrections not already included. Tables XI and XII give the numerical values of the various contributions.

The Dirac value is (Breit, 1928)

\[
g_D = -\frac{2}{3} \left[ 1 + 2 \sqrt{1 - (Z\alpha)^2} - 2 \left[ 1 - \frac{1}{3} (Z\alpha)^2 - \frac{1}{12} (Z\alpha)^4 - \frac{1}{24} (Z\alpha)^6 + \cdots \right] \right],
\]

\[(149)\]

where the only uncertainty is due to the uncertainty in \( \alpha \).

Radiative corrections are given by

\[
\Delta g_{\text{rad}} = -2 \sum_{n=1}^{\infty} c_n^{(2n)} (Z\alpha)^n \frac{\alpha}{2\pi},
\]

\[(150)\]

where the limits

\[
\lim_{Z\alpha \to 0} c_n^{(2n)} (Z\alpha) = C_n^{(2n)}
\]

are given in Eq. (99).

The first term is (Faustov, 1970; Grotch, 1970; Close and Osborn, 1971; Pachucki, Jentschura, and Yerokhin, 2004; Pachucki et al., 2005)

### Table XII. Theoretical contributions and total value for the g-factor of hydrogenic silicon 28 based on the 2014 recommended values of the constants

<table>
<thead>
<tr>
<th>Contribution</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dirac ( g_D )</td>
<td>(-1.993 023 571 557(3))</td>
<td>Eq. (149)</td>
</tr>
<tr>
<td>( \Delta g_{\text{He}}^{(2s)} )</td>
<td>(-0.002 328 917 47(5))</td>
<td>Eq. (157)</td>
</tr>
<tr>
<td>( \Delta g_{\text{He}}^{(2p)} )</td>
<td>(0.000 000 234 81(1))</td>
<td>Eq. (160)</td>
</tr>
<tr>
<td>( \Delta g_{\text{He}}^{(3s)} )</td>
<td>(0.000 003 552(17))</td>
<td>Eq. (164)</td>
</tr>
<tr>
<td>( \Delta g_{\text{He}}^{(4s)} )</td>
<td>(-0.000 000 029 66)</td>
<td>Eq. (166)</td>
</tr>
<tr>
<td>( \Delta g_{\text{He}}^{(5s)} )</td>
<td>(0.000 000 111)</td>
<td>Eq. (167)</td>
</tr>
<tr>
<td>( \Delta g_{\text{He}}^{(6s)} )</td>
<td>(-0.000 000 000 000)</td>
<td>Eq. (168)</td>
</tr>
<tr>
<td>( \Delta g_{\text{He}}^{(7s)} )</td>
<td>(-0.000 000 087 629)</td>
<td>Eqs. (169), (170)</td>
</tr>
<tr>
<td>( \Delta g_{\text{He}}^{(8s)} )</td>
<td>(-0.000 000 000 408(1))</td>
<td>Eq. (172)</td>
</tr>
<tr>
<td>( g_{(28\text{Si}^{1+})} )</td>
<td>(-1.995 348 958(17))</td>
<td>Eq. (173)</td>
</tr>
</tbody>
</table>
Values for the remainder function $R_{SE}(Z\alpha)$ are based on extrapolations from numerical calculations at higher $Z$ (Yerokhin, Indelicato, and Shabaev, 2002, 2004; Pachucki, Jentschura, and Yerokhin, 2004); see also Yerokhin and Jentschura (2008, 2010). In CODATA-10, the values for carbon and oxygen were taken directly from Pachucki, Jentschura, and Yerokhin (2004). The value for silicon in Eq. (156) is obtained here by extrapolation of the data in Yerokhin, Indelicato, and Shabaev (2004) with a fitting function of the form $a + (Z\alpha)^b + \ln(Z\alpha)^c + d \ln^2(Z\alpha)^e$. We thus have

$$C_{e,SE}^{(2)}(6\alpha) = 0.500 183 606 65 (80),$$

$$C_{e,SE}^{(2)}(14\alpha) = 0.501 312 630 (11).$$

(157)

The lowest-order vacuum-polarization correction consists of a wave-function correction and a potential correction, each of which can be separated into a lowest-order Uehling potential contribution and a Wichmann-Kroll higher-order contribution. The wave-function correction is (Beier, 2000; Beier et al., 2000; Karshenboim, 2000; Karshenboim, Ivanov, and Shabaev, 2001a, 2001b)

$$C_{e,VPwf}^{(2)}(6\alpha) = -0.000 001 840 3431 (43),$$

$$C_{e,VPwf}^{(2)}(14\alpha) = -0.000 051 091 98 (22).$$

(158)

For the potential correction, the Uehling contribution vanishes (Beier et al., 2000), and for the Wichmann-Kroll part, we take the value of Lee et al. (2005), which has a negligible uncertainty from omitted binding corrections for the present level of accuracy. This gives

$$C_{e,VP}^{(2)}(6\alpha) = 0.000 000 008 201 (11),$$

$$C_{e,VP}^{(2)}(14\alpha) = 0.000 000 5467 (11).$$

(159)

The total vacuum polarization is the sum of Eqs. (158) and (159):

$$C_{e,VP}^{(2)}(6\alpha) = C_{e,VPwf}^{(2)}(6\alpha) + C_{e,VP}^{(2)}(6\alpha) = -0.000 001 832 142 (12),$$

$$C_{e,VP}^{(2)}(14\alpha) = C_{e,VPwf}^{(2)}(14\alpha) + C_{e,VP}^{(2)}(14\alpha) = -0.000 050 5452 (11).$$

(160)

One-photon corrections are the sum of Eqs. (157) and (160):

$$C_{e}^{(2)}(6\alpha) = C_{e,SE}^{(2)}(6\alpha) + C_{e,VP}^{(2)}(6\alpha) = 0.500 181 774 51 (80),$$

$$C_{e}^{(2)}(14\alpha) = C_{e,SE}^{(2)}(14\alpha) + C_{e,VP}^{(2)}(14\alpha) = 0.501 262 085 (11),$$

(161)

which yields

$$\Delta \theta_{\text{rad}}^{(2)} = -2C_{e}^{(2)}(Z\alpha) \frac{(Z\alpha)^4}{4\pi} = -0.002 323 663 924 (4) \text{ for } Z = 6$$

$$= -0.002 328 682 65 (5) \text{ for } Z = 14. \quad (162)$$

The leading binding correction is known to all orders in $\alpha/\pi$ (Eides and Groth, 1997; Czamecki, Melnikov, and Yelkhovsky, 2001):

$$C_{e}^{(2)}(Z\alpha) = C_{e}^{(2)}\left(1 + \frac{(Z\alpha)^2}{6} + \cdots\right).$$

(163)

To order $(Z\alpha)^4$, the two-photon correction for the ground S state is (Pachucki et al., 2005; Jentschura et al., 2006)

$$C_{e}^{(4)}(Z\alpha) = C_{e}^{(4)}\left(1 + \frac{(Z\alpha)^2}{6} + (Z\alpha)^4 \left[rac{14}{9} \ln(Z\alpha)^{-2} + \frac{991 343}{155 520} \ln k_0 - \frac{4}{3} \ln k_3 + \frac{679 \pi^2}{12 \times 960} \frac{1441 \pi^2}{720} \ln 2 + \frac{1441 \zeta(3)}{480} \right] + O(Z\alpha)^5\right)$$

$$= -0.328 5778 (23) \text{ for } Z = 6$$

$$= -0.329 17 (15) \text{ for } Z = 14, \quad (164)$$

where $C_{e}^{(4)} = -0.328 478 444 0 \ldots$, and where $\ln k_0$ and $\ln k_3$ are given in Eqs. (153) and (154). Pachucki et al. (2005) have estimated the uncertainty due to uncalculated higher-order contributions to be

$$u[C_{e}^{(4)}(Z\alpha)] = 2\left[(Z\alpha)^5 C_{e}^{(4)} R_{SE}(Z\alpha)\right],$$

(165)

which we use as the uncertainty. Since the remainder functions differ only by about 1% for carbon and silicon, the main Z dependence of the uncertainty is given by the factor $(Z\alpha)^5$. We shall assume that the uncertainty of the two-photon correction is completely correlated for the two charged ions. As a consequence, information from the silicon measured value will effectively be included in the theoretical prediction for the carbon value through the least-squares adjustment formalism.

Jentschura (2009) and Yerokhin and Harmann (2013) have calculated two-loop vacuum-polarization diagrams of the same order as the uncertainty in Eq. (164).

Equation (163) gives the leading two terms of the higher-loop contributions. The corrections are
\[ C_e^{(6)}(Z\alpha) = C_e^{(6)} \left( 1 + \frac{(Z\alpha)^2}{6} + \cdots \right) \]
\[ = 1.181\,611 \ldots \quad \text{for } Z = 6 \]
\[ = 1.183\,289 \ldots \quad \text{for } Z = 14, \quad (166) \]

where \( C_e^{(6)} = 1.181\,234\,017 \ldots \).

\[ C_e^{(8)}(Z\alpha) = C_e^{(8)} \left( 1 + \frac{(Z\alpha)^2}{6} + \cdots \right) \]
\[ = -1.912\,678(84) \ldots \quad \text{for } Z = 6 \]
\[ = -1.915\,380(84) \ldots \quad \text{for } Z = 14, \quad (167) \]

where \( C_e^{(8)} = -1.912\,068(84) \), and

\[ C_e^{(10)}(Z\alpha) = C_e^{(10)} \left( 1 + \frac{(Z\alpha)^2}{6} + \cdots \right) \]
\[ = 7.79(34) \ldots \quad \text{for } Z = 6 \]
\[ = 7.80(34) \ldots \quad \text{for } Z = 14, \quad (168) \]

where \( C_e^{(10)} = 7.79(34) \).

Recoil of the nucleus gives a correction proportional to the electron-nucleus mass ratio. It can be written as \( \Delta g_{\text{rec}} = \Delta g_{\text{rec}}^{(0)} + \Delta g_{\text{rec}}^{(2)} + \cdots \), where the two terms are zero and first order in \( \alpha/\pi \), respectively. The first term is \( \text{(Eides and Grotch, 1997; Shabaev and Yerokhin, 2002)} \)

\[ \Delta g_{\text{rec}}^{(0)} = \left\{ - (Z\alpha)^2 + \frac{(Z\alpha)^4}{3 \left[ 1 + \sqrt{1 - (Z\alpha)^2} \right]} \right\} \frac{m_e}{m_N} \]
\[ + (1 + Z)(Z\alpha)^2 \left( \frac{m_e}{m_N} \right)^2 \]
\[ = -0.000\,000\,087\,70 \ldots \quad \text{for } Z = 6 \]
\[ = -0.000\,000\,206\,04 \ldots \quad \text{for } Z = 14, \quad (169) \]

where \( m_N \) is the mass of the nucleus. Mass ratios, based on the current adjustment values of the constants, are \( m_e/m_{^{12}\text{C}^{6+}} = 0.000\,045\,727\,5 \ldots \) and \( m_e/m_{^{28}\text{Si}^{14+}} = 0.000\,019\,613\,6 \ldots \) [see Eqs. (10) and (3)]. For silicon, we use the interpolated value \( P(14\alpha) = 7.162\,23(1) \).

For \( \Delta g_{\text{rec}}^{(2)} \), we have

\[ \Delta g_{\text{rec}}^{(2)} = \frac{\alpha}{\pi} (Z\alpha)^2 \frac{m_e}{m_N} + \cdots \]
\[ = 0.000\,000\,000\,06 \ldots \quad \text{for } Z = 6 \]
\[ = 0.000\,000\,000\,15 \ldots \quad \text{for } Z = 14. \quad (170) \]

The uncertainty in \( \Delta g_{\text{rec}}^{(2)} \) is negligible compared to that of \( \Delta g_{\text{rec}}^{(0)} \).

The uncertainty in \( \Delta g_{\text{rec}}^{(2)} \) is given to lowest order in \( (Z\alpha)^2 \) by \( \text{(Karshenboim, 2000)} \)

\[ \Delta g_{\text{rec}}^{(2)} = \frac{8}{3} (Z\alpha)^4 \frac{R_N}{\lambda_C} \]
\[ \Delta g_{\text{rec}} = -\frac{8}{3} (Z\alpha)^4 \left( \frac{R_N}{\lambda_C} \right)^2, \quad (171) \]

where \( R_N \) is the bound-state nuclear rms charge radius and \( \lambda_C \) is the Compton wavelength of the electron divided by \( 2\pi \).

Glazov and Shabaev (2002) have calculated additional corrections within perturbation theory. Scaling their results with the squares of updated values for the nuclear radii \( R_N = 2.4703(22) \) fm and \( R_N = 3.1223(24) \) fm from the compilation of Angeli (2004) for \(^{12}\text{C} \) and \(^{28}\text{Si} \) respectively yields

\[ \Delta g_{\text{rec}} = -0.000\,000\,000\,408(1) \quad \text{for } ^{12}\text{C}, \]
\[ \Delta g_{\text{rec}} = -0.000\,000\,020\,53(3) \quad \text{for } ^{16}\text{Si}. \quad (172) \]

Tables XI and XII list the contributions discussed above and totals given by

\[ g^{(12}\text{C}^{5+}) = -2.001\,041\,590\,183(26), \]
\[ g^{(28}\text{Si}^{13+}) = -1.995\,348\,958\,1(17). \quad (173) \]

For the purpose of the least-squares adjustment, we write the theoretical expressions for the \( g \)-factors as

\[ g^{(12}\text{C}^{5+}) = g_C(\alpha) + \delta_C, \]
\[ g^{(28}\text{Si}^{13+}) = g_Si(\alpha) + \delta_Si, \quad (174) \]

where the first term on the right-hand side of each expression gives the calculated value along with its functional dependence on \( \alpha \). The second term contains the theoretical uncertainty in the calculated value, except for the component due to uncertainty in \( \alpha \), which is taken into account by the least-squares algorithm through the first term. We thus have

\[ \delta_C = 0.0(2.6) \times 10^{-11}, \quad (175) \]
\[ \delta_Si = 0.0(1.7) \times 10^{-9}. \quad (176) \]

In each case, the uncertainty is dominated by uncalculated two-loop higher-order terms, which are expected to be mainly proportional to \( (Z\alpha)^5 \). For the one-loop self-energy, approximately 85% of the remainder scales as \( (Z\alpha)^5 \) between C and Si [see Eqs. (155) and (156)]. As a conservative estimate, we shall assume that 80% of the uncertainty scales as \( (Z\alpha)^5 \) in the case of the two-loop uncertainty. As a result, information from measurement of the Si \( g \)-factor will provide information about the two-loop uncertainty in C and vice versa through the covariance of the deltas in the least-squares adjustment. The covariance of \( \delta_C \) and \( \delta_Si \) is

\[ u(\delta_C, \delta_Si) = 3.4 \times 10^{-20}, \quad (177) \]

which corresponds to a correlation coefficient of

\[ r(\delta_C, \delta_Si) = 0.79. \]

2. Measurements of \( g^{(12}\text{C}^{5+}) \) and \( g^{(28}\text{Si}^{13+}) \)

As discussed at the start of Sec. V.D, recent measurements of the electron \( g \)-factor in hydrogenic carbon and silicon together with theory provide a value of \( A_l(e) \) with an uncertainty so small that the data used in the CODATA 2010 adjustment to determine \( A_l(e) \) are no longer competitive and need not be considered. As indicated at the end of that section, the experimental quantities actually determined are the ratios of the electron spin precession (or spin-flip) frequency in
hydrogenic carbon and silicon ions to the cyclotron frequency of the ions, both in the same magnetic flux density. The result used in the 2014 adjustment for hydrogenic silicon is

\[
\frac{\omega_{{c(28\text{Si}^{13}\text{+})}}}{\omega_{{c(28\text{Si}^{13}\text{+})}}} = 3912.866\,064\,84(19) \times [4.8 \times 10^{-11}] . \tag{178}
\]

This value, determined by the group at the Max-Planck-Institut für Kernphysik (MPIK), Heidelberg, Germany, is based on additional information including a detailed uncertainty budget provided to the Task Group by the experimenters (Sturm, 2015). The information was needed to calculate the covariance between the silicon result and the MPIK carbon result discussed below. The value in Eq. (178) differs slightly from that published by the MPIK group (Sturm et al., 2013) as a consequence of the group’s reassessment of several small corrections for systematic effects (Sturm, 2015). The net fractional correction applied to the uncorrected ratio to obtain the final ratio given by Sturm et al. (2013) is \(-638.9 \times 10^{-12}\) while the net fractional correction applied to obtain Eq. (178) is \(-678.5 \times 10^{-12}\). The largest of these corrections by far, \(-659(33) \times 10^{-12}\), is due to image charge and the next largest, \(-20(10) \times 10^{-12}\), is due to frequency pulling. The statistical relative uncertainty of eight individual measurements is \(33 \times 10^{-12}\). We identify the result in Eq. (178) as MPIK-15.

Both the silicon and carbon ratios were obtained using the MPIK triple cylindrical Penning trap operating at \(B = 3.8 \, \text{T}\) and in thermal contact with a liquid helium bath. This trap is similar in design and operation to the double Penning trap used to directly measure \(\mu_B/\mu_N\) as discussed in Sec. V.C. The first stage of the experiment occurs in the creation trap in which ions are created; the second stage is carried out in the analysis trap where the spin state of a single ion is determined; the third stage occurs in the precision trap where the cyclotron frequency is measured and a spin flip is attempted by applying a 105 GHz microwave signal. The ion is transferred back to the analysis trap where the spin state of the ion is again determined, thereby determining if the spin-flip attempt in the precision trap was successful. The process is then repeated. The development of the trap and associated measurement techniques over a number of years that has allowed uncertainties below 5 parts in 10^11 to be achieved are discussed in several papers (Blaum et al., 2009; Sturm et al., 2010; Sturm, Wagner, Schabinger, and Blaum, 2011; Ulmer, Blaum et al., 2011) and results using \(28\text{Si}^{13+}\) that reflect experimental improvements have been reported (Sturm, Wagner, Schabinger, Zatorski et al., 2011; Schabinger et al., 2012; Sturm et al., 2013).

For hydrogenic carbon we use

\[
\frac{\omega_{{c(12\text{C}^{5}\text{+})}}}{\omega_{{c(12\text{C}^{5}\text{+})}}} = 4376.210\,500\,87(12) \times [2.8 \times 10^{-11}] , \tag{179}
\]

which is also based on additional information supplied to the Task Group (Sturm et al., 2015) at the same time as that for silicon and similar in nature. The high-accuracy carbon result was first published in a letter to Nature (Sturm et al., 2014) and differs slightly from Eq. (179) but the value subsequently published in the detailed report on the experiment is the same (Köhler et al., 2015). The net fractional correction applied to the uncorrected ratio is \(-283.3 \times 10^{-12}\), with the largest components being for image charge and frequency pulling at \(-282.4, \times 10^{-12}\) and \(2.20 \times 10^{-12}\), respectively. The statistical relative uncertainty of the individual measurements is \(23 \times 10^{-12}\). The detailed report gives a comprehensive discussion of the MPIK trap including many possible systematic effects and their uncertainties. We identify the result in Eq. (179) as MPIK-15.

The carbon and silicon frequency ratios in Eqs. (178) and (179) are correlated. Based on the detailed uncertainty budgets for the two experiments supplied to the Task Group (Sturm, 2015) the correlation coefficient is

\[
[r_{{\omega_{{c(12\text{C}^{5}\text{+})}}/\omega_{{c(12\text{C}^{5}\text{+})}}}, \omega_{{c(28\text{Si}^{13}\text{+})}}/\omega_{{c(28\text{Si}^{13}\text{+})}}}] = 0.347, \tag{180}
\]

which is mostly due to the image charge correction.

The frequency ratios are related to the ion and electron masses by

\[
\frac{\omega_{{c(12\text{C}^{5}\text{+})}}}{\omega_{{c(12\text{C}^{5}\text{+})}}} = -g_{{12\text{C}^{5}\text{+}}}/10A_{{e}}(e) \left[ 12 - 5A_{{e}} + \frac{\Delta E_{{c(12\text{C}^{5}\text{+})}}}{m_{{e}}c^2} \right] , \tag{181}
\]

and

\[
\frac{\omega_{{c(28\text{Si}^{13}\text{+})}}}{\omega_{{c(28\text{Si}^{13}\text{+})}}} = -g_{{28\text{Si}^{13}\text{+}}}/14A_{{e}}(e) A_{{c(28\text{Si}^{13}\text{+})}} , \tag{182}
\]

where \(A_{{c(28\text{Si}^{13}\text{+})}}\) is taken to be an adjusted constant and is related to the input datum \(A_{{c(28\text{Si})}}\) by [see Eq. (3)]

\[
A_{{c(28\text{Si}^{13}\text{+})}} = A_{{c(28\text{Si})}} - 13A_{{e}}(e) + \frac{\Delta E_{{c(28\text{Si}^{13}\text{+})}}}{m_{{e}}c^2} . \tag{183}
\]

With the aid of Eq. (4) this becomes the observational equation for \(A_{{c(28\text{Si}^{13}\text{+})}}\), which is \(B19\) in Table XXIV. Because \(A_{{c(12\text{C})}} = 12\) exactly, such an additional observational equation is unnecessary for carbon; Eq. (181) becomes the observational equation for \(\omega_{{c(12\text{C}^{5}\text{+})}}/\omega_{{c(12\text{C}^{5}\text{+})}}\) simply by using Eq. (4) to modify its last term (see \(B15\) in Table XXIV).

The silicon frequency ratio yields for the relative atomic mass of the electron

\[
A_{{e}}(e) = 0.000\,548\,579\,909\,19(46) \times [8.3 \times 10^{-10}] , \tag{184}
\]

and the carbon frequency ratio gives

\[
A_{{e}}(e) = 0.000\,548\,579\,909\,070(17) \times [3.1 \times 10^{-11}] . \tag{185}
\]

If both data are used, we obtain

\[
A_{{e}}(e) = 0.000\,548\,579\,909\,069(16) \times [2.9 \times 10^{-11}] . \tag{186}
\]

The slight shift in value and reduction in uncertainty in Eq. (186) as compared to Eq. (185) is due to the information about the higher-order terms in the theory resulting from the silicon measurement. If the covariance in the theory given by Eq. (177) is taken to be zero, the result from using both

measurements is the same as the result based only on the carbon datum, within the uncertainty displayed in the equations.

We note that in the 2010 adjustment, the uncertainty of \( A_i(p) \) was much lower than that of \( A_i(e) \). As a result, the ratio \( A_i(e)/A_i(p) \) from the analysis of antiprotonic helium transition frequencies together with the value for \( A_i(p) \) provided a competitive value for \( A_i(e) \). Now that \( A_i(e) \) is more accurately known, the value of \( A_i(e) \) combined with the ratio \( A_i(e)/A_i(p) \) provides a new value for \( A_i(p) \). However, despite improvement in the theory of antiprotonic helium (Korobov, Hilico, and Karr, 2014), the uncertainty of the derived value of \( A_i(p) \) is much too large for it to be used in the 2014 adjustment.

VI. Magnetic-Moment Ratios and the Muon-Electron Mass Ratio

Free-particle magnetic-moment ratios can be obtained from experiments that measure moment ratios in bound states by applying theoretical corrections relating the free moment ratios to the bound moment ratios.

The magnetic moment of a nucleus with spin \( I \) is

\[
\mu = g e \frac{\hbar}{2m_p} I,
\]

where \( g \) is the \( g \)-factor of the nucleus, \( e \) is the elementary charge, and \( m_p \) is the proton mass. The magnitude of the magnetic moment is defined to be

\[
\mu = g\mu_S i,
\]

where \( \mu_S = \hbar/(2m_p) \) is the nuclear magneton, and \( i \) is the maximum spin projection \( I_z \), defined by \( I^2 = i(i + 1)\hbar^2 \).

In the Pauli approximation, the Hamiltonian for a hydrogen atom in the ground state in an applied magnetic flux density \( B \) is

\[
H = \frac{\Delta\omega_h}{\hbar} s \cdot I - g_e(H) \frac{\hbar}{\hbar} s \cdot B - g_p(H) \frac{\hbar}{\hbar} \mu_S I \cdot B,
\]

where \( \Delta\omega_h \) is the ground-state hyperfine frequency, \( s \) is the electron spin as given in Eq. (79), and \( \mu_S \) is given by Eq. (81). The coefficients \( g_e(H) \) and \( g_p(H) \) are bound-state \( g \)-factors and are related to the corresponding free \( g \)-factors \( g_e \) and \( g_p \) by the theoretical corrections given below. The analogous corrections for deuterium, muonium and helium-3 are also given.

A. Theoretical ratios of atomic bound-particle to free-particle \( g \)-factors

Theoretical binding corrections to \( g \)-factors are as follows. References for the calculations are given in previous detailed CODATA reports.

Hydrogen:

\[
\frac{g_e(H)}{g_e} = 1 - \frac{1}{3} (Z\alpha)^2 - \frac{1}{12} (Z\alpha)^4 + \frac{1}{4} (Z\alpha)^2 \left( \frac{\alpha}{\pi} \right)^2 + \frac{1}{2} (Z\alpha)^2 \frac{m_e}{m_p} + \ldots
\]

(190)

\[
\frac{g_p(H)}{g_p} = 1 - \frac{1}{3} a(Z\alpha) - \frac{97}{108} a(Z\alpha)^3 + \frac{1}{6} a(Z\alpha) \frac{m_e}{m_p} \frac{3 + 4a_e}{1 + a_e} + \ldots
\]

(191)

where \( A_1^{(4)} \) is given in Eq. (87), and the proton magnetic-moment anomaly is \( a_e = \mu_p/(\hbar/2m_p) - 1 \approx 1.793 \).

Deuterium:

\[
\frac{g_e(D)}{g_e} = 1 - \frac{1}{3} (Z\alpha)^2 - \frac{1}{12} (Z\alpha)^4 + \frac{1}{4} (Z\alpha)^2 \left( \frac{\alpha}{\pi} \right)^2 + \frac{1}{2} (Z\alpha)^2 \frac{m_e}{m_d} + \ldots
\]

(192)

\[
\frac{g_p(D)}{g_p} = 1 - \frac{1}{3} a(Z\alpha) - \frac{97}{108} a(Z\alpha)^3 + \frac{1}{6} a(Z\alpha) \frac{m_e}{m_d} \frac{3 + 4a_d}{1 + a_d} + \ldots
\]

(193)

where the deuteron magnetic-moment anomaly is \( a_d = \mu_d/(\hbar/2m_d) - 1 \approx -0.143 \).

Muonium (see Sec. VI. B):

\[
\frac{g_e(Mu)}{g_e} = 1 - \frac{1}{3} (Z\alpha)^2 - \frac{1}{12} (Z\alpha)^4 + \frac{1}{4} (Z\alpha)^2 \left( \frac{\alpha}{\pi} \right)^2 + \frac{1}{2} (Z\alpha)^2 \frac{m_e}{m_u} + \ldots
\]

(194)

\[
\frac{g_p(Mu)}{g_p} = 1 - \frac{1}{3} a(Z\alpha) - \frac{97}{108} a(Z\alpha)^3 + \frac{1}{2} a(Z\alpha) \frac{m_e}{m_u} + \ldots
\]

(195)

Helium-3:

\[
\frac{\mu_h(^{3}\text{He})}{\mu_p} = 1 - 59.96743(10) \times 10^{-6},
\]

(196)

which has been calculated by Rudziński, Puchalski, and Pachucki (2009). However, this ratio is not used as an input datum because it is not coupled to any other data, but allows the Task Group to provide a recommended value for the unshielded helium magnetic moment along with other related quantities.

Numerical values for the corrections in Eqs. (190) to (195) are listed in Table XIII; uncertainties are negligible. See
1. Ratio measurements

The experimental magnetic-moment and bound-state magnetic-moment ratios and magnetic-moment shielding corrections that were used as input data in the 2010 adjustment are used again in the 2014 adjustment; they are B30–B35.1, B37, and B38 in Table XVIII, Sec. XIII. A concise cataloging of these data is given in CODATA-10 and each measurement has been discussed fully in at least one of the previous detailed CODATA reports. The observational equations for these data are given in Table XXIV and the adjusted constants in those equations are identified in Table XXVI, both in Sec. XIII. Any relevant correlation coefficients for these data may be found in Table XIX, also in Sec. XIII. The theoretical bound-particle to free-particle $g$-factor ratios in the observational equations, which are taken to be exact because their uncertainties are negligible, are given in Table XIII. The symbol $\mu_p'$ denotes the magnetic moment of a proton in a spherical sample of pure H2O at 25°C surrounded by vacuum; and the symbol $\mu_d'$ denotes the magnetic moment of a helium bound in a $^3$He atom. Although the exact shape and temperature of the gaseous $^3$He sample is unimportant, we assume that it is spherical, at 25°C, and surrounded by vacuum.

In general, the bound magnetic moment of a particle, for example, that of p, d, t, or h, is related to its free value by $\mu(\text{bound}) = (1 - \sigma)\mu(\text{free})$, where $\sigma$ is the nuclear magnetic shielding correction or parameter. For the hydrogen-deuterium molecule HD, $\sigma_p(\text{HD})$ and $\sigma_d(\text{HD})$ are the shielding corrections for the proton and deuteron in HD, respectively. Since $\sigma$ is small, one may define $\sigma_p = \sigma_p(\text{HD}) - \sigma_p(\text{HD})$ and write $\mu_p(\text{HD})/\mu_p(\text{HD}) = [1 + \sigma_p + O(\sigma^2)]\mu_p/\mu_p$. This also applies to the hydrogen-tritium or HT molecule: $\sigma_p = \sigma_p(\text{HT}) - \sigma_p(\text{HT})$ and $\mu_p(\text{HT})/\mu_p(\text{HT}) = [1 + \sigma_p + O(\sigma^2)]\mu_p/\mu_p$.

Two new relevant data have become available since the closing date of the 2010 adjustment. Garbacz et al. (2012) at the University of Warsaw, Warsaw, Poland determined the ratio $\mu_p(\text{HD})/\mu_d(\text{HD})$ by separately measuring, in the same magnetic flux density $B$, the nuclear magnet resonance (NMR) frequencies $\omega_p(\text{HD})$ and $\omega_d(\text{HD})$ of the proton and deuteron in HD. Their result is

$$\frac{\mu_p(\text{HD})}{\mu_d(\text{HD})} = \frac{1}{2} \frac{\omega_p(\text{HD})}{\omega_d(\text{HD})} = 3.257 199 514(21) [6.6 \times 10^{-9}].$$

(197)

The factor 1/2 arises because the spin quantum number for the proton is 1/2 while for the deuteron it is 1. We identify this result, which is taken as an input datum in the 2014 adjustment, as UWars-12; it is item B35.2 in Table XVIII and is the second value of this ratio now available with an uncertainty less than 1 part in 10^6. Its observational equation, B35 in Table XXIV, is the same as for the other value, identified as StPtrsb-03.

The UWars result was obtained using a technique developed at the university and described by Jackowski, Jaszuñski, and Wileczek (2010). The NMR measurements of the frequencies $\omega_p(\text{HD})$ and $\omega_d(\text{HD})$ were carried using a variable gaseous sample that contained HD of a sufficiently low density that the HD-HD molecular interactions were inconsequential and neon of density between 11 mol/L and 2 mol/L so that the observed frequencies could be extrapolated to zero neon density with an uncertainty of 0.5 Hz. The neon was used to increase the pressure of the sample thereby facilitating the NMR measurements. There were no uncertainties of significance from systematic effects (Jackowski, 2015).

The NMR measurements on the HT molecule carried out in St. Petersburg, Russia that led to a value for $\mu(\text{HT})/\mu_p(\text{HT})$ with $\sigma_p = 9.4 \times 10^{-9}$ and which was used as an input datum in the 2010 adjustment have continued (Aleksandrov and Neronov, 2011). The new value reported by Neronov and Aleksandrov (2011), which is in agreement with the earlier value, is

$$\mu_p(\text{HT}) = 1.066 639 8933(21) [2.0 \times 10^{-9}].$$

(198)

This result, input datum B36 in Table XVIII and identified as StPtrsb-11, replaces the earlier result because of its significantly smaller uncertainty. Its observational equation is B36 in Table XXIV.

The reduced uncertainty was achieved by decreasing the magnetic field inhomogeneity across the sample and by measuring the difference $\omega_p(\text{HT}) - \omega_p(\text{HT})$ using a commercial NMR spectrometer operating at 9.4 T. The spectrometer to determine $\omega_p(\text{HT})/\omega_p(\text{HT})$ from the measured value of $\omega_p(\text{HT})/\omega_p(\text{HT})$. The latter frequency ratio was obtained with a specially designed, laboratory-made spectrometer operating at 2.1 T. Use of HD as a buffer gas in the NMR sample was necessary to reduce the diffusion displacement of the HT molecules. The value given in Eq. (198) is the mean of 10 individual values and its assigned uncertainty is the simple standard deviation of these values rather than that of their mean as initially assigned by Neronov and Aleksandrov (2011). This uncertainty, chosen by the Task Group to better reflect possible systematic effects, was discussed with and accepted by Neronov (2015).

For completeness, we briefly mention new and potentially relevant data that were not considered for inclusion in the

TABLE XIII. Theoretical values for various bound-particle to free-particle $g$-factor ratios relevant to the 2014 adjustment based on the 2014 recommended values of the constants

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_e(\text{H})/g_e$</td>
<td>$1 - 17.7054 \times 10^{-6}$</td>
</tr>
<tr>
<td>$g_p(\text{H})/g_p$</td>
<td>$1 - 17.7354 \times 10^{-6}$</td>
</tr>
<tr>
<td>$g_e(D)/g_e$</td>
<td>$1 - 17.7126 \times 10^{-6}$</td>
</tr>
<tr>
<td>$g_d(D)/g_d$</td>
<td>$1 - 17.7461 \times 10^{-6}$</td>
</tr>
<tr>
<td>$g_e(\text{Mu})/g_e$</td>
<td>$1 - 17.5926 \times 10^{-6}$</td>
</tr>
<tr>
<td>$g_d(\text{Mu})/g_d$</td>
<td>$1 - 17.6254 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

Ivanov, Karshenboim, and Lee (2009) for a negligible additional term.
2014 adjustment. More accurate theoretical values for $\sigma_{dp}$ and $\sigma_{tr}$ were reported by Puchalski, Komasa, and Pachucki (2015) but did not become available until well after the 31 December 2014 closing date of the adjustment. They are $\sigma_{dp} = 20.20(2) \times 10^{-9}$ and $\sigma_{tr} = 24.14(2) \times 10^{-9}$ compared with $\sigma_{dp} = 15(2) \times 10^{-9}$ and $\sigma_{tr} = 20(3) \times 10^{-9}$ used in the adjustment (input data B37 and B38 in Table XVIII).

Also, the St. Petersburg NMR researchers reported a value for $\omega_{0}^{\nu}(3{\text{He}})/\omega_{0}^{\nu}(H_{2})$ (Neronov and Seregin, 2012), the NMR frequency ratio of the helion in $^{3}\text{He}$ to that of a proton in $H_{2}$, and also for $\sigma_{tr}(H_{2}) - \sigma_{tr}(H_{2}O)$ (Neronov and Seregin, 2014), the difference in the magnetic screening constants for the proton in $H_{2}$ and $H_{2}O$. However, the unavailability of detailed uncertainty budgets for these two results precluded their consideration for possible inclusion in the adjustment.

### B. Muonium transition frequencies, the muon-proton magnetic-moment ratio $\mu_{\text{m}/\mu_{\text{p}}}$, and muon-electron mass ratio $m_{\text{m}}/M_{\text{e}}$

Muonium ($\mu$) is an atom consisting of a positive muon and an electron in a bound state. Measurements of muonium ground-state hyperfine transitions in a magnetic field provide information on the muon-proton magnetic-moment ratio as well as the muon-electron mass ratio. This information is obtained by an analysis of the Zeeman transition resonances in an applied magnetic flux density.

The theoretical expression for the hyperfine splitting may be factorized into a part that exhibits the main dependence on various fundamental constants and a function $F$ that depends only weakly on them. We write

$$\Delta\nu_{\mu} = \Delta\nu_{\mu}F(\alpha, m_{e}/m_{\mu}),$$

where

$$\Delta\nu_{\mu} = \frac{16}{3} cR_{\mu-c}^{2} \alpha^{2} m_{e}^{2} \left(1 + m_{e}^{2} m_{\mu}^{-1}\right)^{-3}$$

is the Fermi formula. In order to identify the source of the terms, some of the theoretical expressions are for a muon with charge $Ze$ rather than $e$.

#### 1. Theory of the muonium ground-state hyperfine splitting

Presented here is a brief summary of the present theory of $\Delta\nu_{\text{Mu}}$. Complete results of the relevant calculations are given along with references to new work; references to the original literature included in earlier detailed CODATA reports are not repeated.

The general expression for the hyperfine splitting is

$$\Delta\nu_{\mu}(\text{th}) = \Delta\nu_{D} + \Delta\nu_{\text{rad}} + \Delta\nu_{\text{rec}} + \Delta\nu_{-\text{r}} + \Delta\nu_{\text{weak}} + \Delta\nu_{\text{rad}}^{-1},$$

where the terms labeled D, rad, rec, r-r, weak, and had account for the Dirac, radiative, recoi, radiative-recoil, electroweak, and hadronic contributions to the hyperfine splitting, respectively.

The Dirac equation yields

$$\Delta\nu_{D} = \Delta\nu_{F}(1 + a_{\mu}) \left[1 + \frac{3}{2} (Z\alpha)^2 + \frac{17}{8} (Z\alpha)^4 + \cdots\right],$$

where $a_{\mu}$ is the muon magnetic-moment anomaly.

The radiative corrections are

$$\Delta\nu_{\text{rad}} = \Delta\nu_{F}(1 + a_{\mu}) \left[\frac{2(2)}{\pi} \Re^{2} + \frac{4(6)}{\pi} \Re^{6} + \cdots\right],$$

where the functions $D^{(2n)}(Z\alpha)$ are contributions from $n$ virtual photons. The leading term is

$$D^{(2)}(Z\alpha) = A_{1}^{(2)} + \left(\ln 2 - \frac{5}{2}\right)Z\alpha + \left[-\frac{2}{3} \ln^2(Z\alpha)^{-2} + \frac{281}{360} - \frac{8}{3} \ln 2 \ln(Z\alpha)^{-2} + 16.9037 \cdots\right] Z\alpha^2$$

$$+ \left\{\frac{5}{2} \ln 2 - \frac{547}{96}\right\} \ln(Z\alpha)^{-2} \pi(Z\alpha)^3 + G(Z\alpha)(Z\alpha)^3,$$

which agrees with the value $G_{\text{SE}}(\alpha) = -13.8308(43)$, from an earlier calculation by Yerokhin et al. (2005), as well as with other previous estimates. The vacuum-polarization part is (Karshenboim, Ivanov, and Shabaev, 1999, 2000)

$$G_{\text{VP}}(\alpha) = 7.227(9) + \cdots,$$

where the dots denote uncalculated Wichmann-Kroll contributions. For $D^{(4)}(Z\alpha)$, we have

$$D^{(4)}(Z\alpha) = A_{1}^{(4)} + 0.77099(2)\pi Z\alpha$$

$$+ \left[-\frac{1}{3} \ln^2(Z\alpha)^{-2} - 0.6390 \cdots \times \ln(Z\alpha)^{-2} + 10(2.5)\right] Z\alpha^2 + \cdots,$$

where $A_{1}^{(4)}$ is given in Eq. (87), and the coefficient of $\pi Z\alpha$ has been calculated by Mondéjar, Piclum, and Czarnecki (2010).

The next term is

$$D^{(6)}(Z\alpha) = A_{1}^{(6)} + \cdots,$$

where the leading contribution $A_{1}^{(6)}$ is given in Eq. (88), but only partial results of relative order $Z\alpha$ have been calculated (Eides and Shelyuto, 2007). Higher-order functions $D^{(2n)}(Z\alpha)$ with $n > 3$ are expected to be negligible.
The recoil contribution is

\[ \Delta \nu_{\text{rec}} = \Delta \nu_{\text{r}} \left( \frac{3 \alpha}{\pi^2} \right) \frac{m_e}{m_\mu} \left[ -6 \ln^2 \frac{m_e}{m_\mu} + \frac{31}{6} \ln \frac{m_\mu}{m_e} + 68.507(2) \right] \]

\[ = -30.99 \text{ Hz}. \] (211)

Additional radiative-recoil corrections have been calculated, but are negligibly small, less than 0.5 Hz. Uncalculated remaining terms of the same order as those included in Eq. (211) are estimated by Eides and Shelyuto (2014) to be about 10 Hz to 15 Hz.

The electroweak contribution due to the exchange of a Z^0 boson is (Eides, 1996)

\[ \Delta \nu_{\text{weak}} = -65 \text{ Hz}, \] (212)

while for the hadronic vacuum-polarization contribution we have (Nomura and Teubner, 2013)

\[ \Delta \nu_{\text{had}} = 232.7(1.4) \text{ Hz}. \] (213)

A negligible contribution (\approx 0.0065 Hz) from the hadronic light-by-light correction has been given by Karshenboim, Shelyuto, and Vainshtein (2008).

The approach used to evaluate the uncertainty of the theoretical expression for \( \Delta \nu_{\mu} \) is described in detail in CODATA-02. The only change for CODATA-14 is that the probable error estimates of uncertainties that were subsequently multiplied by 1.48 to convert them to standard uncertainties (that is, 1 standard deviation estimates) are now assumed to have been standard uncertainty estimates in the first place and are not multiplied by 1.48. This change was motivated by conversations with Eides (2015).

Four sources of uncertainty in \( \Delta \nu_{\mu}(\text{th}) \) are \( \Delta \nu_{\text{rad}}, \Delta \nu_{\text{r}} \), \( \Delta \nu_{\text{rec}}, \) and \( \Delta \nu_{\text{had}} \) in Eq. (201), although the uncertainty in the latter is now so small that it is of only marginal interest. The total uncertainty in \( \Delta \nu_{\text{rad}} \) is 5 Hz and consists of two components: 4 Hz from an uncertainty of 1 in \( G_{\text{VP}}(\alpha) \) due to the uncalculated Wichmann-Kroll contribution of order \( \alpha(Z\alpha)_3 \), and 3 Hz from the uncertainty 2.5 of the number 10 in the function \( D^{(k)}(Z\alpha) \).

For \( \Delta \nu_{\text{rec}} \), the total uncertainty is 64 Hz and is due to three components: 53 Hz from 2 times the uncertainty 10 of the number 40 in Eq. (209) as discussed in CODATA-02; 34 Hz due to a possible recoil correction of order \( \Delta \nu_{\text{VP}}(m_\mu/m_e) \times (Z\alpha)_3 \ln(m_\mu/m_e) \); and 6 Hz to reflect a possible recoil term of order \( \Delta \nu_{\text{VP}}(m_\mu/m_e) \times (Z\alpha)_3 \ln^2(Z\alpha) \).

The total uncertainty in \( \Delta \nu_{\text{had}} \) is 55 Hz, with 53 Hz from 2 times the uncertainty 10 of the number ≠40 in Eq. (210) as above, and 15 Hz as discussed in connection with the newly included radiative-recoil contribution, Eq. (211). The uncertainty of \( \Delta \nu_{\text{had}} \) is 1.4 Hz from Eq. (213). The final uncertainty in \( \Delta \nu_{\mu}(\text{th}) \) is thus

\[ u[\Delta \nu_{\mu}(\text{th})] = 85 \text{ Hz}. \] (214)

For the least-squares calculations, we use as the theoretical expression for the hyperfine splitting

\[ \Delta \nu_{\mu}(\text{th}) = \Delta \nu_{\mu} \left( R_{\text{eff}}, \alpha, m_\mu/m_e, a_\mu \right) + \delta_{\mu}, \] (215)

where the input datum for the additive correction \( \delta_{\mu} = 0(85) \text{ Hz} \), which accounts for the uncertainty of the theoretical expression, is data item B28 in Table XVIII.

The above theory yields

\[ \Delta \nu_{\mu} = 4463 \text{ 302,868(271) Hz } \left[ 6.1 \times 10^{-8} \right] \] (216)

using values of the constants obtained from the 2014 adjustment without the two measured values of \( \Delta \nu_{\mu} \) discussed in the following section. The main source of uncertainty in this value is the mass ratio \( m_\mu/m_e \).

2. Measurements of muon transition frequencies and values of $\mu_p/\mu_p$ and $m_p/m_e$

The two most precise determinations of muonium Zeeman transition frequencies were carried out at the Clinton P. Anderson Meson Physics Facility at Los Alamos (LAMPF), New Mexico, USA, and were reviewed in detail in CODATA-98. The results are as follows.

Data reported in 1982 by Mariam (1981) and Mariam et al. (1982) are

$$\Delta \nu_{\text{Mu}} = 4.463.302.88(16) \text{ kHz} \quad [3.6 \times 10^{-8}],$$
$$\nu(f_p) = 627.994.77(14) \text{ kHz} \quad [2.2 \times 10^{-7}],$$
$$r[\Delta \nu_{\text{Mu}}, \nu(f_p)] = 0.227,$$

where $f_p$ is 57.972 993 MHz, corresponding to the magnetic flux density of about 1.3616 T used in the experiment, and $r[\Delta \nu_{\text{Mu}}, \nu(f_p)]$ is the correlation coefficient of $\Delta \nu_{\text{Mu}}$ and $\nu(f_p)$. The data reported in 1999 by Liu et al. (1999) are

$$\Delta \nu_{\text{Mu}} = 4.463.302.765(53) \text{ Hz} \quad [1.2 \times 10^{-8}],$$
$$\nu(f_p) = 668.223.166(57) \text{ Hz} \quad [8.6 \times 10^{-8}],$$
$$r[\Delta \nu_{\text{Mu}}, \nu(f_p)] = 0.195,$$

where $f_p$ is 72.320 000 MHz, corresponding to the flux density of approximately 1.7 T used in the experiment, and $r[\Delta \nu_{\text{Mu}}, \nu(f_p)]$ is the correlation coefficient of $\Delta \nu_{\text{Mu}}$ and $\nu(f_p)$. The data in Eqs. (217), (218), (220), and (221) are data items B27.1, B25, B27.2, and B26, respectively, in Table XVIII.

The expression for the magnetic-moment ratio is

$$\frac{\mu_e}{\mu_p} = \frac{\Delta \nu^2_{\text{Mu}} - \nu^2(f_p) + 2 \nu f_p \nu(f_p)}{4 \nu f^2_p - 2 \nu^2 \nu(f_p)} \left( \frac{g_p(\text{Mu})}{g_e} \right)^{-1},$$

where $\Delta \nu_{\text{Mu}}$ and $\nu(f_p)$ are the sum and difference of two measured transition frequencies, $f_p$ is the free proton NMR reference frequency corresponding to the flux density used in the experiment, $g_p(\text{Mu})/g_e$ is the bound-state correction for the muon in muonium given in Table XIII, and

$$s_e = \frac{\mu_e}{\mu_p} \frac{g_p(\text{Mu})}{g_e},$$

where $g_p(\text{Mu})/g_e$ is the bound-state correction for the electron in muonium given in the same table.

The muon to electron mass ratio $m_u/m_e$ and the muon to proton magnetic-moment ratio $\mu_u/\mu_p$ are related by

$$\frac{m_u}{m_e} = \left( \frac{\mu_u}{\mu_p} \right) \left( \frac{\mu_p}{\mu_p} \right)^{-1} \left( \frac{g_p}{g_p} \right).$$

A least-squares adjustment using the LAMPF data, the 2014 recommended values of $R_\infty$, $\mu_p/\mu_p$, $g_p$, and $\mu_u$, together with Eqs. (199), (200) and Eqs. (223) to (225), yields

$$\frac{\mu_u}{\mu_p} = 3.183.345.24(37) \quad [1.2 \times 10^{-7}],$$
$$\frac{m_u}{m_e} = 206.768.526(24) \quad [1.2 \times 10^{-7}],$$
$$\alpha^{-1} = 137.036.0013(79) \quad [5.8 \times 10^{-8}].$$

The muonium value of $\alpha$ in Eq. (228) is compared to other values in Table XX.

The uncertainty of $m_u/m_e$ in Eq. (227) is nearly 5 times as large as the uncertainty of the 2014 recommended value and follows from Eqs. (223) to (225). It has the same relative uncertainty as the moment ratio in Eq. (226). However, taken together the experimental value of and theoretical expression for the hyperfine splitting essentially determine the value of the product $\alpha^2 m_p/m_u$, as is evident from Eqs. (199) and (200), with an uncertainty dominated by the $1.9 \times 10^{-8}$ relative uncertainty in the theory.

On the other hand, in the full least-squares adjustment the value of $\alpha$ is determined by other data which in turn determines the value of $m_u/m_e$ with a significantly smaller uncertainty than that of Eq. (227).

VII. Quotient of Planck Constant and Particle Mass $\hbar/m(X)$ and $\alpha$

A value of the fine-structure constant $\alpha$ can be obtained from a measurement of $\hbar/m(X)$ through the expression

$$\alpha = \left( \frac{2R_\infty A_r(X)}{c A_r(e) m(X)} \right)^{1/2},$$

which follows from the definition of the Rydberg constant $R_\infty = \alpha^2 m_e c^2/2\hbar$, and where $A_r(X)$ is the relative atomic mass of particle $X$ with mass $m(X)$. The relative standard uncertainties of $R_\infty$ and $A_r(e)$ are about $6 \times 10^{-12}$ and $3 \times 10^{-11}$, respectively, and the relative uncertainty of the relative atomic mass of a number of atoms is a few times $10^{-10}$ or less. Hence, a measurement of $\hbar/m(X)$ with $u_\alpha$ of $1 \times 10^{-9}$ can provide a value of $\alpha$ with the highly competitive relative uncertainty of $5 \times 10^{-10}$.

Two values of $\hbar/m(X)$ obtained using atom interferometry techniques were initially included as input data in CODATA-10 and the same values are initially included in CODATA-14. The first value is the result for $\hbar/m(^{133}\text{Cs})$ obtained at Stanford University, Stanford, California, USA, reported by Wicht et al. (2002) with $u_\alpha = 1.5 \times 10^{-8}$. This experiment is discussed in CODATA-06 and the result is input datum B46 in Table XVIII, Sec. XIII, and is labeled StanfordU-02. The value of $\alpha$ inferred from it with $u_\alpha = 7.7 \times 10^{-9}$ is given in Table XX.

The Stanford result for $\hbar/m(^{133}\text{Cs})$ was not included as an input datum in the final adjustment on which the 2010 recommended values are based because of its low weight, and is omitted from the 2014 final adjustment for the same reason. However, it is discussed in order to provide a complete picture of the available data relevant to $\alpha$.

The second value of $\hbar/m(X)$, which is included in the final 2014 adjustment, is the result for $\hbar/m(^{85}\text{Rb})$ determined at LKB in Paris with $u_\alpha = 1.2 \times 10^{-9}$ and reported by Bouchendira.
et al. (2011). The experiment is discussed in CODATA-10 and the result, labeled LKB-11, is datum B48 in Table XVIII; the value of $\alpha$ inferred from it with $u_\alpha = 6.2 \times 10^{-10}$ is given in Table XX. Although it has the second smallest uncertainty of the 14 values of $\alpha$ in that table, its uncertainty is still about 2.6 times that of the value with the smallest uncertainty, that from the Harvard University measurement of $\alpha_*$. Nevertheless, the comparison of the values of $\alpha$ from the two experiments provides a useful test of the QED theory of $\alpha_*$. Such a comparison is discussed in Sec. XV.B of the Summary and Conclusions portion of this report.

We conclude this section by noting that a value of $h/m^{(133)\text{Cs}}$ with $u_h = 4.0 \times 10^{-9}$ can in principle be obtained from the measurement by Lan et al. (2013) of the Compton frequency of the mass of the cesium-133 atom $v_C^{(133)\text{Cs}}$ using atom interferometry since $h/m^{(133)\text{Cs}} = e^2/v_C^{(133)\text{Cs}}$. However, Müller (2015) informed the Task Group that small corrections have recently been identified that were not included in the reported result and consequently it should not be considered. A new result with a highly competitive uncertainty is anticipated.

**VIII. Electrical Measurements**

The principal focus of this portion of the paper is the several moving-coil watt-balance (or simply watt-balance) measurements of $K_2^2R_K = 4/h$ that have become available in the past 4 years, where $K_2 = 2e/h$ is the Josephson constant and $R_K = h/e^2 = \mu_0c^2/2\alpha$ is the von Klitzing constant. Nevertheless, the 13 legacy electrical input data that were initially included in the 2010 adjustment in order to investigate data robustness and the exactness of the relations $K_1 = 2e/h$ and $R_K = h/e^2$ were omitted from the final adjustment on which the 2010 recommended values are based because of their low weight are again included in the 2014 adjustment for the same purpose. These input data are items B39.1 through B43.5 and B45 in Table XVIII, Sec. XIII. They are five measurements of the gyromagnetic ratio of the proton and helium by the low and high field methods, five measurements of $R_K$ using a calculable capacitor, one measurement of $K_1$ using a mercury electrometer and another using a capacitor voltage balance, and one measurement of the Faraday constant using a silver dissolution electrometer. A brief explanation of these different kinds of measurements and a more detailed cataloging of the 13 data are given in CODATA-10 and each measurement has been discussed fully in at least one of the previous four detailed CODATA reports. The observational equations for these 13 data are B39 – B43 and B45 in Table XXIV and the adjusted constants in those equations are in Table XXVI, both in Sec. XIII. Any relevant correlation coefficients for these data are listed in Table XIX, also in Sec. XIII. Table XX or XXI in Sec. XIII.A compares the data among themselves and with other data through the values of either $\alpha$ or $h$ that they infer. Three comments are in order before beginning the discussion of individual watt-balance experiments.

First, some watt-balance researchers find it useful to define the exact, conventional value of the Planck constant $h_{90} = 4/K_2^2R_K = 6.626 068 854 \ldots \times 10^{-34}$ J s to express the results of their watt-balance determinations of $K_2^2R_K$ and hence $h$ (see Table I, Sec. II).

Second, it was discovered that the unit of mass disseminated by the International Bureau of Weights and Measures (BIPM), Sèvres, France, starting about 2000 and extending into 2014 gradually became offset from the SI unit of mass. The latter is defined by assigning to the mass of the international prototype of the kilogram (IPK), which is maintained at the BIPM, the value 1 kg exactly (Stock et al., 2015). The discovery was made during the Extraordinary Calibration Campaign undertaken at the BIPM to ensure that watt-balance measurements of $h$ and the x-ray-crystal-density (XRCD) determinations of $N_A$ (see Sec. IX.B) are closely tied to the IPK in preparation for the adoption of the new SI by the 26th CGPM in 2018. The BIPM working standards used to calibrate client standards had lost mass and in 2014 were offset from the mass of the IPK by about 35 μg. As a consequence, some watt-balance values of $h$ had to be decreased by 35 parts in $10^9$ and the XRCD values of $N_A$ had to be increased by a similar amount.

Our third comment has to do with the measurement of $g$, the local acceleration due to gravity. The basic watt-balance equation is $UI = m_gg$, where $U$ is the voltage induced between the terminals of a coil moving in a magnetic flux density $B$ with velocity $v$; and $I$ is the current in the coil in the same flux density $B$ when the force on the coil due to $I$ and $B$ just balances the weight $m_gg$ of a standard of mass $m_g$. Although commercial absolute free-fall gravimeters that use optical interferometry to measure the position of a falling reflector are capable of determining $g$ with a relative uncertainty of a few parts in $10^9$, recently there has been concern about the correction due to the finite speed of the propagation of light, or so-called “speed of light correction.” This correction for a 20 cm drop is usually about 1 part in $10^9$. In particular, Rothleitner, Niebauer, and Francis (2014) claim that the correction normally applied for this effect is too large by a third. However, Baumann et al. (2015) have recently carried out a very extensive theoretical and experimental investigation of the problem and have convincingly shown that this is not the case.

The resulting values of $K_2^2R_K$ from the seven watt-balance experiments we consider are items B44.1–B44.7 in Table XVIII, Sec. XIII. This quantity is the actual input datum employed in least-squares calculations using as its observational equation $K_2^2R_K \equiv 4/h$. The resulting values of $h$ are compared with each other and with inferred values of $h$ from other experiments in Table XXI, Sec. XIII.A.

**A. NPL watt balance**

There are two watt-balance results from the National Physical Laboratory (NPL), Teddington, UK, both with relative standard uncertainties $u_h = 2.0 \times 10^{-7}$. NPL-90, item B44.1 in Table XVIII, is discussed in CODATA-98 and is from the first truly high-accuracy watt-balance experiment carried out (Kibble, Robinson, and Belliss, 1990). The design of the NPL Mark I apparatus was unique in that the moving coil consisted of two flat rectangular coils above one another in a vertical plane and hung between the poles of a conventional electromagnet.
Item B44.5, NPL-12, is discussed in CODATA-10 and was obtained using the NPL Mark II watt balance. This apparatus has cylindrical symmetry about a vertical axis and employs a horizontal circular coil hung in the gap between two concentric annular permanent magnets (Robinson, 2012); some details of the balance are given in CODATA-98. Just prior to the transfer of the apparatus to the National Research Council (NRC), Ottawa, Canada, in the summer of 2009, Robinson (2012) identified two possible systematic effects in the weighing mode of the experiment. Lack of time necessitated taking them into account by including a comparatively large additional uncertainty component in the experiment’s uncertainty budget, which in turn led to the final comparatively large uncertainty of the final result. Although this value has not been corrected for the BIPM mass-standard problem discussed above, it is of little consequence because of the small size of the correction compared with the final uncertainty. The correlation coefficient of the NPL-90 and NPL-12 results is 0.0025 and thus they are only slightly correlated (Robinson, 2012).

B. METAS watt balance

Reported by Eichenberger et al. (2011), the result from the Federal Institute for Metrology (METAS), Bern-Wabern, Switzerland, item B44.3 with identification METAS-11, is discussed in CODATA-10. Its relative uncertainty of $2.9 \times 10^{-7}$ was too large for it to be included in the 2010 final adjustment but is initially included in the 2014 adjustment for tests of data robustness and the exactness of the Josephson and quantum-Hall-effect relations $K_J = 2e/h$ and $R_K = h/e^2$. No correction for the mass-standard problem has been made to this result; because of the comparatively large uncertainty of METAS-11, it is of no consequence.

C. LNE watt balance

The Laboratoire National de Métrie et d’Essais (LNE), Trappes, France, initiated its watt-balance project in 2001. The various elements of the LNE balance were developed with continued characterization and improvements of each. The first result became available in a preprint in December 2014 and is

$$h = 6.6260688(17) \times 10^{-34} \text{ J s } \quad [2.6 \times 10^{-7}],$$  \hspace{1cm} (230)

which is equivalent to

$$K_J^2 R_K = 6.0367619(15) \times 10^{13} \text{ J}^{-1} \text{ s}^{-1} \quad [2.6 \times 10^{-7}].$$  \hspace{1cm} (231)

The result was subsequently published by Thomas et al. (2015), but although the value of $h$ remained the same as in the preprint, the relative uncertainty was increased to $3.1 \times 10^{-7}$ with a corresponding increase in the absolute uncertainty. The preprint uncertainty is used in all calculations, but with no significant consequence because of its comparatively large size. Indeed, because of its low weight, the LNE result as given in Eq. (231), which is item B44.7 with identification LNE-15 in Table XVIII, is omitted from the 2014 final adjustment. It should also be noted that Thomas et al. (2015) have not corrected the LNE result for the BIPM mass-standard shift, which in the LNE case is only ~3.7 parts in $10^9$.

The LNE watt balance uses a cylindrical geometry and a permanent magnet as does the NPL Mark II balance. One of its unique features is that during the moving-coil mode, the balance beam and its suspension used in the weighing mode is moved as a single element, which avoids using the balance beam to move the coil. Details of the balance, which was operated in air to obtain its first result but has the capability to operate in vacuum, are given in the paper by Thomas et al. (2015) and the references cited therein. The two largest uncertainty components, in parts in $10^7$, are 2.4 for the voltage measurements and 1.2 for the velocity measurement, which includes the correction for the refractive index of air and the verticality of the laser beams.

D. NIST watt balance

There are two watt-balance results from NIST to be considered. NIST-98, item B44.2 with $u_r = 8.7 \times 10^{-8}$, is discussed in CODATA-98 and was obtained using the second generation NIST watt balance called NIST-2 (Williams et al., 1998). In this apparatus an axially symmetric radial magnetic flux density is generated by a specially designed, 1.5 m high magnet consisting of upper and lower superconducting solenoids and smaller compensating windings mounted in a Dewar; the moving coil is circular, mounted horizontally in air, and encircles the Dewar.

After the publication of this result the NIST researchers not only renovated the facility in which the apparatus was used but completely reconstructed it with little remaining of the earlier NIST-2 watt balance, the major exception being the superconducting magnet. In the new watt balance, called NIST-3 and discussed in CODATA-06, the entire balance mechanism and coil are in vacuum. The new apparatus was subsequently used to obtain a result for $K_J^2 R_K$ that turned out to be identical to the 1998 result but with $u_r = 3.6 \times 10^{-8}$ (Steiner et al., 2007). This result, with identification NIST-07 and discussed in CODATA-06, was included in both the 2006 and 2010 final adjustments. However, in the 2006 final adjustment, because of the inconsistencies among several data that contributed to the determination of $h$, including NIST-98 and NIST-07 and the measurement of the molar volume of natural silicon by the International Avogadro Coordination (IAC), their uncertainties were increased by the expansion factor 1.5 to reduce the relevant residuals to less than 2. Similar inconsistencies were present in 2010, especially between NIST-07 and the newly reported XRCD result for $N_A$ by the IAC denoted IAC-11. As a consequence, the expansion factor used in the 2010 final adjustment was increased to 2.

NIST researchers were well aware of this problem and of the disagreement of NIST-07 with the NRC watt-balance result NRC-12 reported in early 2012 by Steele et al. (2012). They were also aware of the fact that NIST-3 had produced stable values of $h$ from October 2004 to March 2010 when the value suddenly changed for no apparent reason. To address these issues, NIST experimenters carried out six series of new measurements lasting between 3 days and 3 weeks starting in...
December 2012 and ending in November 2013 with the aim of producing an independent value. To this end, the measurements were conducted blindly and NIST-3 was first closely inspected and a number of significant changes made to it in order to improve its performance, as described by Schlamminger et al. (2014). The result of this effort as reported in the latter paper and denoted NIST-14 is in reasonable agreement with NRC-12 and IAC-11, and its uncertainty $u_t = 4.5 \times 10^{-8}$ is less than 5 parts in $10^8$.

To answer the question “What is the best value of $K_f^2 R_k$, and hence $h$, that can be deduced from 10 years of NIST-3 data?” Schlamminger et al. (2015) thoroughly reviewed all such data after correcting the new 2012 to 2013 data downward by the fractional amount $35 \times 10^{-9}$ to account for the BIPM mass-standard shift. They divide the data into three epochs, 2004 to 2009, 2010 to 2011, and 2012 to 2013, calculate the result for each epoch, and take as the best value the average of the three. For the uncertainty they take the relative uncertainty $4.5 \times 10^{-8}$ of NIST-14 and combine it in quadrature with an additional component of $3.5 \times 10^{-8}$, which is one-half the approximate $7 \times 10^{-8}$ fractional difference between NIST-07 and the average of the three individual values. This additional component is to account for the lack of understanding of the cause of the 7 parts in $10^9$ difference. Thus following Schlamminger et al. (2015), we employ for the NIST-3 result

$$h = h_{NRC}[1 + 77(57) \times 10^{-9}],$$

which is equivalent to

$$h = 6.626 069 36(38) \times 10^{-34} \text{ J s } [5.7 \times 10^{-8}],$$

$$K_f^2 R_k = 6.036 761 43(34) \times 10^{33} \text{ J}^{-1} \text{ s}^{-1} [5.7 \times 10^{-8}],$$

where this last value is item B44.4 in Table XVIII with identification NIST-15. The NIST-98 and NIST-15 values of $K_f^2 R_k$ are correlated and Schlamminger et al. (2015) estimate their correlation coefficient to be 0.09. This coefficient is included in Table XIX and is used in all relevant calculations.

E. NRC watt balance

The entire NPL Mark II apparatus was dismantled and shipped to NRC in the summer of 2009 where in due course it was reassembled and recommissioned in a newly constructed laboratory. A first result with $u_t = 6.5 \times 10^{-8}$ was obtained and reported by Steele et al. (2012). This result includes experimentally measured corrections for the effect of the stretching of the beryllium copper flexures that support the moving coil under load, and for the effect of the tilting of the support base of the balance when the mass lift is loaded and unloaded. These are the effects that Robinson (2012) had identified but because of a lack of time could only take into account through a comparatively large additional uncertainty component. The corrections could not be retroactively applied to the NPL-12 result, because a new set of flexures were installed after the balance arrived at NRC as a result of an accident that damaged the original flexures. Subsequently, as described by Sanchez et al. (2013), modifications were made to the balance that reduced these effects to a negligible level.

NRC experimenters continued to make important improvements to the NRC watt balance and subsequently carried out four measurement campaigns between September and December 2013 using four different mass standards (Sanchez et al., 2014). They are a 1 kg gold-plated copper cylinder, a 500 g diamond turned silicon cylinder, a 500 g gold-plated piece of copper, and a 250 g piece of silicon. The data set for each consists of 142, 111, 107, and 148 data points, obtained over 14, 11, 15 and 15, days, respectively. The Type A (statistical) uncertainty for the four values of $h/h_{NRC}$ obtained from each of the four mass standards is taken to be the standard deviation of the mean of each day’s result from that standard. The Type B uncertainty (from systematic effects) for each value of $h/h_{NRC}$ is based on an uncertainty budget containing 51 components distributed over seven major categories. For the result from the Au-Cu 1 kg mass standard, the four largest in parts in $10^9$ are 9.0 for the mass of the standard, 6.9 for resistance, 5.9 for alignment, and 5.7 for gravimetry. The latter two topics are discussed in detail by Liard et al. (2014) and Sanchez and Wood (2014). The total relative uncertainty for $h/h_{NRC}$ obtained from the 14 individual values determined using this mass standard is $14.4 \times 10^{-9}$. The 35 parts in $10^9$ reduction of the NRC value of $h/h_{NRC}$ initially reported by Sanchez et al. (2014) due to the BIPM mass-standard shift is documented by Sanchez et al. (2015) and it is the value given therein that we take as the final result of the NRC experiment:

$$h = h_{NRC}[1 + 189(18) \times 10^{-9}],$$

which is equivalent to

$$h = 6.626 070 11(12) \times 10^{-34} \text{ J s } [1.8 \times 10^{-8}],$$

$$K_f^2 R_k = 6.036 760 76(11) \times 10^{33} \text{ J}^{-1} \text{ s}^{-1} [1.8 \times 10^{-8}],$$

where this last value is input datum B44.6 in Table XVIII with identification NRC-15. Although the value for $h/h_{NRC}$ and its uncertainty given by Sanchez et al. (2014) were obtained from the four individual values by a somewhat unusual method, an alternate analysis based on the calculation of a weighted mean of correlated values yields essentially the same value and uncertainty (Wood, 2013). As can be seen from Table XXI, the NRC result has the smallest uncertainty of any single determination of $h$.

For completeness, we note that Xu et al. (2016) at the National Institute of Metrology (NIM), Beijing, PRC, recently published a value for $h$ with $u_t = 2.6 \times 10^{-8}$ in agreement with other values but obtained using the generalized joule balance method. This approach, under development at NIM since 2007, is a variant of the watt-balance approach; the reported value is a consequence of the ongoing NIM investigation of the
feasibility of using a joule balance to achieve a competitive uncertainty.

IX. Measurements Involving Silicon Crystals

The three naturally occurring isotopes of silicon are $^{28}\text{Si}$, $^{29}\text{Si}$, and $^{30}\text{Si}$, and for natural silicon the amount-of-substance fractions $\chi(\text{Si})$ of these isotopes are approximately 0.92, 0.05, and 0.03, respectively. Here we discuss experimental results involving nearly perfect natural silicon single crystals as well as nearly perfect highly enriched silicon single crystals for which $\chi(\text{Si}) \approx 0.99996$.

A. Measurements with natural silicon

The natural silicon results employed in the 2010 adjustment are used in the 2014 adjustment without change. Natural silicon experimental data have been discussed in previous CODATA reports including CODATA-10. The measured quantities of interest are the {220} lattice spacing $d_{220}(X)$ of a number of different crystals $X$ determined in meters using a combined x-ray and optical interferometer or XRCD; and the fractional differences $[d_{220}(X) - d_{220}(\text{ref})]/ d_{220}(\text{ref})$, where ref is a reference crystal, determined using a lattice comparator based on x-ray double crystal nondispersive diffractometry. The eight natural crystals of interest are denoted WASO 4.2a, WASO 04, WASO 17, NRLM3, NRLM4, MO*, ILL, and N, and $d_{220}(X)$ of each is taken to be an adjustable constant (variable) in our least-squares calculations. For simplicity, the simplified forms W4.2a, W04, W17, NR3, and NR4 are used in quantity symbols for the first five crystals.

The CODATA-14 input data for the {220} lattice spacings of MO*, WASO 04, and WASO 4.2a are listed in Table XVIII and are data items B60, B61, B62.1, and B62.2, respectively; their identifications are either INRIM-08, INRIM-09, or PTB-81. The input data for the fractional differences of the various crystals of interest are items B50–B59 in the same table and are labeled NIST-99, NIST-97, NIST-06, PTB-98, or PTB-03. The correlation coefficients of these data are given in Table XIX and their observational equations may be found in Table XXIV. Item B58, the fractional difference between the {220} lattice spacing of an ideal natural silicon crystal $d_{220}^i$ and $d_{220}(\text{W04})$, is discussed in CODATA-06 following Eq. (312). The laboratories for which INRIM, NIST, and PTB, as well as for FSU and NMJJ in subsequent paragraphs, are identifiers may be found in the list of symbols and abbreviations near the end of this paper.

The copper $\text{K}_{\alpha_1}$ x unit, symbol $\text{xu(CuK}_{\alpha_1})$, the molybdenum $\text{K}_{\alpha_1}$ x unit, symbol $\text{xu(MoK}_{\alpha_1})$, and the ångström star, symbol Å are historic x-ray units used in the past but still of current interest. They are defined by assigning an exact, conventional value to the wavelength of the CuK$_\alpha_1$, MoK$_{\alpha_1}$, and WK$_{\alpha_1}$ x-ray lines when each is expressed in its corresponding unit. These assigned wavelengths for $\lambda(\text{CuK}_{\alpha_1})$, $\lambda(\text{MoK}_{\alpha_1})$, and $\lambda(\text{WK}_{\alpha_1})$ are 1537.400 xu(CuK$_{\alpha_1}$), 707.400 xu(MoK$_{\alpha_1}$), and 0.2090 010 0 Å,$^*$, respectively. The four experimental input data relevant to these units, which are the measured ratios of CuK$_{\alpha_1}$, MoK$_{\alpha_1}$, and WK$_{\alpha_1}$ wavelengths to the {220} lattice spacings of WASO 4.2a and N, are items B68–B71 in Table XVIII; they are labeled either FSU/PTB-91, NIST-73, or NIST-79. To obtain recommended values in meters for the units $\text{xu(CuK}_{\alpha_1})$, $\text{xu(MoK}_{\alpha_1})$, and Å,$^*$, they are taken as adjusted constants; the input data are then expressed in terms of these constants and the appropriate {220} lattice spacing of the silicon crystal used to obtain them. The resulting observational equations for these input data are given in Table XXIV.

B. Determination of $N_A$ with enriched silicon

The IAC project to determine $N_A$ using the x-ray-crystal-density (XRCD) method was initiated in 2004 and is being carried out by a group of researchers from a number of different institutions, mostly national metrology institutes.

The first enriched silicon result for $N_A$ from the IAC project, formally published in 2011, was included as an input datum in the 2010 final adjustment and is discussed in CODATA-10. In the IAC work the silicon samples are highly polished, highly pure, and nearly crystallographically perfect spheres of nominal mass 1 kg initially designated AVO28-S5 and AVO28-S8. The basic equation for the XRCD determination of $N_A$ using a perfect silicon crystal is

$$N_A = \frac{A_x(\text{Si}) M_u}{\sqrt{8} d_{220}^3 \rho(\text{Si})}. \quad (238)$$

In the IAC experiment, the macroscopic silicon mass density $\rho(\text{Si})$ is obtained from the relation $\rho(\text{Si}) = m_s/V_s$, where $m_s$ is the mass of the sphere and is determined by weighing, and $V_s = \langle 4 \pi d_s^3 / 6 \rangle$ is the volume of the sphere and is obtained from $d_s$, the sphere’s mean diameter, which is determined by optical interferometry. The lattice spacing $d_{220}$ of the silicon boule from which the sphere was fabricated is measured with an XRCD using representative silicon samples from the boule. The mean relative atomic mass of the silicon atoms $A_x(\text{Si})$ is determined from representative samples by measuring the amount-of-substance ratios $R_{28/28} = n(28\text{Si})/n(28\text{Si})$ and $R_{30/28} = n(30\text{Si})/n(28\text{Si})$ using isotopic dilution mass spectrometry and calculating $A_x(\text{Si})$ from the well-known values of $A_x(\text{Si})$.

Two other aspects of the experiment are equally important. Equation (238) applies only to a pure silicon sphere. In practice, the surface of a sphere is contaminated with a physisorbed water layer, a chemisorbed water layer, a carbonaceous layer, and an SiO$_2$ layer. Thus it is necessary to determine the mass and thickness of these layers so that the measured value of the mass of the sphere $m_s$ and the measured value of the mean diameter of the sphere $d_s$ can be corrected for the surface layers and thereby apply to the silicon core. It is also necessary to characterize the material properties of the silicon, for example, its impurities such as interstitial oxygen and substitutional carbon, nonimpurity point defects, dislocations, vacancies, and microscopic voids, and to apply
corrections to the measured mass of the sphere, \{220\} lattice spacing, and mean diameter as appropriate.

The IAC researchers continued their work after the publication of their first result and instituted a number of improvements after carefully examining all significant aspects of the experiment. This effort is described in detail by Azuma et al. (2015) and in the references cited therein. It is beyond the scope of this review to discuss the many advances made, but two are especially noteworthy. Metallic contaminants in the form of Cu, Ni, and Zn silicide compounds were discovered during the course of the work that led to the 2011 result, most likely arising from the polishing process, and had to be taken into account. This led to an increased uncertainty for the required surface-layer correction. To overcome this problem the spheres were reetched and carefully repolished.

The second improvement involves the determination of the amount-of-substance ratios, and thus the value of \(A_\lambda/(Si)\). In the new work the ratios were measured independently at PTB, NMIJ, and NIST using a multicollector inductively coupled plasma mass spectrometer and isotope dilution. The solvent and diluent used in the three institutes was tetramethylammonium hydroxide (TMAH), which significantly reduced the baseline level of the ion currents to be measured compared with the levels usually seen with the normally used NaOH. For this and other reasons discussed in detail by Azuma et al. (2015), the ratios obtained by Yang et al. (2012) using NaOH were not employed in the calculation of the new IAC value of \(N_\lambda\). See Kuramoto et al. (2015), Mana et al. (2015), Massa et al. (2015), Mizushima et al. (2015), Pramann et al. (2015), Waseda et al. (2015), and Zhang et al. (2015).

The two new values of \(N_\lambda\) reported by Azuma et al. (2015) and which are used as input data in the 2014 adjustment are

\[
N_\lambda = 6.02214099(18) \times 10^{23} \text{ mol}^{-1} \quad [3.0 \times 10^{-8}], \quad (239)
\]

\[
N_\lambda = 6.02214076(12) \times 10^{23} \text{ mol}^{-1} \quad [2.0 \times 10^{-8}]. \quad (240)
\]

The first result is the 2011 value used in CODATA-10 increased by 3 parts in \(10^8\) by Azuma et al. (2015) to reflect the recalibration of the mass standards used in its determination as a consequence of the extraordinary calibration campaign against the international prototype of the kilogram discussed in Sec. VIII. The second result is the value reported by Azuma et al. (2015) as a consequence of the extensive IAC efforts of the past 4 years and is the weighted mean of the results obtained from spheres AVO28-S5c and AVO28-S8c. (The additional letter c has been added by the researchers to distinguish the reetched and repolished spheres used in the new work from the spheres used in the earlier work.) The uncertainty assigned to the value for \(N_\lambda\) obtained from sphere AVO28-S5c is 21 parts in \(10^8\), and for the value obtained from sphere AVO28-S8c is 23 parts in \(10^8\). The two largest uncertainty components for AVO28-S5c are 10 parts in \(10^8\) for surface characterization and 16 parts in \(10^8\) for sphere volume as calculated from the mean diameter. It should be noted that the two values of \(N_\lambda\) are correlated; the IAC researchers report the correlation coefficient to be 0.17 (Mana et al., 2015).

The two values of \(N_\lambda\) in Eqs. (239) and (240) are items B63.1 and B63.2 in Table XVIII with identifiers IAC-11 and IAC-15, the values of \(h\) that can be inferred from them are given in Table XXI, and their observational equation, which also shows how \(h\) can be derived from \(N_\lambda\), may be found in Table XXIV. How these results compare with other data and their role in the 2014 adjustment are discussed in Sec. XIII.

### X. Thermal Physical Quantities

Table XIV summarizes the 11 results for the thermal physical quantities \(R, k/h, \text{ and } A_\lambda/R\), the molar gas constant, the quotient of the Boltzmann and Planck constants, and the quotient of the molar polarizability of a gas and the molar gas constant, respectively, that are taken as input data in the 2014 adjustment. They are data items B6.1 – B66 in Table XVIII with correlation coefficients as given in Table XIX and observational equations as given in Table XXVII. Values of the Boltzmann constant \(k\) that can be inferred from these data are given in Table XXII and are graphically compared in Fig. 5.

There are five new input data that contribute to the 2014 determination of the Boltzmann constant, three from acoustic gas thermometry (NPL-13, NIM-13, and LNE-15), one from Johnson noise thermometry (NIM/NIST-15), and one from dielectric-constant gas thermometry (PTB-15). Not every value in Table XIV appears in the cited references. For some, additional digits have been provided to the Task Group to reduce rounding errors; for others, the actual measured value of \(R\) is recovered from the reported value of \(k\) and the Avogadro constant \(N_\lambda\) used by the researchers to calculate \(k\). Finally, some of the input data incorporate changes based upon newly available information, as discussed in Sec. X.A.2.

Since there is no serious discrepant input data for the inferred value of the Boltzmann constant for either the 2010 or 2014 adjustment, the result for \(k\) from refractive index gas thermometry (Schmidt et al., 2007) and for \(k/h\) from Johnson noise thermometry (Benz et al., 2011) that were initially considered but not included in the final 2010 adjustment due to their large uncertainties are not considered for the 2014 adjustment.

#### A. Molar gas constant \(R\), acoustic gas thermometry

The measurement of \(R\) by the method of acoustic gas thermometry (AGT) is based on the following expressions for the square of the speed of sound in a real gas of atoms or molecules in thermal equilibrium at thermodynamic temperature \(T\) and pressure \(p\) and occupying a volume \(V\):

\[
C_v^2(T, p) = A_0(T) + A_1(T)p + A_2(T)p^2 + A_3(T)p^3 + \cdots. \quad (241)
\]

Here \(A_1(T), A_2(T), \text{ etc.}\) are related to the density virial coefficients and their temperature derivatives. In the limit \(p \to 0\), this becomes

---

DCGT: dielectric-constant gas thermometry

helium, is to measure the acoustic resonant frequencies of for an ideal monotonic gas. The basic experimental approach


NPL-79 R AGT, cylindrical, argon 8.314 504(70) J mol⁻¹ K⁻¹ 8.4 × 10⁻⁶

NIST-88 R AGT, spherical, argon 8.314 470(15) J mol⁻¹ K⁻¹ 1.8 × 10⁻⁶

LNE-09 R AGT, quasispherical, helium 8.314 467(23) J mol⁻¹ K⁻¹ 2.7 × 10⁻⁶

NPL-10 R AGT, quasispherical, argon 8.314 468(26) J mol⁻¹ K⁻¹ 3.2 × 10⁻⁶

INRIM-10 R AGT, spherical, helium 8.314 412(63) J mol⁻¹ K⁻¹ 7.5 × 10⁻⁶

NPL-11 R AGT, quasispherical, argon 8.314 455(12) J mol⁻¹ K⁻¹ 1.4 × 10⁻⁶

NIM-13 R AGT, cylindrical, argon 8.314 455(31) J mol⁻¹ K⁻¹ 3.7 × 10⁻⁶

NPL-13 R AGT, quasispherical, argon 8.314 454(75) J mol⁻¹ K⁻¹ 9.0 × 10⁻⁷

LNE-15 R AGT, quasispherical, helium 8.314 461(84) J mol⁻¹ K⁻¹ 1.0 × 10⁻⁶

NIM/NIST-15 k/h JNT, JE and QHE 2013 0.286658(80) × 10¹⁰ Hz K⁻¹ 3.9 × 10⁻⁹

PTB-15 Aᵣ/R DCGT, helium 6.221 128(25) × 10⁻⁶ m¹ K⁻¹ 4.0 × 10⁻⁶

\[ c_s^2(T, 0) = \frac{A_0(T)}{A_r(X)M_a} (242) \]

where \( \gamma_0 = c_p/c_v \) is the ratio of the specific heat capacity of the gas at constant pressure to that at constant volume and is \( 5/3 \) for an ideal monotonic gas. The basic experimental approach to determining the speed of sound of a gas, usually argon or helium, is to measure the acoustic resonant frequencies of a cavity at or near the triple point of water, \( T_{TPT} = 273.16 \) K, at various pressures and extrapolating to \( p = 0 \). The cavities are either cylindrical of fixed or variable length, or spherical, but most commonly quasispherical in the form of a triaxial ellipsoid. This shape removes the degeneracy of the microwave resonances used to measure the volume of the resonator in order to calculate \( c_s^2(T, p) \) from the measured acoustic frequencies and the corresponding acoustic resonator eigenvalues known from theory. The cavities are formed by carefully joining hemispherical cavities.

In practice, the determination of \( R \) by AGT with a relative standard uncertainty of order 1 part in \( 10^6 \) is complex; the application of numerous corrections is required as well as the investigation of many possible sources of error. For a review of the advances made in AGT in the last 25 years, see Moldover et al. (2014).

1. New values

a. NIM 2013. Lin et al. (2013) report a result for the Boltzmann constant from an improved version of an earlier experiment (Zhang et al., 2011) using argon in a single 80 mm long fixed cylindrical cavity measured by two-color optical interferometry. The shape of the cavity has been made more cylindrical and the thermometry improved. Two different grades of argon with measured relative isotopic abundances were used with two different methods of supporting the cavity. Analysis of the acoustic data was improved by accounting for second-order perturbations to the frequencies from the thermoviscous boundary layer.

The reported calculated value of the Boltzmann constant uses the CODATA-10 value of \( N_A \), implying the measured value

\[ R = 8.314 455(31) \text{ J mol}^{-1} \text{ K}^{-1} [3.7 \times 10^{-6}]. \]  

(243)

The largest uncertainty component, 2.9 parts in \( 10^6 \), is due to inconsistent values determined from the various acoustic modes.

b. NPL 2013. de Podesta et al. (2013) used a quasispherical copper triaxial ellipsoid cavity with a nominal radius of 62 mm filled with argon to determine \( R \). The cavity was suspended from the top of a copper container designed to create a nearly isothermal environment. The initial value reported is

\[ R = 8.314 4787(59) \text{ J mol}^{-1} \text{ K}^{-1} [7.1 \times 10^{-7}]. \]  

(244)

The low uncertainty is attributed to the near-perfect shape and surface condition of the cavity achieved by precise diamond turning techniques during fabrication. The largest uncertainty component, 3.5 parts in \( 10^7 \), is from the determination of the molar mass of the argon used in the experiment.

c. LNE 2015. Pitre et al. (2015) used a quasispherical copper triaxial ellipsoid cavity with a nominal radius of 50 mm filled with helium. The experiment was performed in quasi-adiabatic thermal conditions, instead of standard, constant heat-flux conditions. The reported calculated value of the Boltzmann constant uses the CODATA-10 value of \( N_A \), implying the measured value

\[ R = 8.314 4615(84) \text{ J mol}^{-1} \text{ K}^{-1} [1.0 \times 10^{-6}]. \]  

(245)

The dominant source of uncertainty, 6.2 parts in \( 10^7 \), arises from the acoustic frequency measurements.
2. Updated values

The following updates and corrections for the AGT input data are summarized in Moldover, Gavioso, and Newell (2015), along with a detailed description of the correlation coefficients given in Table XIX. The AGT values and uncertainties in Table XIV incorporate all the corrections listed below.

a. Molar mass of argon. During the period from October to December 2014, three important studies on the molar mass of argon \(M(\text{Ar})\) were performed to investigate the difference of 2.74 parts in \(10^6\) between the results from LNE-11 and NPL-13 (Yang et al., 2015). The LNE-11 value is based on \(M(\text{Ar})\) determinations from the Institute for Reference Materials and Measurements (IRMM), Geel, Belgium (Valkiers et al., 2010). The NPL-13 estimate of \(M(\text{Ar})\) is based on a comparison at the Scottish Universities Environmental Research Centre (SUERC), University of Glasgow, Glasgow, Scotland, of the isotopic composition of the experimental gas with the isotopic composition of argon from atmospheric air (de Podesta et al., 2013), assuming the atmospheric air had the same argon isotopic abundance as the analysis performed by Lee et al. (2006) at the Korea Research Institute of Standards and Science (KRISS), Taedoc Science Town, Republic of Korea. The results of these studies were discussed at the meeting of the CIPM Consultative Committee for Thermometry Task Group for the SI (CCT TG-SI) in Eltville, Germany, on 6 February 2015 in conjunction with the 2015 Fundamental Constants Meeting co-organized by the Task Group (Karshenboim, Mohr, and Newell, 2015).

In the first study, the isotopic composition of samples of argon gas previously measured at the IRMM were remeasured. The analysis showed disagreements in values of \(M(\text{Ar})\) of up to 3.5 parts in \(10^6\). In the second study, the isotopic composition of a sample of argon gas used for isotherm 5 of NPL-13 was remeasured. The estimate of \(M(\text{Ar})\) was 2.73 parts in \(10^6\) lower than the corresponding SUERC estimate. In the third study, mass spectrometry measurements were made on a series of argon samples from NIM, INRIM, NPL, NMJJ and LNE on which corresponding speed-of-sound measurements had been made at LNE. The results showed the expected correlation between the two sets of measurements. From this study an inference of the required correction to the SUERC estimate of the NPL-13 isotherm 5 molar mass of \(-3.6\) parts in \(10^6\) was apparent. However, KRISS considered their direct measurement of this sample to have a lower uncertainty. The inference of the required correction to the LNE-11 \(M(\text{Ar})\) was not strong enough to suggest a meaningful change in its value.

Although the estimated relative statistical uncertainty for the KRISS \(M(\text{Ar})\) measurements was 0.61 \(\times 10^{-6}\), KRISS considered it likely that the isotope ratio \(R(^{38}\text{Ar}/^{36}\text{Ar})\) may be in error because it disagrees distinctly with measurements from Lee et al. (2006). For this reason an additional relative uncertainty of \(0.35 \times 10^{-6}\) was added in quadrature to yield an overall relative uncertainty for the KRISS \(M(\text{Ar})\) measurements of \(0.7 \times 10^{-6}\).

Based on the studies at KRISS it was agreed upon by all participants of the CCT TG-SI meeting that for the 2014 CODATA adjustment, the NPL-13 determination of the molar gas constant will rely on the KRISS value and uncertainty of \(M(\text{Ar})\), resulting in a fractional correction of \(-2.73 \times 10^{-6}\) and a total relative uncertainty of \(0.9 \times 10^{-6}\) (de Podesta et al., 2015). It was also agreed that while the LNE-11 value of the molar gas constant will continue to use the determination of \(M(\text{Ar})\) from IRMM, the relative uncertainty will be that of KRISS, namely \(0.7 \times 10^{-6}\). Similarly since the previous NPL-10 result used the \(M(\text{Ar})\) value from IRMM, the relative uncertainty component for \(M(\text{Ar})\) for NPL-10 was increased to \(0.7 \times 10^{-6}\).

b. Molar mass of helium. Based on the measured \(^3\text{He}/^4\text{He}\) abundance ratios spanning \(0.05 \times 10^{-6}\) to \(0.5 \times 10^{-6}\) from samples taken from 12 natural gas wells in the USA (Aldrich and Nier, 1948), the expected reduction in \(^4\text{He}\) due to naturally occurring \(^3\text{He}\) is fractionally from 0.012 \(\times 10^{-6}\) to 0.12 \(\times 10^{-6}\). The two gas analyses given in the paper by Gavioso et al. (2015) are consistent with this expectation (see Sec. X.D). In contrast, the ratio of speed-of-sound measurements using two different, commercially produced, highly purified helium samples given in the paper reporting LNE-15 differed by the surprisingly large value of 0.44 parts in \(10^6\). This observation was taken into account by including an additional uncertainty component of 0.5 parts in \(10^6\). Based upon the possibility that the concentration of \(^3\text{He}\) may be higher than expected, an additional relative uncertainty component of \(0.5 \times 10^{-6}\) was incorporated in the LNE-09 result. It should finally be noted that certain natural gases in Taiwan have \(^3\text{He}/^4\text{He}\) abundance ratios as large as 3.8 \(\times 10^{-6}\) (Sano, Wakita, and Huang, 1986). Clearly, future helium-based low-uncertainty AGT determinations of \(R\) must measure the \(^3\text{He}\) concentration in the gas samples used.

c. Thermal conductivity of argon. An improved estimate for the thermal conductivity of argon became available for the 2014 CODATA adjustment [see supplementary data in Moldover et al. (2014)]. A change in thermal conductivity affects the estimate for the thermal boundary layer thickness close to the wall of the cavity, resulting in a correction for all resonant frequencies at all pressures. For the 2014 CODATA adjustment, these corrections are applied to the lowest uncertainty results for AGT using argon. In parts in \(10^6\), the corrections to NIST-88 (Moldover, 2015), LNE-11 (Pitre, 2015), and NPL-13 (de Podesta et al., 2015) are \(-0.16\), \(-0.16\), and \(-0.192\), respectively, with inconsequential decreases in the uncertainties.

B. Quotient \(k/h\), Johnson noise thermometry

The Nyquist theorem predicts that, with a fractional error of less than 1 part in \(10^6\) at frequencies less than 10 MHz and temperatures greater than 250 K,

\[
\langle U^2 \rangle = 4 kT R \Delta f. \tag{246}
\]

Here \(\langle U^2 \rangle\) is the mean-square voltage, or Johnson noise voltage, in a measurement bandwidth of frequency \(\Delta f\) across the terminals of a resistor of resistance \(R\), in thermal equilibrium at thermodynamic temperature \(T\). If \(\langle U^2 \rangle\) is measured in...
terms of the Josephson constant $K_j = 2e/h$ and $R_c$ in terms of the von Klitzing constant $R_K = h/e^2$, then the measurement yields a value of $k/h$.

Continuing the pioneering work of Benz et al. (2011), Qu et al. (2015) report an improved determination of the Boltzmann constant using such a method. The reported value is

$$k_0 = 1.3806513(53) \times 10^{-23} \text{ J K}^{-1} \quad [3.9 \times 10^{-6}],$$

(247)

where $k_0$ in the SI unit J/K is the result of a Johnson noise experiment when the voltage and resistance are measured in conventional electrical units defined by $K_{j-90}$ and $R_{K-90}$. Following the analysis given in CODATA-98 [see Eqs. (29d) and (317)], it can be shown that $k_0/\hbar_0 = k/h$. Using the value of $\hbar_0$ (see the introduction to Sec. VIII), the measured value of $k/h$ is

$$k/h = 2.0836658(80) \times 10^{10} \text{ Hz K}^{-1} \quad [3.9 \times 10^{-6}],$$

(248)
as given in Table XIV.

In the Qu et al. (2015) experiment, digitally synthesized pseudonoise voltages $V_Q$ are generated by means of a pulse-biased Josephson junction array. These known voltages are compared to the unknown thermal-noise voltages $V_t$ generated by a specially designed 200 $\Omega$ resistor in a well regulated thermal cell at or near $T_{TPW}$. Since the spectral density of the noise voltage of a 200 $\Omega$ resistor at 273.16 K is only 1.74 nV/$\sqrt{\text{Hz}}$, it is measured using a low-noise, two-channel, cross-correlation technique that enables the resistor signal to be extracted from uncorrelated amplifier noise of comparable amplitude and spectral density. The final result is based on measurements integrated over a bandwidth of 575 kHz and a total integration time of about 33 d.

The dominant uncertainty contributions of 3.2 parts in 10$^6$ and 1.8 parts in 10$^6$ are from the statistical uncertainty of the $(V_R^2/V_Q)^2$ ratio measurement and the ambiguity associated with the spectral mismatch model, respectively.

C. Quotient $A_e/R$, dielectric-constant gas thermometry

The virial expansion of the equation of state for a real gas of amount of substance $n$ in a volume $V$ is

$$p = \rho RT[1 + \rho B(T) + \rho^2 C(T) + \rho^3 D(T) + \cdots],$$

(249)

where $\rho = n/V$ is the amount-of-substance density of the gas at thermodynamic temperature $T$, and $B(T)$, $C(T)$, etc. are the virial coefficients. The Clausius-Mossotti equation is

$$\frac{\varepsilon - 1}{\varepsilon + 2} = \rho A_e[1 + \rho B_e(T) + \rho^2 C_e(T) + \rho^3 D_e(T) + \cdots],$$

(250)

where $\varepsilon = \varepsilon/\varepsilon_0$ is the relative dielectric constant (relative permittivity) of the gas, $\varepsilon$ is its dielectric constant, $\varepsilon_0$ is the exactly known electric constant, $A_e$ is the molar polarizability of the atoms, and $B_e(T)$, $C_e(T)$, etc., are the dielectric virial coefficients. By appropriately combining Eqs. (249) and (250), an expression is obtained from which $A_e/R$ can be experimentally determined by measuring $\varepsilon$, at a known constant temperature such as $T_{TPW}$ and at different pressures and extrapolating to zero pressure.

In practice, dielectric-constant gas thermometry measures the fractional change in capacitance of a specially constructed capacitor, first without helium gas and then with helium gas at a known pressure. The static electric polarizability of a gas atom $\alpha_0$, $A_e$, $R$, and $k$ are related by $A_e/R = \alpha_0/3\varepsilon_0k$, which shows that if $\alpha_0$ is known sufficiently well from theory, then a competitive value of $k$ can be obtained if the quotient $A_e/R$ can be measured with a sufficiently small uncertainty.

Piszczatowski et al. (2015) have calculated the static electric polarizability of $^4$He in atomic units to be

$$\alpha_0(^4\text{He}) = 1.38376077(14) \text{ a.u.} \quad [1.0 \times 10^{-7}],$$

(251)

from which the static electric polarizability of $^4$He in SI units is

$$\alpha_0(^4\text{He}) = 4\pi\varepsilon_0\alpha_0(^4\text{He}),$$

(252)

where $\alpha_0$ and $\varepsilon_0$ are the Bohr radius and electric constant, respectively.

Superseding previous preliminary results (Fellmuth et al., 2011; Gaiser and Fellmuth, 2012), Gaiser et al. (2013) report a final value of $A_e/R$ from dielectric-constant gas thermometry with an updated uncertainty given by Gaiser, Zandt, and Fellmuth (2015):

$$A_e/R = 6.221128(25) \times 10^{-8} \text{ m}^3 \text{ K}^{-1} \quad [4.0 \times 10^{-6}].$$

(253)
The dominant uncertainty components are the fitted coefficient from the 10 isotherms (statistical) and the effective compressibility of the capacitor assembly at 2.6 parts in 10$^6$ and 2.4 parts in 10$^6$, respectively.

D. Other data

For completeness we note the following result that became available only after the 31 December 2014 closing date of the 2014 adjustment. Gavioso et al. (2015) obtained the very competitive value $R = 8.3144743(88) \text{ J mol}^{-1} \text{ K}^{-1} \ [1.06 \times 10^{-6}]$ using acoustic gas thermometry with a misaligned spherical cavity with a nominal radius of 90 mm filled with helium. This value is 1.47 parts in 10$^6$ larger than the 2014 CODATA value.

E. Stefan-Boltzmann constant $\sigma$

The Stefan-Boltzmann constant is related to $c$, $h$, and $k$ by $\sigma = 2\pi^5k^4/15h^2c^2$, which, with the aid of the relations $k = R/N_A$ and $N_Ah = cA_e(c)M_d\alpha^2/2R_c$, can be expressed in terms of the molar gas constant and other adjusted constants as

$$\sigma = \frac{32\pi^5h}{15c^6} \left( \frac{R_cR}{A_e(c)M_d\alpha^2} \right)^4.$$  

(254)
Since no competitive directly measured value of \( \sigma \) is available for the 2014 adjustment, the 2014 recommended value is obtained from this equation.

### XI. Newtonian Constant of Gravitation

Table XV summarizes the 14 measured values of the Newtonian constant of gravitation of interest in the 2014 adjustment. Because the values are independent of the other data relevant to the current adjustment, and because there is no known quantitative theoretical relationship between \( G \) and other adjusted constants, they contribute only to the determination of the 2014 recommended value of \( G \). The calculation of this value is discussed in Sec. XIII.B.1.

While three new values for \( G \) have become available for the 2014 CODATA adjustment, the data remain discrepant. The first is a competitive result from the International Bureau of Weights and Measures (BIPM) (Quinn et al., 2013, 2014) obtained using a similar but completely rebuilt apparatus as was used to obtain the BIPM 2001 result. The second, based on a unique technique involving atom interferometry, is from the European Laboratory for Non-linear Spectroscopy (LENS) (Prevedelli et al., 2014; Rosi et al., 2014). Although not competitive, the conceptually different approach could help identify errors that have proved elusive in other experiments. The third is from the University of California, Irvine (Newman et al., 2014) and is a highly competitive result from data collected over a 7-year span using a cryogenic torsion balance.

The previously reported measurements of \( G \) as discussed in past Task Group reports remain unchanged with one exception. It was discovered that the reported correlation coefficient between the 2005 and 2009 result from the Huazhong University of Science and Technology, HUST-05 and HUST-09, was unphysical. As described below, a reexamination of the uncertainty analysis has led to slight reductions in the HUST-05 value and in the correlation coefficient of the two results.

For simplicity, in the following text, we write \( G \) as a numerical factor multiplying \( G_0 \), where

\[
G_0 = 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2}.
\]  

#### A. Updated values

1. **Huazhong University of Science and Technology**

The initially assigned covariance of the HUST-05 and HUST-09 values of \( G \) exceeded the variance of the HUST-09 value which had the smaller uncertainty of the two. As a result the weighted mean of the two values was outside the interval between them, which is unphysical (Cox et al., 2006; Bich, 2013).

In collaboration with the HUST researchers, the uncertainty budgets and corrections of both the HUST-05 and HUST-09 measurements were reviewed. Upon further examination, the 2010 correlation coefficient between HUST-05 and HUST-09 of 0.234 contained a misassigned contribution due to fiber anelasticity of 0.098. While the suspension fibers in both experiments were 25 \( \mu \)m diameter tungsten wire, the individual wires used were different. Moreover the HUST-05...
anelasticity correction was estimated from the pendulum quality factor \( Q \) as predicted by Kuroda (1995), whereas for HUST-09 it was directly measured (Luo et al., 2009). After careful reevaluation of all correlations and removing the anelasticity correction, the correlation coefficient between HUST-05 and HUST-09 was reduced to 0.134.

Since the \( Q \) for the HUST-05 torsion pendulum was approximately \( 3.6 \times 10^6 \), the positive bias due to fiber anelasticity was originally neglected. For the 2014 adjustment, the HUST-05 value has been reduced by 8.8 parts in \( 10^6 \) based on the Kuroda correction with the result that the HUST-05 input datum is now

\[
G = 6.67222(87)G_0 \quad [1.3 \times 10^{-4}]. \tag{256}
\]

### B. New values

#### 1. International Bureau of Weights and Measures

A new result from the BIPM, labeled BIPM-14, has been reported by Quinn et al. (2013, 2014) using the same principles of a flexure strip torsion balance operating in either of two different modes as in the previous BIPM experiment: compensation mode (cm) and deflection mode (dm) (Quinn et al., 2001). However, almost all of the apparatus was rebuilt or replaced. Extensive tests were performed and improvements made on key parameters, including test and source mass coordinates, calibration of angle measurements, calibration of ac voltage and capacitance electrical instruments, timing measurements for period of oscillation, and precision of torque measurements. With all identified errors taken into account, the results for the two modes are

\[
G_{\text{cm}} = 6.67515(41)G_0 \quad [6.1 \times 10^{-5}], \tag{257}
\]

\[
G_{\text{dm}} = 6.67586(36)G_0 \quad [5.4 \times 10^{-5}]. \tag{258}
\]

The largest uncertainty component for each mode is 47 parts in \( 10^6 \) from angle measurements, but the angle measurements are anticorrelated between the two modes. Taking into consideration all correlations, the reported weighted mean and uncertainty from the two modes is

\[
G = 6.67554(16)G_0 \quad [2.4 \times 10^{-5}]. \tag{259}
\]

which agrees well with the 2001 BIPM result. This agreement is noteworthy, because only the source masses and their carousel in the original apparatus were not replaced; the source masses were reduced in height and remeasured, and the experiment was also rebuilt in a different BIPM laboratory. Quinn et al. (2014) conclude that the 2014 and 2001 BIPM values of \( G \) are not correlated.

#### 2. European Laboratory for Non-Linear Spectroscopy, University of Florence

A novel measurement technique to measure \( G \) using atom interferometry instead of a precision mechanical balance has recently been completed by the European Laboratory for Non-Linear Spectroscopy at the University of Florence (Prevedelli et al., 2014; Rosi et al., 2014). Labeled LENS-14, the experiment combines two vertically separated atomic clouds forming a double atom-interferometer-gravity gradiometer that measures the change in the gravity gradient when a well-characterized source mass is displaced.

The experimental design uses a double differential configuration that greatly reduces the sensitivity to common-mode spurious signals. Two atomic rubidium clouds are launched in the vertical direction with a vertical separation of approximately 30 cm in a juggling sequence. The two clouds are simultaneously interrogated by the same Raman three-pulse interferometric sequence. The difference in the phase shifts between the upper and lower interferometers measures the gravity gradient. The gravity gradient is then modulated by the symmetric placement of the 516 kg tungsten source mass in two different vertical positions around the double atom interferometer. To further cancel common-mode spurious effects the two photon recoil used to split and recombine the wave packets in the interferometers is reversed.

The value of \( G \) is extracted by calculating the source mass gravitational potential and the phase shift for single-atom trajectories, carrying out Monte Carlo simulations of the atomic cloud, and estimating other corrections not taken into account by the Monte Carlo simulation. The final result is

\[
G = 6.67191(99)G_0 \quad [1.5 \times 10^{-4}]. \tag{260}
\]

The leading uncertainty components arise from the determination of the atomic cloud size, center, and launch direction, and the tungsten source mass position, and in parts in \( 10^6 \) are 61, 38, 36, and 38, respectively. Although the final uncertainty is not presently competitive, determinations of \( G \) using atom interferometry could be more competitive in the future.

#### 3. University of California, Irvine

A highly competitive result from data collected over a 7 year span using a cryogenic torsion balance operating below 4 K in a dynamic mode with two orientations for the source mass has recently been reported by researchers from the University of California, Irvine (Newman et al., 2014), labeled UCI-14. The advantages of cryogenic operation are a much higher torsion pendulum \( Q \), which greatly reduces the systematic bias predicted by Kuroda (1995), much lower thermal noise acting on the balance, greatly reduced fiber-property dependence on temperature variation, excellent temperature control, easy to maintain high vacuum, and ease of including effective magnetic shielding with superconducting material. The source mass is a pair of copper rings that produces an extremely uniform gravity gradient over a large region centered on the torsion balance test mass. However, by necessity it is located 40 cm from the test mass (i.e. outside the vacuum dewar), thus greatly reducing the period-change signal of the torsion balance. The torsion balance test mass is a thin fused silica
plate as pioneered by Gundlach and Merkowitz (2000) that, when combined with the ring source masses, minimizes the sensitivity to test mass shape, mass distribution, and placement.

Over the 7 year span three fibers were used. Fiber 1 was an as-drawn CuBe fiber, fiber 2 a heat-treated CuBe fiber, and fiber 3 an as-drawn 5056 aluminium-alloy fiber. It was observed that the Birge ratio for the data within each run was much larger than expected. A total of 27 variants of data analysis methods were used for each fiber to test the robustness of the data, with the resulting 27 values of $G$ varying over a range of 14, 24, and 20 parts in $10^6$ for fibers 1, 2, and 3, respectively. The final analysis uses an unweighted average for each run, a weighted average over runs for each fiber with a Birge ratio uncertainty expansion, and an outlier identification protocol. An additional uncertainty component equal to half of the range of $G$ values determined during a robustness test is included in the final values of $G$ from the three fibers, which are

\[
G_1 = 6.674350(97)G_0 \left[1.5 \times 10^{-5}\right], \quad (261) \\
G_2 = 6.67408(15)G_0 \left[2.2 \times 10^{-5}\right], \quad (262) \\
G_3 = 6.67455(13)G_0 \left[2.0 \times 10^{-5}\right]. \quad (263)
\]

Instead of an averaged value of $G$ from the three fibers as published by Newman et al. (2014), the Task Group decided to use a weighted mean of the three values with a correlated relative uncertainty of 8.6 parts in $10^6$ between each pair of fibers due to uncertainties associated with the source and test masses. The final UCI-14 input datum is

\[
G = 6.67435(13)G_0 \left[1.9 \times 10^{-5}\right], \quad (264)
\]

where the uncertainty is taken to be the average of the three uncertainties as assigned by the researchers rather than that of the weighted mean since it better reflects the researchers view of the reliability of their measurements.

### XII. Electroweak Quantities

There are a few cases in the 2014 adjustment, as in previous adjustments, where an inexact constant that is used in the analysis of input data is not treated as an adjusted quantity, because the adjustment has a negligible effect on its value. Three such constants, used in the calculation of the theoretical expression for the electron magnetic-moment anomaly $a_e$, are the mass of the tau lepton $m_\tau$, the Fermi coupling constant $G_F$, and sine squared of the weak mixing angle $\sin^2\theta_W$; they are obtained from the most recent report of the Particle Data Group (Olive et al., 2014):

\[
m_\tau = 1776.82(16) \text{ MeV} \quad [9.0 \times 10^{-5}], \quad (265) \\
G_F/\hbar c = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2} \quad [5.1 \times 10^{-7}], \quad (266) \\
\sin^2\theta_W = 0.2223(21) \quad [9.5 \times 10^{-3}]. \quad (267)
\]

We use the definition $\sin^2\theta_W = 1 - (m_W/m_Z)^2$, where $m_W$ and $m_Z$ are, respectively, the masses of the $W^\pm$ and $Z^0$ bosons, because it is employed in the calculation of the electroweak contributions to $a_e$ (Zarnecki, Krause, and Marciano, 1996). The Particle Data Group’s recommended value for the mass ratio of these bosons is $m_W/m_Z = 0.8819(12)$, which leads to the value of $\sin^2\theta_W$ given above.

The values of these constants are the same as used in CODATA-10, which were taken from the 2010 Particle Data Group report (Nakamura et al., 2010), except that for $G_F/\hbar c$. The 2014 value exceeds the 2010 value by the fractional amount $1.3 \times 10^{-5}$ and its uncertainty is about one eighth that of the 2010 value. The value for $G_F/\hbar c$ is taken from p. 139 of Olive et al. (2014).

### XIII. Analysis of Data

The input data discussed in the previous sections are analyzed in this section, and based on that analysis the data used to determine the 2014 CODATA recommended values of the constants are selected. We closely follow the approach used in CODATA-10. The input data are given in Tables XVI, XVII, XVIII, and XIX. For ease of presentation the relevant covariances among the data are given in the form of correlation coefficients, but the actual covariances are used in all calculations. There are 15 types of input data with two or more values and the data of the same type generally agree among themselves; that is, there are no differences between like data that exceed $2\sigma_{\text{data}}$, the standard uncertainty of the difference. The major exception is the values of the Newtonian constant of gravitation $G$. These are listed in Table XXVII and because the $G$ data are independent of all other data, they are treated separately in Sec. XIII.B.1. A minor exception is the difference between items B44.2 and B44.6, the NIST-98 and NRC-15 watt-balance values of $K_{J_2}^p R_K$; for these $\sigma_{\text{data}} = 2.04$.

#### A. Comparison of data through inferred values of $\alpha$, $h$, and $k$

The extent to which the data agree is shown in this section by directly comparing values of $\alpha$, $h$, and $k$ that can be inferred from different experiments. However, the inferred value is for comparison purposes only; the datum from which it is obtained, not the inferred value, is used in the least-squares calculations.

Table XX and Figs. 1 and 2 compare values of $\alpha$ calculated from the indicated input data. They are obtained using the appropriate observational equation for the corresponding input datum as given in Table XXIV and the 2014 recommended values of the constants other than $\alpha$ that enter that equation. The table and figures show that a large majority of the values of $\alpha$ agree, and thus the data from which they are obtained agree; of the 91 differences between the 14 values of $\alpha$, there are only eight that exceed $2\sigma_{\text{data}}$ and these are in the range 2.02 to 2.60. Six are between $\alpha$ from item B39.1, the NIST-89 result for $F_{\alpha}^{0.90}(lo)$, and $\alpha$ from B22.1, B22.2, B43.1, B43.3, B46, and B48. The other two are between $\alpha$ from item...
<table>
<thead>
<tr>
<th>Item number</th>
<th>Input datum</th>
<th>Value</th>
<th>Relative standard uncertainty $^a\delta\nu$</th>
<th>Identification</th>
<th>Sec.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>$\delta\nu(1S_{1/2})$</td>
<td>0.0(2.5) kHz</td>
<td>$[7.5 \times 10^{-13}]$</td>
<td>Theory</td>
<td>IV.A.1.1</td>
</tr>
<tr>
<td>A2</td>
<td>$\delta\nu(2S_{1/2})$</td>
<td>0.0(31) kHz</td>
<td>$[3.8 \times 10^{-13}]$</td>
<td>Theory</td>
<td>IV.A.1.1</td>
</tr>
<tr>
<td>A3</td>
<td>$\delta\nu(3S_{1/2})$</td>
<td>0.000(91) kHz</td>
<td>$[2.5 \times 10^{-13}]$</td>
<td>Theory</td>
<td>IV.A.1.1</td>
</tr>
<tr>
<td>A4</td>
<td>$\delta\nu(4S_{1/2})$</td>
<td>0.000(39) kHz</td>
<td>$[1.9 \times 10^{-13}]$</td>
<td>Theory</td>
<td>IV.A.1.1</td>
</tr>
<tr>
<td>A5</td>
<td>$\delta\nu(5S_{1/2})$</td>
<td>0.000(15) kHz</td>
<td>$[1.6 \times 10^{-13}]$</td>
<td>Theory</td>
<td>IV.A.1.1</td>
</tr>
<tr>
<td>A6</td>
<td>$\delta\nu(6S_{1/2})$</td>
<td>0.000(63) kHz</td>
<td>$[1.2 \times 10^{-13}]$</td>
<td>Theory</td>
<td>IV.A.1.1</td>
</tr>
<tr>
<td>A7</td>
<td>$\delta\nu(2P_{1/2})$</td>
<td>0.000(28) kHz</td>
<td>$[3.5 \times 10^{-14}]$</td>
<td>Theory</td>
<td>IV.A.1.1</td>
</tr>
<tr>
<td>A8</td>
<td>$\delta\nu(2P_{3/2})$</td>
<td>0.000(38) kHz</td>
<td>$[1.9 \times 10^{-14}]$</td>
<td>Theory</td>
<td>IV.A.1.1</td>
</tr>
<tr>
<td>A9</td>
<td>$\delta\nu(2D_{3/2})$</td>
<td>0.000(28) kHz</td>
<td>$[3.5 \times 10^{-14}]$</td>
<td>Theory</td>
<td>IV.A.1.1</td>
</tr>
<tr>
<td>A10</td>
<td>$\delta\nu(2D_{5/2})$</td>
<td>0.000(44) kHz</td>
<td>$[8.5 \times 10^{-15}]$</td>
<td>Theory</td>
<td>IV.A.1.1</td>
</tr>
<tr>
<td>A11</td>
<td>$\delta\nu(12D_{3/2})$</td>
<td>0.000(13) kHz</td>
<td>$[5.7 \times 10^{-15}]$</td>
<td>Theory</td>
<td>IV.A.1.1</td>
</tr>
<tr>
<td>A12</td>
<td>$\delta\nu(12D_{5/2})$</td>
<td>0.000(35) kHz</td>
<td>$[1.7 \times 10^{-14}]$</td>
<td>Theory</td>
<td>IV.A.1.1</td>
</tr>
<tr>
<td>A13</td>
<td>$\delta\nu(6D_{3/2})$</td>
<td>0.000(10) kHz</td>
<td>$[1.1 \times 10^{-14}]$</td>
<td>Theory</td>
<td>IV.A.1.1</td>
</tr>
<tr>
<td>A14</td>
<td>$\delta\nu(6D_{5/2})$</td>
<td>0.000(44) kHz</td>
<td>$[8.5 \times 10^{-15}]$</td>
<td>Theory</td>
<td>IV.A.1.1</td>
</tr>
<tr>
<td>A15</td>
<td>$\delta\nu(8D_{3/2})$</td>
<td>0.000(13) kHz</td>
<td>$[5.7 \times 10^{-15}]$</td>
<td>Theory</td>
<td>IV.A.1.1</td>
</tr>
<tr>
<td>A16</td>
<td>$\delta\nu(8D_{5/2})$</td>
<td>0.000(35) kHz</td>
<td>$[1.7 \times 10^{-14}]$</td>
<td>Theory</td>
<td>IV.A.1.1</td>
</tr>
<tr>
<td>A17</td>
<td>$\delta\nu(10S_{1/2})$</td>
<td>0.0(2.3) kHz</td>
<td>$[6.9 \times 10^{-13}]$</td>
<td>Theory</td>
<td>IV.A.1.1</td>
</tr>
<tr>
<td>A18</td>
<td>$\delta\nu(10D_{3/2})$</td>
<td>0.0(29) kHz</td>
<td>$[3.5 \times 10^{-13}]$</td>
<td>Theory</td>
<td>IV.A.1.1</td>
</tr>
<tr>
<td>A19</td>
<td>$\delta\nu(10D_{5/2})$</td>
<td>0.000(36) kHz</td>
<td>$[1.7 \times 10^{-13}]$</td>
<td>Theory</td>
<td>IV.A.1.1</td>
</tr>
<tr>
<td>A20</td>
<td>$\delta\nu(8S_{1/2})$</td>
<td>0.000(60) kHz</td>
<td>$[1.2 \times 10^{-13}]$</td>
<td>Theory</td>
<td>IV.A.1.1</td>
</tr>
<tr>
<td>A21</td>
<td>$\delta\nu(8D_{3/2})$</td>
<td>0.000(44) kHz</td>
<td>$[8.5 \times 10^{-15}]$</td>
<td>Theory</td>
<td>IV.A.1.1</td>
</tr>
<tr>
<td>A22</td>
<td>$\delta\nu(12D_{5/2})$</td>
<td>0.000(13) kHz</td>
<td>$[5.6 \times 10^{-15}]$</td>
<td>Theory</td>
<td>IV.A.1.1</td>
</tr>
<tr>
<td>A23</td>
<td>$\delta\nu(4D_{5/2})$</td>
<td>0.000(35) kHz</td>
<td>$[1.7 \times 10^{-14}]$</td>
<td>Theory</td>
<td>IV.A.1.1</td>
</tr>
<tr>
<td>A24</td>
<td>$\delta\nu(4D_{7/2})$</td>
<td>0.000(44) kHz</td>
<td>$[8.5 \times 10^{-15}]$</td>
<td>Theory</td>
<td>IV.A.1.1</td>
</tr>
<tr>
<td>A25</td>
<td>$\delta\nu(12D_{5/2})$</td>
<td>0.000(13) kHz</td>
<td>$[5.7 \times 10^{-15}]$</td>
<td>Theory</td>
<td>IV.A.1.1</td>
</tr>
</tbody>
</table>

$^a$The values in brackets are relative to the frequency equivalent of the binding energy of the indicated level.
B40, the KR/VN-98 result for $\Gamma'_x^{\nu_{\text{h-90}}} (\text{lo})$, and $\alpha$ from items B43.1 and B43.3.

The inconsistency of these two gyromagnetic ratios has been discussed in previous CODATA reports and is not a serious issue because their self-sensitivity coefficients $S_\nu$ (a measure of their weights in an adjustment, see Sec. XIII.B) are less than 0.01. Therefore the two ratios are omitted from the final adjustment on which the 2014 CODATA recommended values are based. This was also the case in the 2006 and 2010 adjustments. They are initially considered again, as are other data, to test data robustness and the exactness of the relations $K_3 = 2e/h$ and $R_K = h/e^2$ (see Sec. XIII.B.3).

Because of the large uncertainties of most of the values of $\alpha$ compared with the uncertainties of those from B22.2, the HarvU-08 result for $\alpha_e$, and B48, the LKB-11 result for $h/m( ^{87}\text{Rb})$, the 2014 recommended value of $\alpha$ is essentially determined by these two input data. Figure 2 compares them through their inferred values of $\alpha$ and shows how their consistency has changed since 2010, but not because the measured values of $\alpha_e$ and $h/m( ^{87}\text{Rb})$ have changed. Rather, it is because the $\alpha$ QED theoretical expression and the value of $A_e (\gamma)$ required to determine $\alpha$ from $h/m( ^{87}\text{Rb})$ have changed. The UWash-87 result for $\alpha_e$, the $h/m( ^{133}\text{Cs})$ result, the three $\Gamma'_x^{\nu_{\text{h-90}}} (\text{lo})$ results, and the five $R_K$ results are omitted from the 2010 final adjustment because of their low weights and are omitted from the 2014 final adjustment for the same reason.

Table XXI and Figs. 3 and 4 compare values of $\hbar$ obtained from the indicated input data. They show that the vast majority of the values of $\hbar$ agree, and thus the data from which they are obtained agree; of the 91 differences between $\hbar$ values, only one exceeds $2\Delta_{\text{diff}}$. The two values are from input datum B44.2 and B44.6, the NIST-98 and NRC-15 watt-balance results for $K_3^2 R_K$, but the difference is only $2.04\Delta_{\text{diff}}$. The first five values of $\hbar$ in the table, each with relative standard uncertainty

| Table XVI. Correlation coefficients $r(x_i, x_j)>0.0001$ of the input data related to $R_\nu$ in Table XVI. For simplicity, the two items of data to which a particular correlation coefficient corresponds are identified by their item numbers in Table XVI |
|---------------------------------|---------------------------------|
| $r(A1, A2) = 0.9905$           | $r(A6, A19) = 0.7404$           |
| $r(A1, A3) = 0.9900$           | $r(A6, A20) = 0.9851$           |
| $r(A1, A4) = 0.9873$           | $r(A7, A8) = 0.0237$           |
| $r(A1, A5) = 0.7640$           | $r(A9, A10) = 0.0237$          |
| $r(A1, A6) = 0.7627$           | $r(A11, A12) = 0.0006$         |
| $r(A1, A17) = 0.9754$          | $r(A11, A21) = 0.9999$         |
| $r(A1, A18) = 0.9666$          | $r(A11, A22) = 0.0003$         |
| $r(A1, A19) = 0.9619$          | $r(A12, A21) = 0.0003$         |
| $r(A1, A20) = 0.7189$          | $r(A12, A22) = 0.9999$         |
| $r(A2, A3) = 0.9897$           | $r(A13, A14) = 0.0006$         |
| $r(A2, A4) = 0.9870$           | $r(A13, A15) = 0.0006$         |
| $r(A2, A5) = 0.7638$           | $r(A13, A16) = 0.0006$         |
| $r(A2, A6) = 0.7625$           | $r(A13, A23) = 0.9999$         |
| $r(A2, A17) = 0.9666$          | $r(A13, A24) = 0.0003$         |
| $r(A2, A18) = 0.9754$          | $r(A13, A25) = 0.0003$         |
| $r(A2, A19) = 0.9616$          | $r(A14, A15) = 0.0006$         |
| $r(A2, A20) = 0.7187$          | $r(A14, A16) = 0.0006$         |
| $r(A3, A4) = 0.9864$           | $r(A14, A23) = 0.0003$         |
| $r(A3, A5) = 0.7633$           | $r(A14, A24) = 0.0003$         |
| $r(A3, A6) = 0.7620$           | $r(A14, A25) = 0.0003$         |
| $r(A3, A7) = 0.9651$           | $r(A15, A16) = 0.0006$         |
| $r(A3, A8) = 0.9648$           | $r(A15, A23) = 0.0003$         |
| $r(A3, A9) = 0.9611$           | $r(A15, A24) = 0.9999$         |
| $r(A3, A20) = 0.7183$          | $r(A15, A25) = 0.0003$         |
| $r(A4, A5) = 0.7613$           | $r(A16, A23) = 0.0003$         |
| $r(A4, A6) = 0.7600$           | $r(A16, A24) = 0.0003$         |
| $r(A4, A7) = 0.9625$           | $r(A16, A25) = 0.9999$         |
| $r(A4, A8) = 0.9622$           | $r(A17, A18) = 0.9897$         |
| $r(A4, A9) = 0.9755$           | $r(A17, A19) = 0.9859$         |
| $r(A4, A20) = 0.7163$          | $r(A17, A20) = 0.7368$         |
| $r(A5, A6) = 0.5881$           | $r(A18, A19) = 0.9856$         |
| $r(A5, A7) = 0.7448$           | $r(A18, A20) = 0.7366$         |
| $r(A5, A8) = 0.7445$           | $r(A19, A20) = 0.7338$         |
| $r(A5, A9) = 0.7417$           | $r(A21, A22) = 0.0002$         |
| $r(A5, A20) = 0.5543$          | $r(A23, A24) = 0.0001$         |
| $r(A6, A7) = 0.7435$           | $r(A23, A25) = 0.0001$         |
| $r(A6, A8) = 0.7433$           | $r(A24, A25) = 0.0002$         |
TABLE XVIII. Summary of principal input data for the determination of the 2014 recommended values of the fundamental constants ($R_m$ and $G$ excepted)

<table>
<thead>
<tr>
<th>Item number</th>
<th>Input datum</th>
<th>Value</th>
<th>Relative standard uncertainty $u_r$</th>
<th>Identification</th>
<th>Sec. and Eq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>$A_1(n)$</td>
<td>1.008 664 915 85(49)</td>
<td>4.9 x 10^{-10}</td>
<td>AME-12</td>
<td>IIIA</td>
</tr>
<tr>
<td>B2</td>
<td>$A_1(^1H)$</td>
<td>1.007 825 032 231(93)</td>
<td>9.3 x 10^{-11}</td>
<td>AME-12</td>
<td>IIIA</td>
</tr>
<tr>
<td>B3</td>
<td>$\Delta E_{1s}(^1H^+)/\hbar c$</td>
<td>1.096 787 717 430(10) x 10^{-7} m^{-1}</td>
<td>9.1 x 10^{-13}</td>
<td>ASD-14</td>
<td>IIIB</td>
</tr>
<tr>
<td>B4</td>
<td>$A_1(^3H)$</td>
<td>3.016 049 2779(24)</td>
<td>7.9 x 10^{-10}</td>
<td>AME-12</td>
<td>IIIA</td>
</tr>
<tr>
<td>B5</td>
<td>$\Delta E_{1s}(^1H^+)/\hbar c$</td>
<td>1.097 185 4390(13) x 10^{-7} m^{-1}</td>
<td>1.2 x 10^{-9}</td>
<td>ASD-14</td>
<td>IIIB</td>
</tr>
<tr>
<td>B6</td>
<td>$A_1(^4He)$</td>
<td>4.002 603 254 130(63)</td>
<td>1.6 x 10^{-11}</td>
<td>AME-12</td>
<td>IIIA</td>
</tr>
<tr>
<td>B7</td>
<td>$\Delta E_{1s}(^4He^+)/\hbar c$</td>
<td>6.372 195 4487(28) x 10^{-7} m^{-1}</td>
<td>4.4 x 10^{-10}</td>
<td>ASD-14</td>
<td>IIIB</td>
</tr>
<tr>
<td>B8</td>
<td>$\omega_d(\delta)/\omega_d(^{12}C^{14}_N)$</td>
<td>0.992 966 654 743(20)</td>
<td>2.0 x 10^{-11}</td>
<td>UWash-15</td>
<td>ILC (5)</td>
</tr>
<tr>
<td>B9</td>
<td>$\omega_d(\delta)/\omega_d(^{13}C^{14}_N)$</td>
<td>1.326 365 862 193(19)</td>
<td>1.4 x 10^{-11}</td>
<td>UWash-15</td>
<td>ILC (6)</td>
</tr>
<tr>
<td>B10</td>
<td>$\Delta E_{1s}(^1C^{14}_N)/\hbar c$</td>
<td>8.308 962(72) x 10^{-7} m^{-1}</td>
<td>8.7 x 10^{-7}</td>
<td>ASD-14</td>
<td>IIIB</td>
</tr>
<tr>
<td>B11</td>
<td>$\omega_d(^{12}S^{13}_N)/\omega_d(^{12}He^+)$</td>
<td>3.912 866 064 84(19)</td>
<td>4.8 x 10^{-11}</td>
<td>FSU-15</td>
<td>IIIA</td>
</tr>
<tr>
<td>B12</td>
<td>$A_1(^{12}Si)$</td>
<td>27.976 926 534 65(44)</td>
<td>1.6 x 10^{-11}</td>
<td>AME-12</td>
<td>IIIA</td>
</tr>
<tr>
<td>B20</td>
<td>$\Delta E_{1s}(^{28}Si^{14}_N)/\hbar c$</td>
<td>420.608(19) x 10^{-7} m^{-1}</td>
<td>4.4 x 10^{-5}</td>
<td>ASD-14</td>
<td>IIIB</td>
</tr>
<tr>
<td>B21</td>
<td>$\sigma_{\delta}$</td>
<td>0.0(1.7) x 10^{-9}</td>
<td>[8.3 x 10^{-10}]</td>
<td>Theory</td>
<td>V.D.1 (176)</td>
</tr>
<tr>
<td>B22.1</td>
<td>$\mu_\rho/\mu_N$</td>
<td>2.792 847 3498(93)</td>
<td>3.3 x 10^{-9}</td>
<td>UMZ-14</td>
<td>V.C (141)</td>
</tr>
<tr>
<td>B30</td>
<td>$\mu_1(^1H)/\mu_N(H)$</td>
<td>-658.210 7058(66)</td>
<td>1.0 x 10^{-8}</td>
<td>MIT-72</td>
<td>VLA.1</td>
</tr>
<tr>
<td>B31</td>
<td>$\mu_1(^3D)/\mu_N(D)$</td>
<td>-4.664 345 292(50) x 10^{-4}</td>
<td>1.1 x 10^{-8}</td>
<td>MIT-84</td>
<td>VLA.1</td>
</tr>
<tr>
<td>B32</td>
<td>$\mu_1(^1H)/\mu_N(^1H)$</td>
<td>-658.215 9430(72)</td>
<td>1.1 x 10^{-8}</td>
<td>MIT-77</td>
<td>VLA.1</td>
</tr>
<tr>
<td>B33</td>
<td>$\mu_1(^3P)/\mu_N(^3P)$</td>
<td>-0.761 786 1313(33)</td>
<td>4.3 x 10^{-9}</td>
<td>NPL-93</td>
<td>VLA.1</td>
</tr>
<tr>
<td>B34</td>
<td>$\mu_1(^1S)/\mu_N(^1S)$</td>
<td>-0.684 9969(46)</td>
<td>2.4 x 10^{-7}</td>
<td>NPL-93</td>
<td>VLA.1</td>
</tr>
<tr>
<td>B35.1</td>
<td>$\mu_1(^3H)/\mu_N(^3H)$</td>
<td>3.257 199 531(29)</td>
<td>8.9 x 10^{-9}</td>
<td>StPrsh-03</td>
<td>VLA.1</td>
</tr>
<tr>
<td>B35.2</td>
<td>$\mu_1(^3H)/\mu_N(^3H)$</td>
<td>3.257 199 541(24)</td>
<td>6.6 x 10^{-9}</td>
<td>StPrsh-11</td>
<td>VLA.1 (197)</td>
</tr>
<tr>
<td>B36</td>
<td>$\mu_1(^3H)/\mu_N(^3H)$</td>
<td>1.066 639 8393(23)</td>
<td>2.0 x 10^{-9}</td>
<td>StPrsh-3</td>
<td>VLA.1 (198)</td>
</tr>
<tr>
<td>B37</td>
<td>$\sigma_{\phi}$</td>
<td>15(2) x 10^{-9}</td>
<td></td>
<td>StPrsh-3</td>
<td>VLA.1</td>
</tr>
<tr>
<td>B38</td>
<td>$\sigma_\phi$</td>
<td>20(3) x 10^{-9}</td>
<td></td>
<td>StPrsh-3</td>
<td>VLA.1</td>
</tr>
<tr>
<td>B39</td>
<td>$I_{\varphi}(\delta)$</td>
<td>2.675 154 05(30) x 10^{-8} T^{-1}</td>
<td>1.1 x 10^{-7}</td>
<td>NIST-89</td>
<td>VIII</td>
</tr>
<tr>
<td>B40</td>
<td>$I_{\varphi}(\delta)$</td>
<td>2.675 1530(18) x 10^{-8} T^{-1}</td>
<td>6.6 x 10^{-7}</td>
<td>NIM-95</td>
<td>VIII</td>
</tr>
<tr>
<td>B41</td>
<td>$I_{\varphi}(\delta)$</td>
<td>2.037 895 37(37) x 10^{-8} T^{-1}</td>
<td>1.8 x 10^{-7}</td>
<td>KR/NV-98</td>
<td>VIII</td>
</tr>
<tr>
<td>B42</td>
<td>$K_1$</td>
<td>483 597.91(13) GHz V^{-1}</td>
<td>2.7 x 10^{-7}</td>
<td>NMI-98</td>
<td>VIII</td>
</tr>
<tr>
<td>B43</td>
<td>$K_1$</td>
<td>483 597.96(15) GHz V^{-1}</td>
<td>3.1 x 10^{-7}</td>
<td>PTB-91</td>
<td>VIII</td>
</tr>
<tr>
<td>B44</td>
<td>$K_1$</td>
<td>25812.808 31(62) Ω</td>
<td>2.4 x 10^{-8}</td>
<td>NIST-97</td>
<td>VIII</td>
</tr>
<tr>
<td>B45</td>
<td>$K_1$</td>
<td>25812.8071(11) Ω</td>
<td>4.4 x 10^{-8}</td>
<td>NMI-97</td>
<td>VIII</td>
</tr>
<tr>
<td>B46</td>
<td>$K_1$</td>
<td>25812.8092(14) Ω</td>
<td>5.4 x 10^{-8}</td>
<td>NPL-88</td>
<td>VIII</td>
</tr>
<tr>
<td>B47</td>
<td>$K_1$</td>
<td>25812.8084(34) Ω</td>
<td>1.3 x 10^{-7}</td>
<td>NMI-95</td>
<td>VIII</td>
</tr>
<tr>
<td>B48</td>
<td>$K_1$</td>
<td>25812.8081(14) Ω</td>
<td>5.3 x 10^{-8}</td>
<td>LNE-01</td>
<td>VIII</td>
</tr>
</tbody>
</table>
Table XVIII. Summary of principal input data for the determination of the 2014 recommended values of the fundamental constants (\(R_e\) and \(G\) excepted)—Continued

<table>
<thead>
<tr>
<th>Item number</th>
<th>Input datum</th>
<th>Value</th>
<th>Relative standard uncertainty (u_r)</th>
<th>Identification</th>
<th>Sec. and Eq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>B44.1(^b)</td>
<td>(K_1^s R_k)</td>
<td>(6.0367625(12) \times 10^{33} \text{ J}^{-1} \text{s}^{-1})</td>
<td>(2.0 \times 10^{-7})</td>
<td>NPL-90</td>
<td>VIII.A</td>
</tr>
<tr>
<td>B44.2</td>
<td>(K_1^s R_k)</td>
<td>(6.036761\ 85(53) \times 10^{33} \text{ J}^{-1} \text{s}^{-1})</td>
<td>(8.7 \times 10^{-8})</td>
<td>NIST-98</td>
<td>VIII.D</td>
</tr>
<tr>
<td>B44.3(^b)</td>
<td>(K_1^s R_k)</td>
<td>(6.0367167(18) \times 10^{33} \text{ J}^{-1} \text{s}^{-1})</td>
<td>(2.9 \times 10^{-7})</td>
<td>METAS-11</td>
<td>VIII.B</td>
</tr>
<tr>
<td>B44.4</td>
<td>(K_1^s R_k)</td>
<td>(6.036761\ 43(34) \times 10^{33} \text{ J}^{-1} \text{s}^{-1})</td>
<td>(5.7 \times 10^{-8})</td>
<td>NIST-15</td>
<td>VIII.D (234)</td>
</tr>
<tr>
<td>B44.5(^b)</td>
<td>(K_1^s R_k)</td>
<td>(6.0367597(12) \times 10^{33} \text{ J}^{-1} \text{s}^{-1})</td>
<td>(2.0 \times 10^{-7})</td>
<td>NPL-12</td>
<td>VIII.A</td>
</tr>
<tr>
<td>B44.6</td>
<td>(K_1^s R_k)</td>
<td>(6.0367606(11) \times 10^{33} \text{ J}^{-1} \text{s}^{-1})</td>
<td>(1.8 \times 10^{-8})</td>
<td>NRC-15</td>
<td>VIII.E (237)</td>
</tr>
<tr>
<td>B44.7(^a)</td>
<td>(K_1^s R_k)</td>
<td>(6.0367619(15) \times 10^{33} \text{ J}^{-1} \text{s}^{-1})</td>
<td>(2.6 \times 10^{-7})</td>
<td>LNE-15</td>
<td>VIII.C (231)</td>
</tr>
<tr>
<td>B45</td>
<td>(\varphi_{90})</td>
<td>(96.485.369(13) \text{ mg}^{-1})</td>
<td>(1.3 \times 10^{-6})</td>
<td>NIST-80</td>
<td>VIII</td>
</tr>
<tr>
<td>B46</td>
<td>(\nu_m^{(\text{Sn})})</td>
<td>(3.002369.432(46) \times 10^{-9} \text{ m}^2 \text{s}^{-1})</td>
<td>(1.5 \times 10^{-6})</td>
<td>StanfU-02</td>
<td>VII</td>
</tr>
<tr>
<td>B47</td>
<td>(A_6^{(\text{Cs})})</td>
<td>(132.905\ 451.616(86) )</td>
<td>(6.5 \times 10^{-11})</td>
<td>AME-12</td>
<td>III.A</td>
</tr>
<tr>
<td>B48</td>
<td>(\nu_m^{(\text{Rb})})</td>
<td>(4.591359.272(57) \times 10^{-9} \text{ m}^2 \text{s}^{-1})</td>
<td>(1.2 \times 10^{-9})</td>
<td>LKB-11</td>
<td>VII</td>
</tr>
<tr>
<td>B49</td>
<td>(A_4^{(\text{Rb})})</td>
<td>(86.909\ 180.5319(65) )</td>
<td>(7.5 \times 10^{-11})</td>
<td>AME-12</td>
<td>III.A</td>
</tr>
</tbody>
</table>

\(^a\)The values in brackets are relative to the quantities \(g^{(\text{He})^{4+}}, g^{(\text{He})^{5+}}, \alpha_e, \) or \(\Delta \nu_{56}\) as appropriate.

\(^b\)Item not included in the final least-squares adjustment that provides the recommended values of the constants.

\(^c\)Item included in the final least-squares adjustment with an expanded uncertainty.

\(u_r < 10^{-7}\), are compared in Fig. 4. The five input data from which these values are obtained are included in the final adjustment and determine the 2014 recommended value of \(\hbar\).

\(u_r < 10^{-7}\), are compared in Fig. 4. The five input data from which these values are obtained are included in the final adjustment and determine the 2014 recommended value of \(\hbar\). The input data from which the next nine values of \(\hbar\) with \(u_r\) from \(2.0 \times 10^{-7}\) to \(1.6 \times 10^{-6}\) are omitted from the final adjustment because of their low weight.

Table XXII and Fig. 5 compare values of \(k\) obtained from the indicated input data. Although most of the source data are
acoustic gas thermometry measurements of \( R \), values of \( k = R/N_a \) are compared, because \( k \) is one of the defining constants of the new SI (see Sec. I.B.1) and \( u_{\text{ex}}(N_a) \ll u_{\text{ex}}(R) \). The table and figure show that all the values of \( k \) are in excellent agreement, and thus so are the data from which they are obtained; of the 55 differences between \( k \) values, none exceed 2\( \Delta \text{diam} \) and the largest is only 0.96. Moreover, it turns out that only the NPL-79 and INRIM-10 results for \( R \) have insufficient weight to be included in the 2014 final adjustment, although they had sufficient weight to be included in the 2010 final adjustment.

### B. Multivariate analysis of data

Our multivariate analysis of the data employs a well-known least-squares method that allows correlations among the input data to be properly taken into account. Used in the four previous adjustments, it is described in Appendix E of CODATA-98 and the references cited therein. It is recalled from that appendix that a least-squares adjustment is characterized by the number of input data \( N \), number of variables or adjusted constants \( M \), degrees of freedom \( \nu = N - M \), statistic \( \chi^2 \), probability \( \rho(\chi^2|\nu) \) of obtaining an observed value of \( \chi^2 \) that large or larger for the given value of \( \nu \). Birge ratio \( R_B = \sqrt{\chi^2/\nu} \) (\( \chi^2/\nu \) is often called the reduced \( \chi^2 \)), and the normalized residual of the \( i \)th input datum \( r_i = (x_i - \langle x_i \rangle)/u_i \), where \( x_i \) is the input datum, \( \langle x_i \rangle \) its adjusted value, and \( u_i \) its standard uncertainty.

The observational equations for the input data are given in Tables XXIII and XXIV. These equations are written in terms of a particular independent subset of constants (broadly interpreted) called adjusted constants. These are the variables (or unknowns) of the adjustment. The least-squares calculation yields values of the adjusted constants that predict values of the

### Table XIX. Correlation coefficients \( r(x_i, \chi_j) \geq 0.001 \) of the input data in Table XVIII.

For simplicity, the two items of data to which a particular correlation coefficient corresponds are identified by their item numbers in Table XVIII.

<table>
<thead>
<tr>
<th>Primary source</th>
<th>Item number</th>
<th>Identification</th>
<th>Sec. and Eq.</th>
<th>( \alpha^{-1} )</th>
<th>Relative standard uncertainty ( u )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_e )</td>
<td>B22.2</td>
<td>HarvU-08</td>
<td>V.A.2</td>
<td>137.035999 160(33)</td>
<td>2.4 \times 10^{-10}</td>
</tr>
<tr>
<td>( \beta/m(^{87}\text{Rb}) )</td>
<td>B48</td>
<td>LKB-11</td>
<td>VII</td>
<td>137.035998 996(85)</td>
<td>6.2 \times 10^{-9}</td>
</tr>
<tr>
<td>( \alpha_e )</td>
<td>B22.1</td>
<td>UWash-87</td>
<td>V.A.2</td>
<td>137.035998 27(50)</td>
<td>3.7 \times 10^{-9}</td>
</tr>
<tr>
<td>( \beta/m(^{133}\text{Cs}) )</td>
<td>B46</td>
<td>StanU-02</td>
<td>VII</td>
<td>137.036000(11)</td>
<td>7.7 \times 10^{-9}</td>
</tr>
<tr>
<td>( R_K )</td>
<td>B43.1</td>
<td>NIST-97</td>
<td>VIII</td>
<td>137.036037(33)</td>
<td>2.4 \times 10^{-9}</td>
</tr>
<tr>
<td>( \Gamma^\omega_a(\theta) )</td>
<td>B39.1</td>
<td>NIST-89</td>
<td>VIII</td>
<td>137.035979(51)</td>
<td>3.7 \times 10^{-9}</td>
</tr>
<tr>
<td>( R_K )</td>
<td>B43.2</td>
<td>NMI-97</td>
<td>VIII</td>
<td>137.035973(61)</td>
<td>4.4 \times 10^{-9}</td>
</tr>
<tr>
<td>( R_K )</td>
<td>B43.5</td>
<td>LNE-01</td>
<td>VIII</td>
<td>137.036023(73)</td>
<td>5.3 \times 10^{-9}</td>
</tr>
<tr>
<td>( R_K )</td>
<td>B43.3</td>
<td>NPL-88</td>
<td>VIII</td>
<td>137.036083(73)</td>
<td>5.4 \times 10^{-9}</td>
</tr>
<tr>
<td>( \Delta\delta H_u(\theta) )</td>
<td>B27.1, B27.2</td>
<td>LAMPFF</td>
<td>V.I.B.2 (228)</td>
<td>137.036013(79)</td>
<td>5.8 \times 10^{-9}</td>
</tr>
<tr>
<td>( \Gamma^\omega_a(\theta) )</td>
<td>B40</td>
<td>KR/VN-98</td>
<td>VIII</td>
<td>137.035982(82)</td>
<td>6.0 \times 10^{-9}</td>
</tr>
<tr>
<td>( R_K )</td>
<td>B43.4</td>
<td>NIM-95</td>
<td>VIII</td>
<td>137.036004(18)</td>
<td>1.3 \times 10^{-7}</td>
</tr>
<tr>
<td>( \Gamma^\omega_a(\theta) )</td>
<td>B39.2</td>
<td>NIM-95</td>
<td>VIII</td>
<td>137.036006(30)</td>
<td>2.2 \times 10^{-7}</td>
</tr>
<tr>
<td>( \nu_{\text{H}}, \nu_0 )</td>
<td>IV.A.1.m (71)</td>
<td></td>
<td></td>
<td>137.035992(55)</td>
<td>4.0 \times 10^{-7}</td>
</tr>
</tbody>
</table>
input data through their observational equations that best agree with the data themselves in the least-squares sense. The adjusted constants used in the 2014 calculations are given in Tables XXV and XXVI.

The symbol \( \pm \) in an observational equation indicates that an input datum of the type on the left-hand side is ideally given by the expression on the right-hand side containing adjusted constants. But because the equation is one of an overdetermined set that relates a datum to adjusted constants, the two sides are not necessarily equal. The best estimate of the value of an input datum is its observational equation evaluated with the least-squares adjusted values of the adjusted constants on which its observational equation depends. For some input data such as \( \delta_c \) and \( R \), the observational equation is simply \( \delta_c = \delta_c \) and \( R = R \).

The bound-state \( g \)-factor ratios in the observational equations of Table XXIV are treated as fixed quantities with negligible uncertainties (see Table XIII, Sec. VI.A). The frequency \( f_0 \) is not an adjusted constant but is included in the equation for data items \( B25 \) and \( B26 \) to indicate that they are functions of \( f_0 \). Finally, the observational equations for items \( B25 \) and \( B26 \), which are based on Eqs. (223)–(225) of Sec. VI.B.2, include the theoretical expressions for \( a_e \) and \( \Delta \nu_{Mu} \) which are given as observational equations \( B22 \) and \( B27 \) in Table XXIV.

Also recalled from Appendix E of CODATA-98 is the self-sensitivity coefficient \( \delta_i \) for an input datum, which is a measure of the influence of that datum on the adjusted value of the

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**Table XXI.** Inferred values of the Planck constant \( h \) in order of increasing standard uncertainty obtained from the indicated experimental data in Table XVIII.

<table>
<thead>
<tr>
<th>Primary source</th>
<th>Item number</th>
<th>Identification</th>
<th>Sec. and Eq.</th>
<th>( h/(\text{J s}) )</th>
<th>Relative standard uncertainty ( u_h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_i^2 R_k )</td>
<td>B44.6</td>
<td>NRC-15</td>
<td>VII.E (237)</td>
<td>6.626 070 11(12) \times 10^{-34}</td>
<td>1.8 \times 10^{-9}</td>
</tr>
<tr>
<td>( N_9(28\text{Si}) )</td>
<td>B63.2</td>
<td>IAC-15</td>
<td>IX.B (240)</td>
<td>6.626 070 15(13) \times 10^{-34}</td>
<td>2.0 \times 10^{-9}</td>
</tr>
<tr>
<td>( N_9(28\text{Si}) )</td>
<td>B63.1</td>
<td>IAC-11</td>
<td>IX.B (239)</td>
<td>6.626 069 89(20) \times 10^{-34}</td>
<td>3.0 \times 10^{-9}</td>
</tr>
<tr>
<td>( K_i^2 R_k )</td>
<td>B44.4</td>
<td>NIST-15</td>
<td>VIII.D (234)</td>
<td>6.626 069 36(38) \times 10^{-34}</td>
<td>5.7 \times 10^{-9}</td>
</tr>
<tr>
<td>( K_i^2 R_k )</td>
<td>B44.2</td>
<td>NIST-98</td>
<td>VIII.D</td>
<td>6.626 068 91(58) \times 10^{-34}</td>
<td>8.7 \times 10^{-9}</td>
</tr>
<tr>
<td>( K_i^2 R_k )</td>
<td>B44.5</td>
<td>NPL-12</td>
<td>VIII.A</td>
<td>6.626 071 12(13) \times 10^{-34}</td>
<td>2.0 \times 10^{-7}</td>
</tr>
<tr>
<td>( K_i^2 R_k )</td>
<td>B44.1</td>
<td>NPL-90</td>
<td>VIII.A</td>
<td>6.626 068 2(13) \times 10^{-34}</td>
<td>2.0 \times 10^{-7}</td>
</tr>
<tr>
<td>( K_i^2 R_k )</td>
<td>B44.7</td>
<td>LNE-15</td>
<td>VII.C (231)</td>
<td>6.626 068 88(17) \times 10^{-34}</td>
<td>2.6 \times 10^{-7}</td>
</tr>
<tr>
<td>( K_i^2 R_k )</td>
<td>B44.3</td>
<td>METAS-11</td>
<td>VII.B</td>
<td>6.626 069 1(20) \times 10^{-34}</td>
<td>2.9 \times 10^{-7}</td>
</tr>
<tr>
<td>( K_i )</td>
<td>B42.1</td>
<td>NML-89</td>
<td>VIII</td>
<td>6.626 068 4(36) \times 10^{-34}</td>
<td>5.4 \times 10^{-7}</td>
</tr>
<tr>
<td>( K_i )</td>
<td>B42.2</td>
<td>PTB-91</td>
<td>VIII</td>
<td>6.626 067 0(42) \times 10^{-34}</td>
<td>6.3 \times 10^{-7}</td>
</tr>
<tr>
<td>( \Gamma_{p-x_0}(h) )</td>
<td>B41.2</td>
<td>NPL-79</td>
<td>VIII</td>
<td>6.626 073 0(67) \times 10^{-34}</td>
<td>1.0 \times 10^{-6}</td>
</tr>
<tr>
<td>( \Gamma_{p-x_0}(h) )</td>
<td>B45</td>
<td>NIST-80</td>
<td>VIII</td>
<td>6.626 065 8(88) \times 10^{-34}</td>
<td>1.3 \times 10^{-6}</td>
</tr>
<tr>
<td>( \Gamma_{p-x_0}(h) )</td>
<td>B41.1</td>
<td>NIM-95</td>
<td>VIII</td>
<td>6.626 071 1(11) \times 10^{-34}</td>
<td>1.6 \times 10^{-6}</td>
</tr>
</tbody>
</table>
(quantity of which the datum is an example. As in previous adjustments, in general, for an input datum to be included in the final adjustment on which the 2014 recommended values are based, its value of $S_c$ must be greater than 0.01, or 1%, which means that its uncertainty must be no more than about a factor of 10 larger than the uncertainty of the adjusted value of that quantity; see Sec. 1D of CODATA-98 for the justification of this 1% cutoff. However, the exclusion of a datum is not followed if, for example, a datum with $S_c < 0.01$ is part of a group of data obtained in a given experiment where most of the other data have self-sensitivity coefficients $> 0.01$. It is also not followed for $G$, but in this case it is because of the significant disagreement of the available data and hence lack of motivation for anything beyond a simple weighted mean. Indeed, because the $G$ data are independent of all other data and can be treated separately, and because they determine only one variable, in this case the multivariate analysis becomes simply the calculation of their weighted mean.

1. Data related to the Newtonian constant of gravitation $G$

The 14 values of $G$ to be considered are summarized in Table XV of Sec. XI and are discussed in the text accompanying it. For easy reference they are listed in Table XXII of this section and are graphically compared in Fig. 6. The last three results in the table and figure, BIPM-14, LENS-14, and UCI-14, have become available since 2010. Although the BIPM-14 and UCI-14 results have the comparatively small relative standard uncertainties $u_r = 24 \times 10^{-6}$ and $u_r = 19 \times 10^{-6}$, respectively, they have not reduced the quite large inconsistencies among the $G$ data that have plagued them for some 20 years and which are evident in the table and figure.

Indeed, of the 91 differences among the 14 results, 45 are larger than $2u_{\text{diff}}$, 22 are larger than $4u_{\text{diff}}$, and 5 are larger than $10u_{\text{diff}}$. The five largest, $15.1u_{\text{diff}}$, $11.4u_{\text{diff}}$, $10.7u_{\text{diff}}$, $10.5u_{\text{diff}}$, and $10.4u_{\text{diff}}$, are

![Figure 3](image1.png)

**Fig. 3.** Values of the Planck constant $\hbar$ with $u_r < 10^{-6}$ inferred from the input data in Table XVIII and the 2014 CODATA recommended value in chronological order from top to bottom (see Table XXI).

![Figure 4](image2.png)

**Fig. 4.** Values of the Planck constant $\hbar$ with $u_r < 10^{-7}$ inferred from the input data in Table XVIII and the 2014 CODATA recommended value in chronological order from top to bottom (see Table XXI). The input data from which these values are inferred are included in the final adjustment on which the 2014 recommended values are based.

<p>| Table XXII. Inferred values of the Boltzmann constant $k$ in order of increasing standard uncertainty obtained from the indicated experimental data in Table XVIII |
|-----------------|-----------------|-----------------|-----------------|-----------------|</p>
<table>
<thead>
<tr>
<th>Primary source</th>
<th>Item number</th>
<th>Identification</th>
<th>Section</th>
<th>$k/(J K^{-1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>B64.8</td>
<td>NPL-13</td>
<td>X.A</td>
<td>1.3806476(12) $\times 10^{-23}$</td>
</tr>
<tr>
<td>$R$</td>
<td>B64.9</td>
<td>LNE-15</td>
<td>X.A</td>
<td>1.3806487(14) $\times 10^{-23}$</td>
</tr>
<tr>
<td>$R$</td>
<td>B64.6</td>
<td>LNE-11</td>
<td>X.A</td>
<td>1.3806477(19) $\times 10^{-23}$</td>
</tr>
<tr>
<td>$R$</td>
<td>B64.2</td>
<td>NIST-88</td>
<td>X.A</td>
<td>1.3806501(25) $\times 10^{-23}$</td>
</tr>
<tr>
<td>$R$</td>
<td>B64.3</td>
<td>LNE-09</td>
<td>X.A</td>
<td>1.3806497(38) $\times 10^{-23}$</td>
</tr>
<tr>
<td>$R$</td>
<td>B64.4</td>
<td>NPL-10</td>
<td>X.A</td>
<td>1.3806498(44) $\times 10^{-23}$</td>
</tr>
<tr>
<td>$R$</td>
<td>B64.7</td>
<td>NIM-13</td>
<td>X.A</td>
<td>1.3806477(51) $\times 10^{-23}$</td>
</tr>
<tr>
<td>$k/h$</td>
<td>B65</td>
<td>NIM/NIST-15</td>
<td>X.B</td>
<td>1.3806513(53) $\times 10^{-23}$</td>
</tr>
<tr>
<td>$A_e/R$</td>
<td>B66</td>
<td>PTB-15</td>
<td>X.C</td>
<td>1.3806509(55) $\times 10^{-23}$</td>
</tr>
<tr>
<td>$R$</td>
<td>B64.5</td>
<td>INRIM-10</td>
<td>X.A</td>
<td>1.380641(10) $\times 10^{-23}$</td>
</tr>
<tr>
<td>$R$</td>
<td>B64.1</td>
<td>NPL-79</td>
<td>X.A</td>
<td>1.380656(12) $\times 10^{-23}$</td>
</tr>
</tbody>
</table>

The 91 differences among the 14 results, 45 are larger than $2u_{\text{diff}}$, and 22 are larger than $4u_{\text{diff}}$. The five largest, $15.1u_{\text{diff}}$, $11.4u_{\text{diff}}$, $10.7u_{\text{diff}}$, $10.5u_{\text{diff}}$, and $10.4u_{\text{diff}}$, are
residuals \(|r_i| > 2\): JILA-10, BIPM-14, BIPM-01, NIST-82, HUST-09, TR&D-96, LENS-14, HUST-05, and UCI-14; their respective values are −12.5, 9.1, 5.6, −3.7, 3.3, 2.4, 2.2, 2.1, and 2.1.

Because of their comparatively small uncertainties, there is little impact if this calculation is repeated with just the six \(G\) results with \(u_i < 30 \times 10^{-6}\). These are, in order of increasing uncertainty, UWash-00, UZur-06, UCI-14, JILA-10, BIPM-14, and HUST-09. Their weighted mean is \(6.674\,077(52)G_o [7.8 \times 10^{-6}]\), with \(\chi^2 = 258.6, p(258.6|13) \approx 0, \) and \(R_B = 7.19;\) their respective normalized residuals \(r_i\) are 1.9, 1.4, 2.2, −12.4, 9.1, and −3.3. The significant disagreement of the JILA-10 and BIPM-14 results with the four other low-uncertainty results is apparent. Additional calculations have been carried out, for example, one in which the JILA-10, BIPM-01, and BIPM-14 results are omitted. The weighted mean of the remaining 11 data is \(6.674\,121(57)G_o [8.6 \times 10^{-6}]\), with \(\chi^2 = 49.8, p(49.8|13) = 2.9 \times 10^{-6}, \) and \(R_B = 2.2.\) The value of \(G\) is not significantly different from the two other weighted-mean values and deleting the three data increases the \(\chi^2\) probability by 10 orders of magnitude. Nevertheless, for all practical purposes it is still very small.

In 2010 the Task Group decided to take as the recommended value of \(G\) the weighted mean and its uncertainty of the 11 values then available (essentially the first 11 values in Tables XV and XXVII), but after multiplying the initially assigned uncertainty of each value by the factor 14, called the expansion factor. The number 14 was chosen so that the smallest and largest of the 11 values differed from the recommended value by

\[
\begin{align*}
\text{Table XXIII. Observational equations that express the input data related to } R_o \text{ in Table XVI as functions of the adjusted constants in Table XXV. The numbers in the first column correspond to the numbers in the first column of Table XVI. Energy levels of hydrogenic atoms are discussed in Sec. IV.A. Note that } E_i(n_L)/h \text{ is proportional to } cR_o \text{ and independent of } h, \text{ hence } h \text{ is not an adjusted constant in these equations. See Sec. XIII.B for an explanation of the symbol } \delta. \\
\quad \text{Type of input datum} & \quad \text{Observational equation} \\
A1–A16 & \delta_1(n_L) = \delta_1(n_L) \\
A17–A25 & \delta_2(n_L) = \delta_2(n_L) \\
A26–A32, A39, A40 & \nu_4(n_1L_{1j} - n_2L_{2j}) = [E_4(n_1L_{1j}; R_o, \alpha, A_e, A(p), r_p, r_0(n_1L_{1j})) \\
A33–A38 & - E_4(n_1L_{1j}; R_o, \alpha, A_e, A(p), r_p, \delta_0(n_1L_{1j}))]/h \\
A41–A45 & \nu_5(n_1L_{1j} - n_2L_{2j}) - \frac{1}{2}\nu_4(n_1L_{1j} - n_2L_{2j}) = [E_5(n_1L_{1j}; R_o, \alpha, A_e, A(p), r_p, \delta_0(n_1L_{1j})) \\
A46–A47 & - E_5(n_1L_{1j}; R_o, \alpha, A_e, A(p), r_p, \delta_0(n_1L_{1j}))]/h \\
A48 & \nu_6(1S_{1/2} - 2S_{1/2}) - \nu_6(1S_{1/2} - 2S_{1/2}) = [E_6(2S_{1/2}; R_o, \alpha, A_e, A(p), r_0(2S_{1/2})) \\
A49 & - E_6(2S_{1/2}; R_o, \alpha, A_e, A(p), r_0(2S_{1/2}))]/h \\
A50 & \quad r_p = r_0 \\
& \quad r_0 = r_0
\end{align*}
\]
TABLE XXIV. Observational equations that express the input data in Table XVIII as functions of the adjusted constants in Table XXVI. The numbers in the first column correspond to the numbers in the first column of Table XVIII. For simplicity, the lengthier functions are not explicitly given. See Sec. XIII.B for an explanation of the symbol $\omega$.

<table>
<thead>
<tr>
<th>Type of input datum</th>
<th>Observational equation</th>
<th>Sec.</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>$A_i(n) = A_i(n)$</td>
<td>III.A</td>
</tr>
<tr>
<td>B2</td>
<td>$A_i(^1\text{H}) = A_i(p) + A_i(e) - \Delta E_{\text{B}}(^1\text{H})/\sigma A_i(e)/2R_{\text{n}}hc$</td>
<td>III.B</td>
</tr>
<tr>
<td>B3</td>
<td>$\Delta E_{\text{B}}(^1\text{H}) = \Delta E_{\text{B}}(^1\text{H})/hc$</td>
<td>III.B</td>
</tr>
<tr>
<td>B4</td>
<td>$A_i(^1\text{H}) = A_i(t) + A_i(e) - \Delta E_{\text{B}}(^1\text{He})/2R_{\text{n}}hc$</td>
<td>III.B</td>
</tr>
<tr>
<td>B5</td>
<td>$\Delta E_{\text{B}}(^1\text{He}) = \Delta E_{\text{B}}(^1\text{He})/hc$</td>
<td>III.B</td>
</tr>
<tr>
<td>B6</td>
<td>$A_i(^{4}\text{He}) = A_i(a) + 2A_i(e) - \Delta E_{\text{B}}(^{4}\text{He})/\sigma^2 A_i(e)/2R_{\text{n}}hc$</td>
<td>III.B</td>
</tr>
<tr>
<td>B7</td>
<td>$\Delta E_{\text{B}}(^{4}\text{He}) = \Delta E_{\text{B}}(^{4}\text{He})/hc$</td>
<td>III.B</td>
</tr>
<tr>
<td>B8</td>
<td>$\omega_e(d) = 12 - 6A_i(e) + \Delta E_{\text{B}}(^{12}\text{C})/\sigma A_i(e)/2R_{\text{n}}hc$</td>
<td>III.C</td>
</tr>
<tr>
<td>B9</td>
<td>$\omega_e(h) = 12 - 6A_i(e) + \Delta E_{\text{B}}(^{12}\text{C})/\sigma A_i(h)$</td>
<td>III.C</td>
</tr>
<tr>
<td>B10</td>
<td>$\Delta E_{\text{B}}(^{12}\text{C}) = \Delta E_{\text{B}}(^{12}\text{C})/hc$</td>
<td>III.B</td>
</tr>
<tr>
<td>B11</td>
<td>$\omega_{\text{e}}(\text{HD}^+) = A_i(h) + A_i(e) - \Delta E_{\text{B}}(\text{He})/\sigma A_i(e)/2R_{\text{n}}hc$</td>
<td>III.C</td>
</tr>
<tr>
<td>B12</td>
<td>$\omega_{\text{e}}(\text{He}) = A_i(p) + A_i(d) + A_i(e) - \Delta E_{\text{B}}(\text{He})/\sigma A_i(e)/2R_{\text{n}}hc$</td>
<td>III.C</td>
</tr>
<tr>
<td>B13</td>
<td>$E_i(\text{He})/hc = E_i(\text{He})/hc$</td>
<td>III.B</td>
</tr>
<tr>
<td>B14</td>
<td>$E_i(\text{HD}^+) = E_i(\text{HD}^+)/hc$</td>
<td>III.B</td>
</tr>
<tr>
<td>B15</td>
<td>$\omega_{\text{e}}(^{28}\text{Si}) = -\frac{3}{10A_i(e)} [12 - 5A_i(e) + \Delta E_{\text{B}}(^{12}\text{C})/\sigma A_i(e)/2R_{\text{n}}hc]$</td>
<td>V.D.2</td>
</tr>
<tr>
<td>B16</td>
<td>$\Delta E_{\text{B}}(^{12}\text{C}) = \Delta E_{\text{B}}(^{12}\text{C})/hc$</td>
<td>III.B</td>
</tr>
<tr>
<td>B17</td>
<td>$\delta_{e} = \delta_{e}$</td>
<td>V.D.1</td>
</tr>
<tr>
<td>B18</td>
<td>$\omega_{\text{e}}(^{28}\text{Si}) = -\frac{g_{\text{a}}(e) + \delta_{n}}{26A_i(e)}$</td>
<td>V.D.2</td>
</tr>
<tr>
<td>B19</td>
<td>$A_i(^{29}\text{Si}) = A_i(^{29}\text{Si}) + 13A_i(e) - \Delta E_{\text{B}}(^{28}\text{Si})/\sigma A_i(e)/2R_{\text{n}}hc$</td>
<td>III.B</td>
</tr>
<tr>
<td>B20</td>
<td>$\Delta E_{\text{B}}(^{28}\text{Si}) = \Delta E_{\text{B}}(^{28}\text{Si})/hc$</td>
<td>III.B</td>
</tr>
<tr>
<td>B21</td>
<td>$\delta_{s} = \delta_{s}$</td>
<td>V.D.1</td>
</tr>
<tr>
<td>B22</td>
<td>$\delta_{e} = \delta_{e}$</td>
<td>V.A.1</td>
</tr>
<tr>
<td>B23</td>
<td>$\delta_{s} = \delta_{s}$</td>
<td>V.A.1</td>
</tr>
<tr>
<td>B24</td>
<td>$\mathcal{R} = \frac{a_{u}}{1 + a_{u}(\sigma) + \delta_{e} \frac{m_{e}}{m_{p}}}$</td>
<td>V.B.2</td>
</tr>
<tr>
<td>B25, B26</td>
<td>$V(f_{e}) = V(f_{e}^{28}; R_{obs}, \alpha_{u}, \frac{m_{e}}{m_{p}}, a_{u}, \frac{\mu_{e}}{\mu_{p}}, \delta_{s}, \delta_{s})$</td>
<td>V.B.2</td>
</tr>
<tr>
<td>B27</td>
<td>$\Delta V_{\text{obs}} = \Delta V_{\text{obs}}^{28}(R_{obs}, \alpha_{u}, \frac{m_{e}}{m_{p}}, a_{u}, \frac{\mu_{e}}{\mu_{p}}, \delta_{s}, \delta_{s})$</td>
<td>V.B.1</td>
</tr>
<tr>
<td>B28</td>
<td>$\delta_{s} = \delta_{s}$</td>
<td>V.B.1</td>
</tr>
<tr>
<td>B29</td>
<td>$\frac{\mu_{e}}{\mu_{s}} = \frac{\mu_{d}}{\mu_{s}} = (1 + a_{u}(\sigma) + \delta_{e}) \frac{A_{i}(p)}{A_{i}(e)} \frac{\mu_{s}}{\mu_{p}}$</td>
<td>V.C</td>
</tr>
<tr>
<td>B30</td>
<td>$\frac{\mu_{e}}{\mu_{s}}(\text{H}) = \frac{g_{e}}{g_{p}} g_{e}(\text{H}) \frac{1}{\mu_{e}}$</td>
<td>V.A.1</td>
</tr>
<tr>
<td>B31</td>
<td>$\frac{\mu_{e}}{\mu_{s}}(\text{D}) = \frac{g_{e}}{g_{p}} g_{e}(\text{D}) \frac{1}{\mu_{e}}$</td>
<td>V.A.1</td>
</tr>
<tr>
<td>B32</td>
<td>$\frac{\mu_{e}}{\mu_{s}}(\text{H}) = \frac{g_{e}}{g_{p}} \frac{\mu_{e}}{\mu_{p}}$</td>
<td>V.A.1</td>
</tr>
<tr>
<td>B33</td>
<td>$\frac{\mu_{e}}{\mu_{s}} = \frac{\mu_{e}}{\mu_{s}}$</td>
<td>V.A.1</td>
</tr>
<tr>
<td>B34</td>
<td>$\frac{\mu_{e}}{\mu_{s}} = \frac{\mu_{e}}{\mu_{s}}$</td>
<td>V.A.1</td>
</tr>
<tr>
<td>B35</td>
<td>$\frac{\mu_{e}}{\mu_{s}}(\text{HD}) = \frac{1 + \sigma_{p}}{\mu_{e}} \frac{\mu_{s}}{\mu_{p}}$</td>
<td>V.A.1</td>
</tr>
<tr>
<td>B36</td>
<td>$\frac{\mu_{e}}{\mu_{s}}(\text{HT}) = \frac{1 - \sigma_{p}}{\mu_{e}} \frac{\mu_{s}}{\mu_{p}}$</td>
<td>V.A.1</td>
</tr>
<tr>
<td>B37</td>
<td>$\sigma_{p} = \sigma_{p}$</td>
<td>V.A.1</td>
</tr>
</tbody>
</table>
by about twice its uncertainty. This reduced each $|r_i|$ to less than 1. To achieve this level of consistency for the 14 values now available would require an expansion factor of about 16. After due consideration the Task Group decided that it would be more appropriate to follow its usual approach of treating inconsistent data, namely, to choose an expansion factor that reduces each $|r_i|$ to less than 2. It concluded that the resulting uncertainty would better reflect the current situation in light of the low-uncertainty UCI-14 result with $u_t = 19 \times 10^{-6}$, which agrees well with the low-uncertainty UWash-00 and UZur-06 results with $u_t = 14 \times 10^{-6}$ and $u_t = 19 \times 10^{-6}$, respectively. Thus based on an expansion factor of 6.3, the 2014 CODATA recommended value is

$$G = 6.67408(31) \times 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2} \ [47 \times 10^{-6}] \ . \quad (268)$$

(Note that when the same expansion factor is applied to both members of a correlated pair, its square is also applied to their covariance so their correlation coefficient is unchanged. When the expansion factor is applied to one member of a correlated pair, just the expansion factor is applied to the covariance.) For this calculation $\chi^2 = 8.1$, $p(8.1|13) = 0.84$, and $R_B = 0.79$. As might be expected, JILA-10 and BIPM-14 still have the largest values of $|r_i|$; 1.98 and 1.45, respectively. The other 12 values of $|r_i|$ are less than 0.01.

In this calculation $S_i$ is less than 0.01 for five of the 14 input data. If they are omitted, the value for $G$ in Eq. (268) increases by 2 in the last place, but its two-digit uncertainty is unchanged. Although excluding such data is the Task Group’s usual practice, it is not implemented in this case, because of the significant disagreements among the data and the desirability of having the recommended value reflect all the data.

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TABLE XXIV. Observational equations that express the input data in Table XVIII as functions of the adjusted constants in Table XVII. The numbers in the first column correspond to the numbers in the first column of Table XVIII. For simplicity, the lengthier functions are not explicitly given. See Sec. XIII.B for an explanation of the symbol $\tilde{=}$. —Continued

<table>
<thead>
<tr>
<th>Type of input datum</th>
<th>Observational equation</th>
<th>Sec.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_{38}$</td>
<td>$\sigma_{\nu} \equiv \sigma_{\nu}$</td>
<td>VI.A.1</td>
</tr>
<tr>
<td>$B_{39}$</td>
<td>$\Gamma_{\nu, 90}(\text{lo}) \equiv \frac{-K_{\nu, 90}R_{\nu, 90}[1 + \alpha_{\nu}(\nu) + \delta_{\nu}^2][\mu_{\nu}]}{2\mu_{\nu}R_{\nu, 90}}$</td>
<td>VIII</td>
</tr>
<tr>
<td>$B_{40}$</td>
<td>$\Gamma_{\nu, 90}(\text{hi}) \equiv \frac{-K_{\nu, 90}R_{\nu, 90}[1 + \alpha_{\nu}(\nu) + \delta_{\nu}^2][\mu_{\nu}]}{2\mu_{\nu}R_{\nu, 90}}$</td>
<td>VIII</td>
</tr>
<tr>
<td>$B_{41}$</td>
<td>$\Gamma_{\nu, 90}(\text{hi}) \equiv \frac{c[1 + \alpha_{\nu}(\nu) + \delta_{\nu}^2][\mu_{\nu}]}{K_{\nu, 90}R_{\nu, 90}h}$</td>
<td>VIII</td>
</tr>
<tr>
<td>$B_{42}$</td>
<td>$K_{\nu} \equiv \left(\frac{8\alpha}{\mu_{\nu}R_{\nu}}\right)^{1/2}$</td>
<td>VIII</td>
</tr>
<tr>
<td>$B_{43}$</td>
<td>$R_{K} \equiv \frac{12c}{2\alpha}$</td>
<td>VIII</td>
</tr>
<tr>
<td>$B_{44}$</td>
<td>$K_{\nu}^2R_{K} \equiv \frac{4}{h}$</td>
<td>VIII</td>
</tr>
<tr>
<td>$B_{45}$</td>
<td>$\mathcal{F}<em>{\nu, 90} \equiv \frac{cmc</em>{B}(c)a^2}{K_{\nu, 90}R_{\nu, 90}R_{\nu, 90}}$</td>
<td>VIII</td>
</tr>
<tr>
<td>$B_{46}$, $B_{48}$</td>
<td>$\frac{h}{m(X)} \equiv \frac{A_B(c) \cdot \alpha^2}{A_B(X) \cdot 2R_{\nu}}$</td>
<td>VII</td>
</tr>
<tr>
<td>$B_{47}$, $B_{49}$</td>
<td>$A_{B}(X) \equiv A_{B}(X)$</td>
<td>III.A</td>
</tr>
<tr>
<td>$B_{50}$—$B_{59}$</td>
<td>$d_{220(X)}(X) \equiv d_{220(Y)}(Y)$</td>
<td>IX.A</td>
</tr>
<tr>
<td>$B_{60}$—$B_{62}$</td>
<td>$d_{220(X)}(X) \equiv d_{220(X)}(X)$</td>
<td>IX.A</td>
</tr>
<tr>
<td>$B_{63}$</td>
<td>$N_{A} \equiv \frac{cm_{A}(c)a^2}{2R_{\nu}}$</td>
<td>IX.B</td>
</tr>
<tr>
<td>$B_{64}$</td>
<td>$R \equiv R$</td>
<td>X.A</td>
</tr>
<tr>
<td>$B_{65}$</td>
<td>$k \equiv \frac{2\mu_{\nu}R_{\nu}}{cm_{A}(c)\alpha^2}$</td>
<td>X.B</td>
</tr>
<tr>
<td>$B_{66}$</td>
<td>$A_{B} \equiv \frac{\alpha_{B}(X)}{4\pi\epsilon^2a_{B}^4}$</td>
<td>X.C</td>
</tr>
<tr>
<td>$B_{67}$</td>
<td>$N_{A} \equiv \frac{96\pi^{2}R_{\nu}^2}{\alpha_{B}(X)}$</td>
<td>X.C</td>
</tr>
<tr>
<td>$B_{68}$, $B_{69}$</td>
<td>$\frac{\lambda(CuK\alpha)}{d_{220}(X)} \equiv \frac{1537.40044(CuK\alpha)}{d_{220}(X)}$</td>
<td>IX.A</td>
</tr>
<tr>
<td>$B_{70}$</td>
<td>$\frac{\lambda(WK\alpha)}{d_{220}(N)} \equiv \frac{0.2099010 \ \AA}{d_{220}(N)}$</td>
<td>IX.A</td>
</tr>
<tr>
<td>$B_{71}$</td>
<td>$\frac{\lambda(MoK\alpha)}{d_{220}(N)} \equiv \frac{707.831xu(MoK\alpha)}{d_{220}(N)}$</td>
<td>IX.A</td>
</tr>
</tbody>
</table>
TABLE XXV. The 28 adjusted constants (variables) used in the least-squares multivariate analysis of the Rydberg constant data given in Table XVI. These adjusted constants appear as arguments of the functions on the right-hand side of the observational equations of Table XXIII.

<table>
<thead>
<tr>
<th>Adjusted constant</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rydberg constant</td>
<td>$R_o$</td>
</tr>
<tr>
<td>Bound-state proton rms charge radius</td>
<td>$r_p$</td>
</tr>
<tr>
<td>Bound-state deuteron rms charge radius</td>
<td>$r_d$</td>
</tr>
<tr>
<td>Additive correction to $E_0(1S_{1/2})/h$</td>
<td>$\delta h(1S_{1/2})$</td>
</tr>
<tr>
<td>Additive correction to $E_0(1S_{1/2})/h$</td>
<td>$\delta h(2S_{1/2})$</td>
</tr>
<tr>
<td>Additive correction to $E_0(3S_{1/2})/h$</td>
<td>$\delta h(3S_{1/2})$</td>
</tr>
<tr>
<td>Additive correction to $E_0(4S_{1/2})/h$</td>
<td>$\delta h(4S_{1/2})$</td>
</tr>
<tr>
<td>Additive correction to $E_0(6S_{1/2})/h$</td>
<td>$\delta h(6S_{1/2})$</td>
</tr>
<tr>
<td>Additive correction to $E_0(8S_{1/2})/h$</td>
<td>$\delta h(8S_{1/2})$</td>
</tr>
<tr>
<td>Additive correction to $E_0(2P_{3/2})/h$</td>
<td>$\delta h(2P_{3/2})$</td>
</tr>
<tr>
<td>Additive correction to $E_0(4P_{3/2})/h$</td>
<td>$\delta h(4P_{3/2})$</td>
</tr>
<tr>
<td>Additive correction to $E_0(6P_{3/2})/h$</td>
<td>$\delta h(6P_{3/2})$</td>
</tr>
<tr>
<td>Additive correction to $E_0(8P_{3/2})/h$</td>
<td>$\delta h(8P_{3/2})$</td>
</tr>
<tr>
<td>Additive correction to $E_0(12D_{3/2})/h$</td>
<td>$\delta h(12D_{3/2})$</td>
</tr>
<tr>
<td>Additive correction to $E_0(4D_{3/2})/h$</td>
<td>$\delta h(4D_{3/2})$</td>
</tr>
<tr>
<td>Additive correction to $E_0(8D_{3/2})/h$</td>
<td>$\delta h(8D_{3/2})$</td>
</tr>
<tr>
<td>Additive correction to $E_0(12D_{3/2})/h$</td>
<td>$\delta h(12D_{3/2})$</td>
</tr>
<tr>
<td>Additive correction to $E_0(15D_{3/2})/h$</td>
<td>$\delta h(15D_{3/2})$</td>
</tr>
<tr>
<td>Additive correction to $E_0(25D_{3/2})/h$</td>
<td>$\delta h(25D_{3/2})$</td>
</tr>
<tr>
<td>Additive correction to $E_0(45D_{3/2})/h$</td>
<td>$\delta h(45D_{3/2})$</td>
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<tr>
<td>Additive correction to $E_0(85D_{3/2})/h$</td>
<td>$\delta h(85D_{3/2})$</td>
</tr>
<tr>
<td>Additive correction to $E_0(8D_{5/2})/h$</td>
<td>$\delta h(8D_{5/2})$</td>
</tr>
<tr>
<td>Additive correction to $E_0(12D_{5/2})/h$</td>
<td>$\delta h(12D_{5/2})$</td>
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<tr>
<td>Additive correction to $E_0(24D_{5/2})/h$</td>
<td>$\delta h(24D_{5/2})$</td>
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<tr>
<td>Additive correction to $E_0(4D_{7/2})/h$</td>
<td>$\delta h(4D_{7/2})$</td>
</tr>
<tr>
<td>Additive correction to $E_0(8D_{7/2})/h$</td>
<td>$\delta h(8D_{7/2})$</td>
</tr>
<tr>
<td>Additive correction to $E_0(12D_{7/2})/h$</td>
<td>$\delta h(12D_{7/2})$</td>
</tr>
</tbody>
</table>

2. Data related to all other constants

Tables XXVIII and XXIX summarize 12 least-squares analyses of the input data in Tables XVI and XVIII, including their correlation coefficients in Tables XVII and XIX; they are discussed in the following paragraphs. Because the adjusted value of $R_o$ is essentially the same for all five adjustments summarized in Table XXVIII and equal to that of adjustment 3 of Table XXIX, the values are not listed in Table XXVIII. (Note that adjustment 3 in Tables XXVIII and XXIX is the same adjustment.)

Adjustment 1. The initial adjustment includes all of the input data, four of which have values of $|r|$ that are problematically larger than 2. They are $B2$, the AME-12 result for $A_{13}$ ($^1H$), $B11$ and $B12$, the FSU-15 results for $\omega_c/(HD^+)/\omega_c/(1^3He^+)$ and $\omega_c/(HD^+)/\omega_c(t)$, and $B39.1$, the NIST-89 result for $I^*_{p=90}(I_0)$. Their respective values of $r_i$ are 2.61, 4.06, 3.57, and 2.20. The NIST-89 datum was discussed above in connection with inferred values of $\alpha$ and because it is of no real concern it is not discussed further. The other three residuals are due to the inconsistency of $B9$, the UWash-15 result for $\omega_c/(h)/(1^3C^+)$, and $B11$. Although $u_c$ is $1.4 \times 10^{-11}$ and $4.8 \times 10^{-11}$ for $B9$ and $B11$, respectively, which are quite small, their inconsistency becomes apparent by comparing the values of $A_f(1^3He)$ that they infer; the result is that the value from $B11$ exceeds that from $B9$ by $3.9\Delta a_0$ (Myers et al., 2015; Zafonte and Van Dyck, 2015). Because the reason for this discrepancy is unknown and both frequency ratios are credible,

<table>
<thead>
<tr>
<th>Adjusted constant</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neutron relative atomic mass</td>
<td>$A_n(n)$</td>
</tr>
<tr>
<td>Electron relative atomic mass</td>
<td>$A_e(e)$</td>
</tr>
<tr>
<td>Proton relative atomic mass</td>
<td>$A_p(p)$</td>
</tr>
<tr>
<td>$^1H^+$ electron removal energy</td>
<td>$\Delta E_b(^1H^+)$</td>
</tr>
<tr>
<td>Triton relative atomic mass</td>
<td>$A_t(t)$</td>
</tr>
<tr>
<td>$^3H^+$ electron removal energy</td>
<td>$\Delta E_b(^3H^+)$</td>
</tr>
<tr>
<td>Alpha particle relative atomic mass</td>
<td>$A_{\alpha}(\alpha)$</td>
</tr>
<tr>
<td>$^4He^+$ electron removal energy</td>
<td>$\Delta E_b(^4He^+)$</td>
</tr>
<tr>
<td>Deuteron relative atomic mass</td>
<td>$A_d(d)$</td>
</tr>
<tr>
<td>Helion relative atomic mass</td>
<td>$A_h(h)$</td>
</tr>
<tr>
<td>$^12C^+$ electron removal energy</td>
<td>$\Delta E_b(^12C^+)$</td>
</tr>
<tr>
<td>$^3He^*$ electron ionization energy</td>
<td>$\Delta E_i(^3He^*)$</td>
</tr>
<tr>
<td>$^1H^*$ electron ionization energy</td>
<td>$\Delta E_i(^1H^*)$</td>
</tr>
<tr>
<td>$^12C^+$ electron removal energy</td>
<td>$\Delta E_b(^12C^+)$</td>
</tr>
<tr>
<td>Additive correction to $q_c(\alpha)$</td>
<td>$A_c^{(28Si)}$</td>
</tr>
<tr>
<td>$^3Si^{13}$ relative atomic mass</td>
<td>$A_{\alpha}(^{28Si}^{13})$</td>
</tr>
<tr>
<td>$^3Si^{13}$ electron removal energy</td>
<td>$\Delta E_b(^{28Si}^{13})$</td>
</tr>
<tr>
<td>Additive correction to $g_s(\alpha)$</td>
<td>$\delta_S$</td>
</tr>
<tr>
<td>Fine-structure constant</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>Additive correction to $a_{\alpha}(\alpha)$</td>
<td>$a_\alpha$</td>
</tr>
<tr>
<td>Muon magnetic-moment anomaly</td>
<td>$a_m/\mu_n$</td>
</tr>
<tr>
<td>Electron-proton magnetic-moment ratio</td>
<td>$\mu_e/\mu_p$</td>
</tr>
<tr>
<td>Additive correction to $\Delta m_{\alpha}(\text{th})$</td>
<td>$\delta_{\alpha\text{th}}$</td>
</tr>
<tr>
<td>Deuteron-electron magnetic-moment ratio</td>
<td>$\mu_d/\mu_p$</td>
</tr>
<tr>
<td>Shielded helion to shielded proton magnetic-moment ratio</td>
<td>$\mu_h/\mu_p$</td>
</tr>
<tr>
<td>Neutron to shielded proton magnetic-moment ratio</td>
<td>$\mu_n/\mu_p$</td>
</tr>
<tr>
<td>Shielding difference of $d$ and $p$ in HD</td>
<td>$\sigma_{dp}$</td>
</tr>
<tr>
<td>Triton-proton magnetic-moment ratio</td>
<td>$\mu_t/\mu_p$</td>
</tr>
<tr>
<td>Shielding difference of $t$ and $p$ in HT</td>
<td>$\sigma_{tp}$</td>
</tr>
<tr>
<td>Electron to shielded proton magnetic-moment ratio</td>
<td>$\mu_e/\mu_p$</td>
</tr>
<tr>
<td>Planck constant</td>
<td>$h$</td>
</tr>
<tr>
<td>$^{13}Cs$ relative atomic mass</td>
<td>$A_{13}(^{13}Cs)$</td>
</tr>
<tr>
<td>$^{87}Rb$ relative atomic mass</td>
<td>$A_{87}(^{87}Rb)$</td>
</tr>
<tr>
<td>$d_{230}$ of Si crystal WASO 17</td>
<td>$d_{230}(^{17}WASO)$</td>
</tr>
<tr>
<td>$d_{230}$ of Si crystal ILL</td>
<td>$d_{230}(^{17}ILL)$</td>
</tr>
<tr>
<td>$d_{230}$ of Si crystal MO*</td>
<td>$d_{230}(^{17}MO^*)$</td>
</tr>
<tr>
<td>$d_{230}$ of Si crystal NR3</td>
<td>$d_{230}(^{17}NR3)$</td>
</tr>
<tr>
<td>$d_{230}$ of Si crystal N</td>
<td>$d_{230}(^{17}N)$</td>
</tr>
<tr>
<td>$d_{230}$ of Si crystal WASSO 4.2a</td>
<td>$d_{230}(^{4.2a}WASSO)$</td>
</tr>
<tr>
<td>$d_{230}$ of Si crystal WASSO 04</td>
<td>$d_{230}(^{0.4}WASSO)$</td>
</tr>
<tr>
<td>$d_{230}$ of an ideal Si crystal</td>
<td>$d_{230}$</td>
</tr>
<tr>
<td>$d_{230}$ of Si crystal NR4</td>
<td>$d_{230}(^{17}NR4)$</td>
</tr>
<tr>
<td>Molar gas constant</td>
<td>$R$</td>
</tr>
<tr>
<td>Static electric dipole polarizability of $^4He$ in atomic units</td>
<td>$a_\sigma(^4He)$</td>
</tr>
<tr>
<td>Copper Koo x unit</td>
<td>$x[u(CuKo)]$</td>
</tr>
<tr>
<td>Ångstrom star</td>
<td>$\AA^*$</td>
</tr>
<tr>
<td>Molybdenum Koo x unit</td>
<td>$x[u(MoKo)]$</td>
</tr>
</tbody>
</table>
the Task Group decided to include both in the final adjustment with a sufficiently large expansion factor so that \( r_7 < 2 \) for both. It was also decided to include the companion ratios \( \omega_c(d)/\omega_c(^{12}\text{C}^+ \Gamma) \) (B8) and \( \omega_c(\text{HD}^+)/\omega_c(t) \) (B12) with \( u_7 = 2.0 \times 10^{-11} \) and \( 4.8 \times 10^{-11} \), respectively, without an expansion factor.

**Adjustment 2.** This adjustment uses all the data with an expansion factor of 2.8 applied to the uncertainties of data B9 and B11, resulting in \( r_7 = 1.95 \) for B11 and \( r_7 = 1.73 \) for B12. This also results in the reduction of \( r_7 \) of B2 from 2.61 to 0.49. The complex relationships, apparent from their observational equations, between input data B2, B8 and B9, and B11 and B12, and the adjusted constants \( A_i(p), A_i(d), \) and \( A_i(h), \) are responsible for the somewhat surprising effect of the expansion factor on \( r_7 \) (see Table XXIV). The expansion factor has no effect on the values of \( \alpha \) and \( h \), as can be seen from Table XXVII.

**Adjustment 3.** Adjustment 3 is the adjustment on which the 2014 CODATA recommended values are based and as such is called the “final adjustment.” It differs from adjustment 2 in that, following the prescription described above, 22 of the initial input data, all from Table XVIII, with values of \( S_r < 0.01 \) in adjustment 2 are omitted. These are B4, B22.1, B39.1 to B44.1, B44.3, B44.5, B44.7 to B46, B64.1, and B64.5. (The range in values of \( S_r \) for the deleted data is 0.0000 to 0.0096, and no datum with a value of \( S_r > 1 \) was “converted” to a value with \( S_r < 1 \) due to the expansion factor.) Further, because \( A_r(^3\text{H}) \), item B4, is deleted as an input datum due to its low weight, the value of \( \Delta E_B(^3\text{H}^+)/hc \), item B5, which is not relevant to any other input datum, is also deleted and omitted as an adjusted constant. The situation is exactly the same for \( h/m(^{133}\text{Cs}) \), item B46, and \( A_r(^{133}\text{Cs}) \), item B47. This brings the total number of omitted data to 24. Table XXVIII shows that deleting them has inconsequential impact on the values of \( \alpha \) and \( h \). The data for the final adjustment are quite consistent, as demonstrated by the value of \( \chi^2; p(42.4/54) = 0.87 \).

**Adjustments 4 and 5.** The purpose of these adjustments is to test the robustness of the 2014 recommended values of \( \alpha \) and \( h \) by omitting the most accurate data that determine these constants. Adjustment 4 differs from adjustment 2 in that the four data that provide values of \( \alpha \) with the smallest uncertainties are deleted, namely, items B22.2, B48, B22.1, and B46, which are the two values of \( u_7 \) and the \( h/m(X) \) values for \(^{133}\text{Cs}\) and \(^{87}\text{Rb}\); see the first four entries of Table XX. [For the same reason as in adjustment 3, in adjustment 4 the value of \( A_r(^{133}\text{Cs}) \) is also deleted as an input datum and \( A_r(^{133}\text{Cs}) \) as an adjusted constant; the same applies to \( A_r(^{87}\text{Rb}) \).] Adjustment 5 differs from adjustment 2 in that the five data that provide values of \( h \) with the smallest uncertainties are deleted, namely, items B44.6, B63.2, B63.1, B44.4, and B44.2, which are three

---

**Table XXVII.** Summary of values of \( G \) used to determine the 2014 recommended value (see also Table XV, Sec. XI).

<table>
<thead>
<tr>
<th>Item number</th>
<th>Value ( (10^{-11} \text{ m}^3 \text{kg}^{-1} \text{s}^{-2}) )</th>
<th>Relative standard uncertainty ( u_i )</th>
<th>Identification</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>6.672 48(43)</td>
<td>6.4 x 10^{-5}</td>
<td>NIST-82</td>
</tr>
<tr>
<td>G2</td>
<td>6.672 9(5)</td>
<td>7.5 x 10^{-5}</td>
<td>TRKD-96</td>
</tr>
<tr>
<td>G3</td>
<td>6.673 98(70)</td>
<td>1.0 x 10^{-4}</td>
<td>LANL-97</td>
</tr>
<tr>
<td>G4</td>
<td>6.674 255(92)</td>
<td>1.4 x 10^{-5}</td>
<td>UWash-00</td>
</tr>
<tr>
<td>G5</td>
<td>6.675 59(27)</td>
<td>4.0 x 10^{-5}</td>
<td>BIPM-01</td>
</tr>
<tr>
<td>G6</td>
<td>6.674 22(98)</td>
<td>1.5 x 10^{-4}</td>
<td>UWup-02</td>
</tr>
<tr>
<td>G7</td>
<td>6.673 87(27)</td>
<td>4.0 x 10^{-5}</td>
<td>MSL-03</td>
</tr>
<tr>
<td>G8</td>
<td>6.672 22(87)</td>
<td>1.3 x 10^{-4}</td>
<td>HUST-05</td>
</tr>
<tr>
<td>G9</td>
<td>6.674 25(12)</td>
<td>1.9 x 10^{-5}</td>
<td>UZer-06</td>
</tr>
<tr>
<td>G10</td>
<td>6.673 49(18)</td>
<td>2.7 x 10^{-5}</td>
<td>HUST-09</td>
</tr>
<tr>
<td>G11</td>
<td>6.672 34(14)</td>
<td>2.1 x 10^{-5}</td>
<td>JILA-10</td>
</tr>
<tr>
<td>G12</td>
<td>6.675 54(16)</td>
<td>2.4 x 10^{-5}</td>
<td>BIPM-14</td>
</tr>
<tr>
<td>G13</td>
<td>6.671 91(99)</td>
<td>1.5 x 10^{-4}</td>
<td>LENS-14</td>
</tr>
<tr>
<td>G14</td>
<td>6.674 35(13)</td>
<td>1.9 x 10^{-5}</td>
<td>UCI-14</td>
</tr>
</tbody>
</table>

*Correlation coefficients: \( r(G1, G3) = 0.351; r(G8, G10) = 0.134.\)
Table XXVIII. Summary of the results of some of the least-squares adjustments used to analyze the input data given in Tables XVI, XVII, XVIII, and XIX. The values of \( \alpha \) and \( h \) are those obtained in the adjustment, \( N \) is the number of input data, \( M \) is the number of adjusted constants, \( \chi^2 = N - M \) is the degrees of freedom, and \( R_B = \sqrt{\chi^2/\nu} \) is the Birge ratio. See the text for an explanation and discussion of each adjustment, but in brief, adjustment 1 is all the data; 2 is the same as 1 except with the uncertainties of the two cyclotron frequency ratios related to the helion multiplied by 2.8; 3 is 2 with the low-weight input data deleted and is the adjustment on which the 2014 recommended values are based; 4 is 2 with the input data that provide the most accurate values of \( \alpha \) deleted; and 5 is with the input data that provide the most accurate values of \( h \) deleted as well as the low-weight data for \( \alpha \).

Table XXIX. Summary of the results of some of the least-squares adjustments used to analyze the input data related to \( R_{\alpha} \). The values of \( R_{\alpha}, r_p, \) and \( r_d \) are those obtained in the indicated adjustment, \( N \) is the number of input data, \( M \) is the number of adjusted constants, \( \chi^2 = N - M \) is the degrees of freedom, and \( R_B = \sqrt{\chi^2/\nu} \) is the Birge ratio. See the text for an explanation and discussion of each adjustment, but in brief, adjustment 3 is but the scattering data for the nuclear radii are omitted; 7 is 3, but with only the hydrogen data included (but not the isotope shift); 8 is 7 with the \( r_d \) datum deleted; 9 and 10 are similar to 7 and 8, but for the deuterium data; 11 is 3 with the muonic Lamb-shift value of \( r_p \) included; and 12 is 11, but without the scattering values of \( r_p \) and \( r_d \).

watt-balance values of \( K_\alpha^2 R_k \) and two XRCO-enriched silicon values of \( N_\alpha \); see the first five entries of Table XXI. The results of these two adjustments are reasonable: Table XXVIII shows that the value of \( \alpha \) from the less accurate \( \alpha \)-related data used in adjustment 4, and the value of \( h \) from the less accurate \( h \)-related data used in adjustment 5 agree with the correspondingly recommended values from adjustment 3.

Adjustments 6 to 12. The purpose of the seven adjustments summarized in Table XXIX is to investigate the data that determine the recommended values of \( R_{\alpha}, r_p, \) and \( r_d \). Results from adjustment 3, the final adjustment, are included in the table for reference purposes. We begin with a discussion of adjustments 6 to 10, which are derived from adjustment 3 by deleting selected input data. We then discuss adjustments 11 and 12, which examine the impact of the proton rms charge radius derived from the measurement of the Lamb shift in muonic hydrogen discussed in Sec. IV.A.3.c and given in Eq. (78). Note that the value of \( R_{\alpha} \) depends only weakly on the data in Table XVIII.

In adjustment 6, the electron scattering values of \( r_p \) and \( r_d \), data items A49 and A50 in Table XVI, are deleted from adjustment 3. Thus, the values of these two quantities from adjustment 6 are based solely on \( H \) and \( D \) spectroscopic data and are called the spectroscopic values of \( r_p \) and \( r_d \):

\[
r_p = 0.8759(77) \text{ fm,} \quad (269)\\
\]

\[
r_d = 2.1416(31) \text{ fm.} \quad (270)
\]

It is evident from a comparison of the results of this adjustment and adjustment 3 that the scattering values of the radii play a comparatively small role in determining the 2014 recommended values of \( R_{\alpha}, r_p, \) and \( r_d \).

Adjustment 7 is based on only hydrogen data, including the scattering values of \( r_p \) but not the difference between the \( 1S_{1/2} - 2S_{1/2} \) transition frequencies in \( H \) and \( D \), item A48 in Table XVI, known as the isotope shift. Adjustment 8 differs from adjustment 7 in that the scattering value of \( r_p \) is deleted. Adjustments 9 and 10 are similar to 7 and 8 but are based on only deuterium data; that is, adjustment 9 includes the scattering value of \( r_d \) but not the isotope shift, while for adjustment 10 the scattering value is deleted. The results of these four adjustments show the dominant role of the hydrogen data and the importance of the isotope shift in determining the recommended value of \( r_d \). Further, the four values of \( R_{\alpha} \) from these adjustments agree with the 2014 recommended value, and the two values of \( r_p \) and of \( r_d \) also agree with their respective recommended values: the largest difference from the recommended value for the eight results is \( 1.4\mu_{\text{diff}} \).
Adjustment 11 differs from adjustment 3 in that it includes the muonic hydrogen value \( r_p = 0.840 \) 87(39) fm, and adjustment 12 differs from adjustment 11 in that the two scattering values of the nuclear radii are deleted. Because the muonic hydrogen value is significantly smaller and has a significantly smaller uncertainty than the purely spectroscopic value of adjustment 6 as well as the scattering value, it has a major impact on the results of adjustments 11 and 12, as can be seen from Table XXIX: for both adjustments the value of \( R_\alpha \) shifts down by over 5 standard deviations and its uncertainty is reduced by a factor greater than 6. Moreover, and not surprisingly, the values of \( r_p \) and of \( r_\alpha \) from both adjustments are significantly smaller than the recommended values and have significantly smaller uncertainties. The inconsistency between the muonic hydrogen result for \( r_p \) and the spectroscopic and scattering results is further demonstrated by the comparatively low probability of \( \chi^2 \) for adjustment 11: \( P(72.8|55) = 0.0054 \). The 2014 recommended value of \( r_p \) and the purely spectroscopic value, which is that from adjustment 6, exceed the muonic hydrogen value by 5.6\( \Delta \)diff and 4.5\( \Delta \)diff, respectively.

The impact of the muonic hydrogen value of \( r_p \) can also be seen by examining for adjustments 3, 11, and 12 the normalized residuals and self-sensitivity coefficients of the principal experimental data that determine \( R_\alpha \), namely, items A26.1 to A50 in Table XVI. In brief, \( |r_i| \) for these data in the final adjustment range from near 0 to 1.13 for item A50, the \( r_\alpha \) scattering result, with the vast majority being less than 1. For the three greater than 1, \( |r_i| \) is 1.03, 1.02, and 1.02. The value of \( S_r \) is 1.00 for items A26.1 and A26.2 together, which are the two hydrogen \( 1S_{1/2} - 2S_{1/2} \) transition frequencies; it is also 1.00 for A48, the H-D isotope shift. For item A49, the scattering value of \( r_p \), it is 0.31. Most others are a few percent, although some values of \( S_r \) are near 0.

The situation is markedly different for adjustment 12. First, \( |r_i| \) for item A30, the hydrogen transition frequency involving the \( 8D_{3/2} \) state, is 3.12 compared to 1.03 in adjustment 3; and items A41, A42, and A43, deuterium transitions involving the \( 8S_{1/2}, 8D_{3/2}, \) and \( 8D_{5/2} \) states, are now 2.54, 2.47, and 3.12, respectively, compared to 0.56, 0.34, and 0.86. Further, ten other transitions have residuals in the range 1.04 to 1.80. As a result, with this proton radius, the predictions of the theory for hydrogen and deuterium transition frequencies are not generally consistent with the experiments. Equally noteworthy is the fact that, although \( S_r \) for items A26.1 and A26.2 together and A48 remain equal to 1.00, for all other transition frequencies \( S_r \) is less than 0.01, which means that they play an inconsequential role in determining \( R_\alpha \). The results for adjustment 11, which includes the scattering values of the nuclear radii as well as the muonic hydrogen value, are similar.

Because of the impact of the latter value on the internal consistency of the \( R_\alpha \) data and its continued disagreement with the spectroscopic and scattering values, the Task Group decided, as it did for the 2010 adjustment, that it was premature to include it as an input datum in the 2014 final adjustment; it was deemed more prudent to continue to wait and see if further research can resolve what has come to be called the “proton radius puzzle”; see Sec. IV.A.3.c for additional discussion.

3. Test of the Josephson and quantum-Hall-effect relations

As in the three previous CODATA adjustments, the exactness of the relations \( K_J = 2e/h \) and \( R_K = h/e^2 \) is investigated by writing

\[
K_J = \frac{2e}{h} \left( 1 + \epsilon_j \right) = \left( \frac{8\alpha}{\mu_B e^2 h} \right)^{1/2} \left( 1 + \epsilon_j \right), \quad (271)
\]

\[
R_K = \frac{h}{e^2} \left( 1 + \epsilon_K \right) = \frac{4\mu_B c^2}{2\alpha} \left( 1 + \epsilon_K \right), \quad (272)
\]

where \( \epsilon_j \) and \( \epsilon_K \) are unknown correction factors taken to be additional adjusted constants. Replacing the relations \( K_J = 2e/h \) and \( R_K = h/e^2 \) in the analysis leading to the observational equations in Table XXIV with the generalizations in Eqs. (271) and (272) leads to the modified observational equations given in Table XXX.

Although the NIM/NIST-15 result for \( k/h \) item B65, was obtained using the Josephson and quantum-Hall effects, it is not included in the tests of the relations \( K_J = 2e/h \) and \( R_K = h/e^2 \), because of its comparatively large uncertainty.

The results of six different adjustments are summarized in Table XXXI. An entry of 0 in the \( \epsilon_K \) column means that it is assumed \( R_K = h/e^2 \) in the corresponding adjustment; similarly, an entry of 0 in the \( \epsilon_j \) column means that it is assumed \( K_J = 2e/h \) in the corresponding adjustment. The following remarks apply to the six adjustments.

Adjustment (i) differs from adjustment 2 summarized in Table XXVIII only in that the assumption \( K_J = 2e/h \) and \( R_K = h/e^2 \) is relaxed. For this adjustment, \( N = 153, M = 77, \nu = N - M = 76, \chi^2 = 64.1, p(64.1|76) = 0.83, \) and \( R_p = 0.92 \). Examination of the table shows that \( \epsilon_K \) is consistent with

<table>
<thead>
<tr>
<th>Type of input datum</th>
<th>Generalized observational equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B_{39'} )</td>
<td>( \Gamma_{P,90}(10) = -\frac{K_{J,90}R_{K,90}[1 + a_0(\alpha, \delta_0)]^a}{2\mu_B R_{\alpha}(1 + \epsilon_j)(1 + \epsilon_K)} \mu_B^{-1} \mu_p^{-1} )</td>
</tr>
<tr>
<td>( B_{40'} )</td>
<td>( \Gamma_{P,90}(10) = \frac{K_{J,90}R_{K,90}[1 + a_0(\alpha, \delta_0)]^a}{2\mu_B R_{\alpha}(1 + \epsilon_j)(1 + \epsilon_K)} \mu_B^{-1} \mu_p^{-1} )</td>
</tr>
<tr>
<td>( B_{41'} )</td>
<td>( \Gamma_{P,90}(11) = -\frac{[1 + a_0(\alpha, \delta_0)]^2}{K_{J,90}R_{K,90}R_{\alpha}h} \mu_B^{-1} \mu_p^{-1} )</td>
</tr>
<tr>
<td>( B_{43} )</td>
<td>( R_K = \frac{\mu_B^2 c^2}{2\alpha} \left( 1 + \epsilon_K \right) )</td>
</tr>
<tr>
<td>( B_{42} )</td>
<td>( \epsilon_K = \left( \frac{8\alpha}{\mu_B e^2 h} \right)^{1/2} \left( 1 + \epsilon_j \right) )</td>
</tr>
<tr>
<td>( B_{44} )</td>
<td>( K_J^2 R_K^2 = \frac{4}{h} \left( 1 + \epsilon_j \right)^2 \left( 1 + \epsilon_K \right) )</td>
</tr>
<tr>
<td>( B_{45} )</td>
<td>( \chi_{10} = \frac{cM_A(\epsilon_0)^2}{K_{J,90}R_{K,90}R_{\alpha}h} \left( 1 + \epsilon_j \right)(1 + \epsilon_K) )</td>
</tr>
</tbody>
</table>

TABLE XXXI. Summary of the results of several least-squares adjustments to investigate the relations $K_1 = (2e/h)(1 + \epsilon_J)$ and $R_K = (h/2\epsilon^2)(1 + \epsilon_K)$. See the text for an explanation and discussion of each adjustment, but in brief, adjustment (i) uses all the data, (ii) assumes $K_1 = 2e/h$ (that is, $\epsilon_J = 0$) and obtains $\epsilon_K$ from the five measured values of $R_K$, (iii) is based on the same assumption and obtains $\epsilon_K$ from the two values of the proton gyromagnetic ratio and one value of the helion gyromagnetic ratio, (iv) is (iii) but assumes $R_K = h/2\epsilon^2$ (that is, $\epsilon_K = 0$) and obtains $\epsilon_J$ in place of $\epsilon_K$, (v) and (vi) are based on the same assumption and obtain $\epsilon_J$ from the measured values given in Table XVIII for the quantities indicated.

<table>
<thead>
<tr>
<th>Adj.</th>
<th>Data included</th>
<th>$10^6 \epsilon_K$</th>
<th>$10^6 \epsilon_J$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>All</td>
<td>2.2(1.8)</td>
<td>-0.9(1.5)</td>
</tr>
<tr>
<td>(ii)</td>
<td>$R_K$</td>
<td>2.8(1.8)</td>
<td>0</td>
</tr>
<tr>
<td>(iii)</td>
<td>$I'_{p,b=90}(lo)$</td>
<td>-25.5(9.3)</td>
<td>0</td>
</tr>
<tr>
<td>(iv)</td>
<td>$I'_{p,b=90}(lo)$</td>
<td>0</td>
<td>-25.5(9.3)</td>
</tr>
<tr>
<td>(v)</td>
<td>$I'<em>{p=90}(hi)$, $K_1$, $K_2^2 R_K$, $E</em>\infty$</td>
<td>8.2(71.9)</td>
<td></td>
</tr>
<tr>
<td>(vi)</td>
<td>$I'<em>{p=90}(hi)$, $K_1$, $K_2^2 R_K$, $E</em>\infty$, $N_A$</td>
<td>0.7(1.2)</td>
<td></td>
</tr>
</tbody>
</table>

0 within 1.2 times its uncertainty of $1.8 \times 10^{-8}$, while $\epsilon_J$ is consistent with 0 within its uncertainty of $1.5 \times 10^{-8}$.

Adjustments (ii) and (iii) focus on $\epsilon_K$; $\epsilon_J$ is set equal to 0 and values of $\epsilon_K$ are calculated from data whose observational equations are independent of $h$. Adjustment (ii) uses the five results for $R_K$, items B43.1 to B43.5, and (iii) uses the three low-field gyromagnetic ratio results, items B39.1, B39.2, and B40 [the three together are denoted by $I'_{p=90}(lo)$]. We see from Table XXXI that the values of $\epsilon_K$ resulting from the two adjustments not only have opposite signs but disagree significantly: their difference is $3.0_{\Delta \text{diff}}$. Their disagreement reflects the fact that while the five inferred values of $a$ from $R_K$ are consistent among themselves and with the highly accurate value from $\alpha$, the inferred value from the NIST-89 result for $I'_{p=90}(lo)$ (item B39.1), which has the smallest uncertainty of the three low-field gyromagnetic ratios, is not (see Table XX, Sec. XIII.A).

Adjustments (iv) to (vi) focus on $\epsilon_J$; $\epsilon_K$ is set equal to 0 and values of $\epsilon_J$ are calculated from data whose observational equations, with the exception of adjustment (iv), are dependent on $h$. Because $\epsilon_J$ and $\epsilon_K$ enter the observational equations for the gyromagnetic ratios in the symmetric form $(1 + \epsilon_J)(1 + \epsilon_K)$, the numerical result from adjustments (iii) and (iv) are identical. Although $\epsilon_J$ from adjustment (iv) has the opposite sign of $\epsilon_J$ from adjustments (v) and (vi), it agrees with $\epsilon_J$ from adjustment (v) because of that result’s extremely large uncertainty. However, it does disagree with the adjustment (vi) result: their difference is $2.8_{\Delta \text{diff}}$.

In summary, we recall that rather limited conclusions could be drawn from the similar analysis presented in CODATA-10, because of the disagreement between the NIST-07 watt-balance measurement of $h$ and the IAC-11 XRCD enriched silicon measurement of $N_A$. With the resolution of the disagreement by the replacement of the earlier NIST-15 result with NIST-15 and the agreement of its inferred value of $h$ with other values (see Table XXI, Sec. XIII.A), the remaining issue is the values of $\epsilon_K$ and $\epsilon_J$ from $I'_{p,b=90}(lo)$. However, these three data are dominated by the NIST-89 $I'_{p=90}(lo)$ result and as discussed in Sec. XIII.A, the inconsistency of this datum has been of concern in past adjustments and because of its low weight is not included in the 2006 and 2010 final adjustments, or in the 2014 final adjustment. It can thus be concluded that the current data show the Josephson and quantum-Hall-effect relations to be exact within 2 parts in $10^8$.

One way to test the universality of the Josephson and quantum-Hall-effect relations is to investigate their material dependence. Recently, Ribeiro-Palau et al. (2015) obtained agreement between the quantized Hall resistance in a graphene (two-dimensional graphite) device and a GaAs/AlGaAs heterostructure device well within the 8.2 parts in $10^{11}$ uncertainty of their measurement. This is slightly better than the previous best graphene-GaAs/AlGaAs comparison, which obtained agreement within the 8.7 parts in $10^{11}$ uncertainty of the experiment (Janssen et al., 2012). Another way is to “close the metrology triangle” by using a single electron tunneling (SET) device that generates a quantized current $I = ef$ when an alternating voltage of frequency $f$ is applied to it. The current $I$ is then compared to a current obtained from Josephson and quantum-Hall-effect devices. The status of such efforts was briefly discussed in the same section of CODATA-10 and little has changed since. Two relevant papers not referenced in CODATA-10 are by Devoille et al. (2012) and Scherer and Camarota (2012).

XIV. The 2014 CODATA Recommended Values

A. Calculational details

The 151 input data and their many correlation coefficients initially considered for inclusion in the 2014 CODATA adjustment of the values of the constants are given in Tables XVI, XVII, XVIII, and XIX. The 2014 recommended values are based on adjustment 3, the final adjustment, summarized in Table XXVIII and discussed in the associated text. Adjustment 3 omits 22 of the 151 initially considered input data, namely, items B4, B22.1, B39.1 to B44.1, B44.3, B44.4, B44.7 to B46, B64.1, and B64.5, because of their low weight (self-sensitivity coefficient $S_c$ less than 0.01). However, because the observational equation for $A_1(\bar{3}^1\text{H})$, item B4, depends on $\Delta E_{\bar{3}^1\text{H}}/h c$ and item B4 is deleted due to its low weight, the value of $\Delta E_{\bar{3}^1\text{H}}/hc$, item B5, is also deleted as an adjusted constant. The same statement applies to $h/m_{\text{133Cs}}$, item B46, and $A_1(\text{133Cs})$, item B47. Further, the initial uncertainties of two input data, items B11 and B12, are multiplied by the expansion factor 2.8. As a consequence, the normalized residual $r_i$ of each as well as that of item B2 is reduced to below 2.

Each input datum in this final adjustment has a self-sensitivity coefficient $S_c$ greater than 0.01, or is a subset of the data of an experiment or series of experiments that provide an input datum or input data with $S_c > 0.1$. Not counting such input data with $S_c < 0.01$, the five data with $|r_i| > 1.2$ are B11, B12, B44.2, B44.4, and B48; their values of $r_i$ are 1.95, 1.71, 1.96, 1.84, and 1.68, respectively.

The 2014 recommended values are calculated from the set of best estimated values, in the least-squares sense, of 75 adjusted constants, including $G$, and their variances and covariances.
TABLE XXXII. An abbreviated list of the CODATA recommended values of the fundamental constants of physics and chemistry based on the 2014 adjustment

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Numerical value</th>
<th>Unit</th>
<th>Relative std. uncert. uᵣ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed of light in vacuum</td>
<td>c, c₀</td>
<td>299 792 458</td>
<td>m s⁻¹</td>
<td>Exact</td>
</tr>
<tr>
<td>Magnetic constant</td>
<td>μ₀</td>
<td>4π × 10⁻⁷</td>
<td>N A⁻²</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>= 12.566 370 614 ... × 10⁻⁷</td>
<td>N A⁻²</td>
<td>Exact</td>
</tr>
<tr>
<td>Electric constant 1/μₑc²</td>
<td>ε₀</td>
<td>8.854 187 817 ... × 10⁻¹²</td>
<td>F m⁻¹</td>
<td></td>
</tr>
<tr>
<td>Newtonian constant of gravitation</td>
<td>G</td>
<td>6.674 08(31) × 10⁻¹¹</td>
<td>m³ kg⁻¹ s⁻²</td>
<td>4.7 × 10⁻⁵</td>
</tr>
<tr>
<td>Planck constant</td>
<td>h</td>
<td>6.626 070 400(81) × 10⁻³⁴</td>
<td>J s</td>
<td>1.2 × 10⁻⁸</td>
</tr>
<tr>
<td>Planck constant/2π</td>
<td>h</td>
<td>1.054 571 800(13) × 10⁻³⁴</td>
<td>J s</td>
<td>1.2 × 10⁻⁸</td>
</tr>
<tr>
<td>Elementary charge</td>
<td>e</td>
<td>1.602 176 6208(98) × 10⁻¹⁹</td>
<td>C</td>
<td>6.1 × 10⁻⁹</td>
</tr>
<tr>
<td>Magnetic flux quantum h/2e</td>
<td>Φ₀</td>
<td>2.067 833 831(13) × 10⁻¹⁵</td>
<td>Wb</td>
<td>6.1 × 10⁻⁹</td>
</tr>
<tr>
<td>Conductance quantum 2e²/h</td>
<td>Gₚ</td>
<td>7.748 091 7310(18) × 10⁻⁵</td>
<td>S</td>
<td>2.5 × 10⁻¹⁰</td>
</tr>
<tr>
<td>Electron mass</td>
<td>mₑ</td>
<td>9.109 383 56(11) × 10⁻³¹</td>
<td>kg</td>
<td>1.2 × 10⁻⁸</td>
</tr>
<tr>
<td>Proton mass</td>
<td>mₚ</td>
<td>1.672 621 898(21) × 10⁻²³</td>
<td>kg</td>
<td>1.2 × 10⁻⁸</td>
</tr>
<tr>
<td>Proton-electron mass ratio</td>
<td>mₑ/mₚ</td>
<td>1836.152 673 89(17)</td>
<td>9.5 × 10⁻¹¹</td>
<td></td>
</tr>
<tr>
<td>Fine-structure constant e²/4πε₀hc</td>
<td>α</td>
<td>7.297 352 566(4) × 10⁻³</td>
<td>2.3 × 10⁻¹⁰</td>
<td></td>
</tr>
<tr>
<td>inverse fine-structure constant</td>
<td>α⁻¹</td>
<td>137.035 999(13) × 10⁻¹</td>
<td>2.3 × 10⁻¹⁰</td>
<td></td>
</tr>
<tr>
<td>Rydberg constant a²mₑc/2h</td>
<td>Rₑ</td>
<td>10 973 731.568 508(65)</td>
<td>m⁻¹</td>
<td>5.9 × 10⁻¹²</td>
</tr>
<tr>
<td>Avogadro constant</td>
<td>Nₐ, L</td>
<td>6.022 140 857(74) × 10²³</td>
<td>mol⁻¹</td>
<td>1.2 × 10⁻⁸</td>
</tr>
<tr>
<td>Faraday constant NₑF</td>
<td>F</td>
<td>96.485 352 89(59)</td>
<td>C mol⁻¹</td>
<td>6.2 × 10⁻⁹</td>
</tr>
<tr>
<td>Molar gas constant</td>
<td>R</td>
<td>8.314 599(8)</td>
<td>J mol⁻¹ K⁻¹</td>
<td>5.7 × 10⁻⁷</td>
</tr>
<tr>
<td>Boltzmann constant R/Nₐ</td>
<td>k</td>
<td>1.380 648 52(79) × 10⁻²³</td>
<td>J K⁻¹</td>
<td>5.7 × 10⁻⁷</td>
</tr>
<tr>
<td>Stefan-Boltzmann constant (π²/60)k²/hc²</td>
<td>σ</td>
<td>5.670 367(13) × 10⁻⁸</td>
<td>W m⁻² K⁻⁴</td>
<td>2.5 × 10⁻⁶</td>
</tr>
<tr>
<td>Electron volt (e/C) J</td>
<td>eV</td>
<td>1.602 176 6208(98) × 10⁻¹⁹</td>
<td>J</td>
<td>6.1 × 10⁻⁹</td>
</tr>
<tr>
<td>(Unified) atomic mass unit ṁₑ/mₑ⁻¹(¹²C)</td>
<td>u</td>
<td>1.660 539 040(20) × 10⁻²⁷</td>
<td>kg</td>
<td>1.2 × 10⁻⁸</td>
</tr>
</tbody>
</table>

Together with (i) those constants that have exact values such as μ₀ and c; and (ii) the values of mₑ, Gₚ, and sin²θₑ given in Sec. XII of this report. See Sec. V.B of CODATA-98 for details.

B. Tables of values

Tables XXXII, XXXIII, XXXIV, XXXV, XXXVI, XXXVII, XXXVIII, and XXXIX give the 2014 CODATA recommended values of the basic constants and conversion factors of physics and chemistry and related quantities. They are identical in form and content to their 2010 counterparts in that no constants are added or deleted.

It should be noted that the values of the four helion-related constants are calculated from the adjusted constant μₑ/μₚ and the theoretically predicted shielding correction σₚ = 59.967 43(10) × 10⁻⁶ due to Rudzinski, Puchalski, and Pachucki (2009) using the relation μₑ = μ₀(1 − σₚ); see Sec. VI.A.

Table XXXII is a highly abbreviated list of the values of the constants and conversion factors most commonly used. Table XXXIII is a much more extensive list of values categorized as follows: UNIVERSAL; ELECTROMAGNETIC; ATOMIC AND NUCLEAR; and PHYSICOCHEMICAL. The ATOMIC AND NUCLEAR category is subdivided into 11 subcategories: General; Electroweak; Electron; e; Muon, μ; Tau, τ; Proton, p; Neutron, n; Deuteron, d; Triton, t; Helion, h; and Alpha particle, α. Table XXXIV gives the variances, covariances, and correlation coefficients of a selected group of constants. (Use of the covariance matrix is discussed in Appendix E of CODATA-98.) Table XXXV gives the internationally adopted values of various quantities; Table XXXVI lists the values of a number of x-ray-related quantities; Table XXXVII lists the values of various non-SI units; and Tables XXXVIII and XXXIX give the values of various energy equivalents.

All of the values given in these tables are available on the website of the Fundamental Constants Data Center of the NIST Physical Measurement Laboratory at http://physics.nist.gov/ constants. In fact, this electronic version of the 2014 CODATA recommended values of the constants enables users to obtain the correlation coefficient of any two constants listed in the tables. It also allows users to automatically convert the value of an energy-related quantity expressed in one unit to the corresponding value expressed in another unit (in essence, an automated version of Tables XXXVIII and XXXIX).

XV. Summary and Conclusion

Here we (i) compare the 2014 to the 2010 recommended values of the constants and identify those new results that have contributed most to the changes in the 2010 values; (ii) present several conclusions that can be drawn from the 2014 recommended values and the input data from which they are obtained; and (iii) identify new experimental and theoretical work that can advance our knowledge of the values of the constants. Topic (iii) is relevant to the plan of the 26th General Conference on Weights and Measures (CGPM) to adopt at its
<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Numerical value</th>
<th>Unit</th>
<th>Relative std. uncert. ( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>UNIVERSAL</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Speed of light in vacuum</td>
<td>( c, c_0 )</td>
<td>299 792 458</td>
<td>( \text{m s}^{-1} )</td>
<td>Exact</td>
</tr>
<tr>
<td>Magnetic constant</td>
<td>( \mu_0 )</td>
<td>( 4\pi \times 10^{-7} )</td>
<td>( \text{N A}^{-2} )</td>
<td>Exact</td>
</tr>
<tr>
<td>Electric constant</td>
<td>( \varepsilon_0 )</td>
<td>( 8.854 187 817 \times \times 10^{-12} )</td>
<td>( \text{F m}^{-1} )</td>
<td>Exact</td>
</tr>
<tr>
<td>Characteristic impedance of vacuum ( \mu_0 c )</td>
<td>( Z_0 )</td>
<td>376 730 313 461</td>
<td>( \Omega )</td>
<td>Exact</td>
</tr>
<tr>
<td>Newtonian constant of gravitation</td>
<td>( G )</td>
<td>( 6.674 08(31) \times \times 10^{-11} )</td>
<td>( \text{m}^3 \text{kg}^{-1} \text{s}^{-2} )</td>
<td>( 4.7 \times 10^{-5} )</td>
</tr>
<tr>
<td>Planck constant</td>
<td>( h )</td>
<td>6.626 070 400(81)</td>
<td>( \text{J} )</td>
<td>( 1.2 \times 10^{-8} )</td>
</tr>
<tr>
<td>Planck mass ( (\hbar/G)^{1/2} )</td>
<td>( m_p )</td>
<td>1.105 571 800(13)</td>
<td>( \text{J} )</td>
<td>( 1.2 \times 10^{-8} )</td>
</tr>
<tr>
<td>Planck temperature ( (\hbar c^3/G)^{1/2}/k )</td>
<td>( T_p )</td>
<td>1.220 910(29)</td>
<td>( \text{GeV} )</td>
<td>( 2.3 \times 10^{-5} )</td>
</tr>
<tr>
<td>Planck length ( h/\sqrt{m_p c} = (\hbar G/c^3)^{1/2} )</td>
<td>( l_p )</td>
<td>1.616 229(38)</td>
<td>( \text{m} )</td>
<td>( 2.3 \times 10^{-5} )</td>
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<tr>
<td>Planck time ( l_p c/(\hbar G/c^3)^{1/2} )</td>
<td>( t_p )</td>
<td>5.391 16(13)</td>
<td>( \text{s} )</td>
<td>( 2.3 \times 10^{-5} )</td>
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<tr>
<td><strong>ELECTROMAGNETIC</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elementary charge</td>
<td>( e )</td>
<td>1.602 176 6208(98)</td>
<td>( \text{C} )</td>
<td>( 6.1 \times 10^{-9} )</td>
</tr>
<tr>
<td>Magnetic flux quantum ( h/2e )</td>
<td>( \Phi_0 )</td>
<td>2.067 833 831(13)</td>
<td>( \text{Wb} )</td>
<td>( 6.1 \times 10^{-9} )</td>
</tr>
<tr>
<td>Conductance quantum ( 2e^2/h )</td>
<td>( G_0 )</td>
<td>7.748 091 7310(18)</td>
<td>( \Omega )</td>
<td>( 2.3 \times 10^{-10} )</td>
</tr>
<tr>
<td>inverse of conductance quantum</td>
<td>( G_0^{-1} )</td>
<td>12 906 403 7278(29)</td>
<td>( \Omega )</td>
<td>( 2.3 \times 10^{-10} )</td>
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<tr>
<td>Josephson constant ( *2e/h )</td>
<td>( K_j )</td>
<td>483 597 8525(30)</td>
<td>( \text{Hz} )</td>
<td>( 6.1 \times 10^{-9} )</td>
</tr>
<tr>
<td>von Klitzing constant ( *h/e^2 = \mu_0 c/2\alpha )</td>
<td>( R_K )</td>
<td>25 812 807 4555(59)</td>
<td>( \Omega )</td>
<td>( 2.3 \times 10^{-10} )</td>
</tr>
<tr>
<td>Bohr magneton ( e\hbar/2m_e )</td>
<td>( \mu_B )</td>
<td>927 400 9994(57)</td>
<td>( \text{JT}^{-1} )</td>
<td>( 6.2 \times 10^{-9} )</td>
</tr>
<tr>
<td>Nuclear magneton ( e\hbar/2m_p )</td>
<td>( \mu_N )</td>
<td>5.050 783 699(31)</td>
<td>( \text{JT}^{-1} )</td>
<td>( 6.2 \times 10^{-9} )</td>
</tr>
<tr>
<td><strong>ATOMIC AND NUCLEAR</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fine-structure constant ( e^2/4\pi\varepsilon_0\hbar c )</td>
<td>( \alpha )</td>
<td>7.297 352 5664(17)</td>
<td>( 2.3 \times 10^{-10} )</td>
<td></td>
</tr>
<tr>
<td>inverse fine-structure constant</td>
<td>( \alpha^{-1} )</td>
<td>137.035 999 139(31)</td>
<td>( 2.3 \times 10^{-10} )</td>
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<tr>
<td>Rydberg constant ( a^2m_e c/2\hbar )</td>
<td>( R_a )</td>
<td>10 973 731 568 508(65)</td>
<td>( \text{m}^{-1} )</td>
<td>( 5.9 \times 10^{-12} )</td>
</tr>
<tr>
<td>Bohr radius ( a/4\pi R_a )</td>
<td>( \alpha_0 )</td>
<td>0.529 177 210 67(12)</td>
<td>( \text{m} )</td>
<td>( 2.3 \times 10^{-10} )</td>
</tr>
<tr>
<td>Hartree energy ( e^2/4\pi\varepsilon_0\hbar c ) ( = 2R_a\hbar c = a^2m_e c^2 )</td>
<td>( E_h )</td>
<td>4.359 744 650(54)</td>
<td>( \text{J} )</td>
<td>( 1.2 \times 10^{-8} )</td>
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<tr>
<td>Quantum of circulation</td>
<td>( h/2m_e )</td>
<td>3.636 947 5486(17)</td>
<td>( \text{m}^2 \text{s}^{-1} )</td>
<td>( 4.5 \times 10^{-10} )</td>
</tr>
<tr>
<td></td>
<td>( h/m_e )</td>
<td>7.273 895 079(33)</td>
<td>( \text{m}^2 \text{s}^{-1} )</td>
<td>( 4.5 \times 10^{-10} )</td>
</tr>
<tr>
<td><strong>Electroweak</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fermi coupling constant</td>
<td>( G_F/(\hbar c)^3 )</td>
<td>1.166 3787(6)</td>
<td>( \text{GeV}^{-2} )</td>
<td>( 5.1 \times 10^{-7} )</td>
</tr>
<tr>
<td>Weak mixing angle ( \theta_W )</td>
<td>( \sin^2 \theta_W = s_W^2 = 1 - (m_w/m_Z)^2 )</td>
<td>( 0.2223(21) )</td>
<td>( 9.5 \times 10^{-3} )</td>
<td></td>
</tr>
<tr>
<td>Electron, ( e^- )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electron mass</td>
<td>( m_e )</td>
<td>9.109 383 56(11)</td>
<td>( \text{kg} )</td>
<td>( 1.2 \times 10^{-8} )</td>
</tr>
<tr>
<td></td>
<td>( 5.485 799 090(70) \times 10^{-4} )</td>
<td>( u )</td>
<td>( 2.9 \times 10^{-11} )</td>
<td></td>
</tr>
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</table>
### CODATA RECOMMENDED VALUES: 2014

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Numerical value</th>
<th>Unit</th>
<th>Relative std. uncert. $u_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy equivalent</td>
<td>$m_e c^2$</td>
<td>$8.187 \times 10^5 (65) \times 10^{-14}$</td>
<td>J</td>
<td>$1.2 \times 10^{-8}$</td>
</tr>
<tr>
<td>Electron-muon mass ratio</td>
<td>$m_e/m_{\mu}$</td>
<td>$4.836 \times 10^{-3}$</td>
<td>kg</td>
<td>$2.2 \times 10^{-8}$</td>
</tr>
<tr>
<td>Electron-tau mass ratio</td>
<td>$m_e/m_\tau$</td>
<td>$9.0 \times 10^{-5}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electron-proton mass ratio</td>
<td>$m_e/m_p$</td>
<td>$5.446 \times 10^{-3}$</td>
<td>kg</td>
<td>$9.5 \times 10^{-11}$</td>
</tr>
<tr>
<td>Electron-neutron mass ratio</td>
<td>$m_e/m_n$</td>
<td>$4.9 \times 10^{-10}$</td>
<td>kg</td>
<td>$4.9 \times 10^{-11}$</td>
</tr>
<tr>
<td>Electron-deuteron mass ratio</td>
<td>$m_e/m_d$</td>
<td>$3.5 \times 10^{-11}$</td>
<td>kg</td>
<td>$4.6 \times 10^{-11}$</td>
</tr>
<tr>
<td>Electron-triton mass ratio</td>
<td>$m_e/m_t$</td>
<td>$3.3 \times 10^{-11}$</td>
<td>kg</td>
<td>$4.9 \times 10^{-11}$</td>
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<tr>
<td>Electron-helion mass ratio</td>
<td>$m_e/m_H$</td>
<td>$3.1 \times 10^{-11}$</td>
<td>kg</td>
<td>$2.2 \times 10^{-8}$</td>
</tr>
<tr>
<td>Electron to alpha particle mass ratio</td>
<td>$m_e/m_\alpha$</td>
<td>$1.2 \times 10^{-11}$</td>
<td>kg</td>
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</tr>
<tr>
<td>Electron charge to mass quotient</td>
<td>$-e/m_e$</td>
<td>$-1.758 \times 10^{11}$</td>
<td>C kg$^{-1}$</td>
<td>$6.2 \times 10^{-9}$</td>
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<tr>
<td>Electron molar mass $N_A m_e$</td>
<td>$M_e$, $M_e$</td>
<td>$5.485 \times 10^{-3}$</td>
<td>kg mol$^{-1}$</td>
<td>$2.9 \times 10^{-11}$</td>
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<tr>
<td>Compton wavelength $h/m_e c$</td>
<td>$\lambda C$</td>
<td>$2.426 \times 10^{-12}$</td>
<td>m</td>
<td>$4.5 \times 10^{-10}$</td>
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<tr>
<td>Compton wavelength $h/m_e c$</td>
<td>$\lambda C/2\pi = \alpha_a = \alpha c^2/4\pi R_m$</td>
<td>$386.159 \times 10^{-15}$</td>
<td>m</td>
<td>$4.5 \times 10^{-10}$</td>
</tr>
<tr>
<td>Classical electron radius $a_0^2$</td>
<td>$r_e$</td>
<td>$2.8179 \times 10^{-15}$</td>
<td>m</td>
<td>$6.8 \times 10^{-10}$</td>
</tr>
<tr>
<td>Thomson cross section $(8\pi/3)(a_0^2 c^2)$</td>
<td>$\sigma_e$</td>
<td>$0.66524 \times 10^{-28}$</td>
<td>m$^2$</td>
<td>$1.4 \times 10^{-9}$</td>
</tr>
<tr>
<td>Electron magnetic moment</td>
<td>$\mu_e$</td>
<td>$9.284 \times 10^{-26}$</td>
<td>J T$^{-1}$</td>
<td>$6.2 \times 10^{-9}$</td>
</tr>
<tr>
<td>to Bohr magneton ratio</td>
<td>$\mu_e/\mu_B$</td>
<td>$1.001 \times 10^{-11}$</td>
<td></td>
<td>$2.7 \times 10^{-13}$</td>
</tr>
<tr>
<td>to nuclear magneton ratio</td>
<td>$\mu_e/\mu_N$</td>
<td>$1.838 \times 10^{-17}$</td>
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<td>$9.5 \times 10^{-11}$</td>
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<tr>
<td>Electron magnetic-moment anomaly</td>
<td>$\mu_e/\mu_B - 1$</td>
<td>$1.159 \times 10^{12}$</td>
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<td>$2.3 \times 10^{-10}$</td>
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<tr>
<td>Electron $g$-factor $-2(1 + a_o)$</td>
<td>$g_e$</td>
<td>$2.002 \times 10^{12}$</td>
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<td>$2.6 \times 10^{-13}$</td>
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<tr>
<td>Electron-muon magnetic-moment ratio</td>
<td>$\mu_e/\mu_{\mu}$</td>
<td>$206.766 \times 10^{12}$</td>
<td></td>
<td>$2.2 \times 10^{-8}$</td>
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<tr>
<td>Electron-proton magnetic-moment ratio</td>
<td>$\mu_e/\mu_p$</td>
<td>$-658.210 \times 10^{12}$</td>
<td></td>
<td>$3.0 \times 10^{-9}$</td>
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<tr>
<td>Electron to proton magnetic-moment ratio</td>
<td>$\mu_e/\mu_p$</td>
<td>$-658.227 \times 10^{12}$</td>
<td></td>
<td>$1.1 \times 10^{-8}$</td>
</tr>
<tr>
<td>Electron-muon magnetic-moment ratio</td>
<td>$\mu_e/\mu_{\mu}$</td>
<td>$960.920 \times 10^{23}$</td>
<td></td>
<td>$2.4 \times 10^{-7}$</td>
</tr>
<tr>
<td>Electron-deuteron magnetic-moment ratio</td>
<td>$\mu_e/\mu_d$</td>
<td>$-2134.923 \times 10^{12}$</td>
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<td>$5.5 \times 10^{-9}$</td>
</tr>
<tr>
<td>Electron to proton magnetic-moment ratio</td>
<td>$\mu_e/\mu_p$</td>
<td>$864.058 \times 10^{12}$</td>
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<td>$1.2 \times 10^{-8}$</td>
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<tr>
<td>Electron gyromagnetic ratio $2\mu_e/c$</td>
<td>$\gamma_e$</td>
<td>$1.760 \times 10^{-11}$</td>
<td></td>
<td>$6.2 \times 10^{-9}$</td>
</tr>
<tr>
<td>to nuclear magneton ratio</td>
<td>$\gamma_e/2\pi$</td>
<td>$28.024 \times 10^{17}$</td>
<td>MHz T$^{-1}$</td>
<td>$6.2 \times 10^{-9}$</td>
</tr>
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</table>

**Muon, $\mu$**

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Numerical value</th>
<th>Unit</th>
<th>Relative std. uncert. $u_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muon mass</td>
<td>$m_\mu$</td>
<td>$1.883 \times 10^{-28}$</td>
<td>kg</td>
<td>$2.5 \times 10^{-8}$</td>
</tr>
<tr>
<td>energy equivalent</td>
<td>$m_\mu c^2$</td>
<td>$1.692 \times 10^{-11}$</td>
<td>J</td>
<td>$2.5 \times 10^{-8}$</td>
</tr>
<tr>
<td>Muon-electron mass ratio</td>
<td>$m_\mu/m_e$</td>
<td>$105.658 \times 10^{24}$</td>
<td>MeV</td>
<td>$2.3 \times 10^{-8}$</td>
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<tr>
<td>Muon-tau mass ratio</td>
<td>$m_\mu/m_\tau$</td>
<td>$5.946 \times 10^{-2}$</td>
<td>kg</td>
<td>$9.0 \times 10^{-5}$</td>
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<tr>
<td>Muon-proton mass ratio</td>
<td>$m_\mu/m_p$</td>
<td>$5.112 \times 10^{-2}$</td>
<td>kg</td>
<td>$2.2 \times 10^{-8}$</td>
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<tr>
<td>Muon-neutron mass ratio</td>
<td>$m_\mu/m_n$</td>
<td>$5.112 \times 10^{-2}$</td>
<td>kg</td>
<td>$9.0 \times 10^{-5}$</td>
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<tr>
<td>Muon molar mass $N_A m_\mu$</td>
<td>$M(\mu)$, $M_\mu$</td>
<td>$1.134 \times 10^{-2}$</td>
<td>kg mol$^{-1}$</td>
<td>$2.2 \times 10^{-8}$</td>
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<tr>
<td>Muon Compton wavelength $h/m_\mu c$</td>
<td>$\lambda_{\mu}$, $\lambda_{\mu}c/2\pi$</td>
<td>$1.867 \times 10^{-2}$</td>
<td>m</td>
<td>$2.2 \times 10^{-8}$</td>
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<tr>
<td>Muon magnetic moment</td>
<td>$\mu_\mu$</td>
<td>$-4.490 \times 10^{-26}$</td>
<td>J T$^{-1}$</td>
<td>$2.3 \times 10^{-8}$</td>
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<tr>
<td>to Bohr magneton ratio</td>
<td>$\mu_\mu/\mu_B$</td>
<td>$-4.841 \times 10^{-26}$</td>
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<tr>
<td>to nuclear magneton ratio</td>
<td>$\mu_\mu/\mu_N$</td>
<td>$-8.890 \times 10^{-26}$</td>
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**Muon-magnetic-moment anomaly $[\mu_\mu]/(e\hbar/2m_\mu) - 1$**

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Numerical value</th>
<th>Unit</th>
<th>Relative std. uncert. $u_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muon $g$-factor $-2(1 + a_o)$</td>
<td>$g_\mu$</td>
<td>$-2.002 \times 10^{13}$</td>
<td></td>
<td>$6.3 \times 10^{-10}$</td>
</tr>
<tr>
<td>Muon-proton magnetic-moment ratio</td>
<td>$\mu_\mu/\mu_p$</td>
<td>$-3.183 \times 10^{12}$</td>
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<td>$2.2 \times 10^{-8}$</td>
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**Tau, $\tau$**

<table>
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<th>Quantity</th>
<th>Symbol</th>
<th>Numerical value</th>
<th>Unit</th>
<th>Relative std. uncert. $u_i$</th>
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<tbody>
<tr>
<td>Tau mass</td>
<td>$m_\tau$</td>
<td>$1.367 \times 10^{-27}$</td>
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<td>$9.0 \times 10^{-5}$</td>
</tr>
<tr>
<td>energy equivalent</td>
<td>$m_\tau c^2$</td>
<td>$1.907 \times 10^{-10}$</td>
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<td>$9.0 \times 10^{-5}$</td>
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</tbody>
</table>
### TABLE XXXIII. The CODATA recommended values of the fundamental constants of physics and chemistry based on the 2014 adjustment—Continued

| Quantity | Symbol | Numerical value | Unit | Relative std. uncert. $u_i$
|----------|-------|-----------------|------|----------------|
| Tau-electron mass ratio | $\mu_e/m_e$ | 3.477(15) | $10^{-8}$ | 9.0
| Tau-muon mass ratio | $\mu_u/m_u$ | 18.87(15) | $10^{-8}$ | 9.0
| Tau-proton mass ratio | $\mu_p/m_p$ | 1.893(17) | $10^{-8}$ | 9.0
| Tau-neutron mass ratio | $\mu_n/m_n$ | 1.891(17) | $10^{-8}$ | 9.0
| Tau molar mass $N_a m_a$ | $M(\tau), M_e$ | $1.907 40(17) \times 10^{-3}$ | kg mol$^{-1}$ | 9.0
| Tau Compton wavelength $\lambda_{\text{CS}}$ | $\lambda_{\text{CS}}$ | 0.697787(63) | $10^{-15}$ | 9.0
| $\lambda_{\text{CS}}/2\pi$ | $\lambda_{\text{CS}}/2\pi$ | 0.111 056(10) | $10^{-15}$ | 9.0
| Proton mass | $m_p$ | $1.672 618 988(12) \times 10^{-27}$ | kg | 1.2
| energy equivalent | $m_p c^2$ | 1.503 277 539(18) | $10^{-10}$ | 9.0
| to Bohr magnetron ratio | $m_p/m_B$ | 938.272 0813(58) | MeV | 6.2
| Proton-neutron mass ratio | $m_p/m_n$ | 1.891 11(17) | $10^{-8}$ | 9.0
| Proton-tau mass ratio | $m_p/m_{\tau}$ | 1.891 11(17) | $10^{-8}$ | 9.0
| Proton-electron mass ratio | $m_p/m_e$ | 1836.152 673(89) | $10^{-10}$ | 9.5
| Proton-muon mass ratio | $m_p/m_{\mu}$ | 8.880 243 8(20) | $10^{-8}$ | 2.2
| Proton-neutron mass ratio | $m_p/m_n$ | 0.998 623 478 44(51) | $10^{-10}$ | 5.1
| Proton charge-to-mass quotient | $e/m_p$ | 9.578 833 226(59) | $10^{-10}$ | 2.6
| Proton molar mass $N_a m_p$ | $M(p), M_p$ | 1.007 276 466 879(91) | $10^{-3}$ | 9.0
| Proton Compton wavelength $\lambda_{\text{CS}}$ | $\lambda_{\text{CS}}$ | 0.210 308 910 109(97) | $10^{-15}$ | 9.0
| $\lambda_{\text{CS}}/2\pi$ | $\lambda_{\text{CS}}/2\pi$ | 0.210 308 910 109(97) | $10^{-15}$ | 9.0
| Proton rms charge radius | $r_p$ | 0.8753(61) | $10^{-15}$ | 7.0
| Proton magnetic moment | $\mu_p$ | 1.410 606 783(97) | $10^{-26}$ | 6.9
| to Bohr magnetron ratio | $\mu_p/m_B$ | 1.521 032 203(46) | $10^{-3}$ | 3.0
| to nuclear magnetron ratio | $\mu_p/m_N$ | 2.792 847 3508(85) | $10^{-3}$ | 3.0
| Proton $g$-factor | $g_p$ | 5.858 694 702(17) | $10^{-3}$ | 3.0
| Proton-neutron magnetic-moment ratio | $\mu_p/m_n$ | $-1.459 898 05(34)$ | $10^{-7}$ | 2.4
| Shielded proton magnetic moment | (H$_2$O, sphere, 25 °C) | $\mu_p^s$ | 1.410 570 547(18) | $10^{-26}$ | 1.3
| to Bohr magnetron ratio | $\mu_p^s/m_B$ | 1.520 993 128(17) | $10^{-3}$ | 1.1
| to nuclear magnetron ratio | $\mu_p^s/m_N$ | 2.792 775 600(30) | $10^{-3}$ | 1.1
| Proton magnetic shielding correction | 1 $-\mu_p^s/m_p$ (H$_2$O, sphere, 25 °C) | $\alpha_p^s$ | 25.69(11) | $10^{-4}$ | 4.4
| Proton gyromagnetic ratio $2\mu_p/e$ | $\gamma_p$ | 2.675 221 900(18) | $10^{-8}$ | 6.9
| $\gamma_p/2\pi$ | $\gamma_p/2\pi$ | 42.577 478 92(29) | $10^{-1}$ | 6.9
| Shielded proton gyromagnetic ratio | (H$_2$O, sphere, 25 °C) | $\gamma_p^s$ | 2.675 153 171(33) | $10^{-8}$ | 1.3
| $\gamma_p^s/2\pi$ | $\gamma_p^s/2\pi$ | 42.576 385 07(53) | $10^{-1}$ | 1.3

<table>
<thead>
<tr>
<th>Neutron</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>
| Neutron mass | $m_n$ | 1.674 272 417(21) | $10^{-27}$ | 1.2
| energy equivalent | $m_n c^2$ | 1.505 349 739(19) | $10^{-10}$ | 9.0
| Neutron-electron mass ratio | $m_n/m_e$ | 1838.683 661 58(90) | $10^{-10}$ | 4.9
| Neutron-muon mass ratio | $m_n/m_{\mu}$ | 8.992 484 08(20) | $10^{-8}$ | 2.2
| Neutron-neckron mass ratio | $m_n/m_n$ | 0.528 700(48) | $10^{-8}$ | 9.0
| Neutron-proton mass ratio | $m_n/m_p$ | 1.001 378 418 98(51) | $10^{-10}$ | 5.1
| Neutron-proton mass difference | $m_n - m_p$ | 2.305 573 77(85) | $10^{-30}$ | 3.7
| energy equivalent | $(m_n - m_p)c^2$ | 2.072 146 37(76) | $10^{-13}$ | 3.7
| Neutron molar mass $N_a m_n$ | $M(n), M_n$ | 1.008 649 185 84(49) | $10^{-3}$ | 4.9
| Neutron Compton wavelength $\lambda_{\text{CS}}$ | $\lambda_{\text{CS}}$ | 1.319 590 904 81(88) | $10^{-15}$ | 6.7
| $\lambda_{\text{CS}}/2\pi$ | $\lambda_{\text{CS}}/2\pi$ | 0.210 019 415 36(14) | $10^{-15}$ | 6.7
| Neutron magnetic moment | $\mu_n$ | $-0.966 236 50(23)$ | $10^{-10}$ | 2.4
| to Bohr magnetron ratio | $\mu_n/m_B$ | $-0.104 875 63(25)$ | $10^{-3}$ | 2.4
| to nuclear magnetron ratio | $\mu_n/m_N$ | $-1.913 042 73(45)$ | $10^{-7}$ | 2.4

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Numerical value</th>
<th>Unit</th>
<th>Relative std. uncert. u_μ</th>
</tr>
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<tbody>
<tr>
<td>Neutron g-factor 2μ_n/μ_N</td>
<td>g_n</td>
<td>-3.826 085 45(90)</td>
<td>2.4 × 10^{-7}</td>
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</tr>
<tr>
<td>Neutron-electron magnetic-moment ratio</td>
<td>μ_n/μ_e</td>
<td>1.040 668 82(25) × 10^{-3}</td>
<td>2.4 × 10^{-7}</td>
<td></td>
</tr>
<tr>
<td>Neutron-proton magnetic-moment ratio</td>
<td>μ_n/μ_p</td>
<td>-0.684 979 34(16)</td>
<td>2.4 × 10^{-7}</td>
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</tr>
<tr>
<td>Neutron to shielded proton magnetic-moment ratio (H_2O, sphere, 25 °C)</td>
<td>μ_n/μ'_p</td>
<td>-0.684 996 94(16)</td>
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<tr>
<td>Neutron gyromagnetic ratio 2μ_n/ℏ</td>
<td>γ_n</td>
<td>1.832 471 72(43) × 10^{8}</td>
<td>s^{-1} T^{-1}</td>
<td>2.4 × 10^{-7}</td>
</tr>
<tr>
<td></td>
<td>γ_n/2π</td>
<td>29.164 6933(69)</td>
<td>MHz T^{-1}</td>
<td>2.4 × 10^{-7}</td>
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<tr>
<td>Deuteron, d</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deuteron mass</td>
<td>m_d</td>
<td>3.343 583 719(41) × 10^{-27}</td>
<td></td>
<td>1.2 × 10^{-8}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.013 533 212 745(40)</td>
<td>kg</td>
<td>2.0 × 10^{-11}</td>
</tr>
<tr>
<td>energy equivalent</td>
<td>m_e c^2</td>
<td>5.002 063 183(37) × 10^{-10}</td>
<td></td>
<td>1.2 × 10^{-8}</td>
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<tr>
<td></td>
<td></td>
<td>1875.612 928(12)</td>
<td>J</td>
<td>6.2 × 10^{-9}</td>
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<tr>
<td>Deuteron-electron mass ratio</td>
<td>m_d/μ_e</td>
<td>3670.482 967 85(13)</td>
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<tr>
<td>Deuteron-proton mass ratio</td>
<td>m_d/μ_p</td>
<td>1.999 007 500 87(19)</td>
<td>9.3 × 10^{-11}</td>
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</tr>
<tr>
<td>Deuteron molar mass N_d m_d</td>
<td>M_d</td>
<td>2.013 533 212 745(40) × 10^{-3}</td>
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<td>2.0 × 10^{-11}</td>
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<tr>
<td>Deuteron rms charge radius</td>
<td>r_d</td>
<td>2.141(25) × 10^{-15}</td>
<td>m</td>
<td>1.2 × 10^{-3}</td>
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<tr>
<td>Deuteron magnetic moment</td>
<td>μ_d</td>
<td>0.433 073 5040(36) × 10^{-26}</td>
<td>J T^{-1}</td>
<td>8.3 × 10^{-9}</td>
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<tr>
<td>to Bohr magneton ratio</td>
<td>μ_d/μ_B</td>
<td>0.466 975 4554(26) × 10^{-3}</td>
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<td>5.5 × 10^{-9}</td>
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<tr>
<td>to nuclear magneton ratio</td>
<td>μ_d/μ_N</td>
<td>0.857 438 2311(48)</td>
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<td></td>
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<tr>
<td>Deuteron g-factor μ_d/μ_N</td>
<td>g_d</td>
<td>0.857 438 2311(48)</td>
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<td>Deuteron-electron magnetic-moment ratio</td>
<td>μ_d/μ_e</td>
<td>-4.664 345 535(26) × 10^{-4}</td>
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<td>5.5 × 10^{-9}</td>
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<td>Deuteron-neutron magnetic-moment ratio</td>
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<td>Triton, t</td>
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<td></td>
</tr>
<tr>
<td>Triton mass</td>
<td>m_t</td>
<td>5.007 356 665(62) × 10^{-27}</td>
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<td>1.2 × 10^{-8}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.015 500 716 32(11)</td>
<td>kg</td>
<td>3.6 × 10^{-11}</td>
</tr>
<tr>
<td>energy equivalent</td>
<td>m_e c^2</td>
<td>4.500 387 735(55) × 10^{-10}</td>
<td></td>
<td>1.2 × 10^{-8}</td>
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<tr>
<td></td>
<td></td>
<td>2808.921 112(17)</td>
<td>J</td>
<td>6.2 × 10^{-9}</td>
</tr>
<tr>
<td>Triton-electron mass ratio</td>
<td>m_t/μ_e</td>
<td>5.496 921 535 88(26)</td>
<td>4.6 × 10^{-11}</td>
<td></td>
</tr>
<tr>
<td>Triton-proton mass ratio</td>
<td>m_t/μ_p</td>
<td>2.993 717 033 48(22)</td>
<td>7.5 × 10^{-11}</td>
<td></td>
</tr>
<tr>
<td>Triton molar mass N_t m_t</td>
<td>M_t</td>
<td>3.015 500 716 32(11) × 10^{-3}</td>
<td>kg mol^{-1}</td>
<td>3.6 × 10^{-11}</td>
</tr>
<tr>
<td>Triton magnetic moment</td>
<td>μ_t</td>
<td>1.504 609 503(12) × 10^{-26}</td>
<td>J T^{-1}</td>
<td>7.8 × 10^{-9}</td>
</tr>
<tr>
<td>to Bohr magneton ratio</td>
<td>μ_t/μ_B</td>
<td>1.622 393 6616(76) × 10^{-3}</td>
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<td>4.7 × 10^{-9}</td>
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<tr>
<td>to nuclear magneton ratio</td>
<td>μ_t/μ_N</td>
<td>2.978 962 460(14)</td>
<td>4.7 × 10^{-9}</td>
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</tr>
<tr>
<td>Triton g-factor 2μ_t/μ_N</td>
<td>g_t</td>
<td>5.957 924 920(28)</td>
<td>4.7 × 10^{-9}</td>
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<td>Helion, h</td>
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<td></td>
<td></td>
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<tr>
<td>Helion mass</td>
<td>m_h</td>
<td>5.006 412 700(62) × 10^{-27}</td>
<td></td>
<td>1.2 × 10^{-8}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.014 932 246 73(12)</td>
<td>kg</td>
<td>3.9 × 10^{-11}</td>
</tr>
<tr>
<td>energy equivalent</td>
<td>m_e c^2</td>
<td>4.499 539 341(55) × 10^{-10}</td>
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<td>1.2 × 10^{-8}</td>
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<td></td>
<td>2808.391 586(17)</td>
<td>J</td>
<td>6.2 × 10^{-9}</td>
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<td>Helion-electron mass ratio</td>
<td>m_h/μ_e</td>
<td>5.495 885 279 22(27)</td>
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</tr>
<tr>
<td>Helion-proton mass ratio</td>
<td>m_h/μ_p</td>
<td>2.993 152 670 46(29)</td>
<td>9.6 × 10^{-11}</td>
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</tr>
<tr>
<td>Helion molar mass N_h m_h</td>
<td>M_h</td>
<td>3.014 932 246 73(12) × 10^{-3}</td>
<td>kg mol^{-1}</td>
<td>3.9 × 10^{-11}</td>
</tr>
<tr>
<td>Helion magnetic moment</td>
<td>μ_h</td>
<td>-1.074 617 522(14) × 10^{-26}</td>
<td>J T^{-1}</td>
<td>1.3 × 10^{-8}</td>
</tr>
<tr>
<td>to Bohr magneton ratio</td>
<td>μ_h/μ_B</td>
<td>-1.158 749 958(14) × 10^{-3}</td>
<td></td>
<td>1.2 × 10^{-8}</td>
</tr>
<tr>
<td>to nuclear magneton ratio</td>
<td>μ_h/μ_N</td>
<td>-2.127 625 308(25)</td>
<td>1.2 × 10^{-8}</td>
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</tr>
<tr>
<td>Helion g-factor 2μ_h/μ_N</td>
<td>g_h</td>
<td>-4.255 250 616(50)</td>
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<tr>
<td>Shielded helion magnetic moment</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(gas, sphere, 25 °C)</td>
<td>μ'_h</td>
<td>-1.074 553 080(14) × 10^{-26}</td>
<td>J T^{-1}</td>
<td>1.3 × 10^{-8}</td>
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<tr>
<td>to Bohr magneton ratio</td>
<td>μ'_h/μ_B</td>
<td>-1.158 671 471(14) × 10^{-3}</td>
<td></td>
<td>1.2 × 10^{-8}</td>
</tr>
<tr>
<td>to nuclear magneton ratio</td>
<td>μ'_h/μ_N</td>
<td>-2.127 497 720(25)</td>
<td>1.2 × 10^{-8}</td>
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</tr>
<tr>
<td>Shielded helion to proton magnetic-moment ratio (gas, 25 °C)</td>
<td>μ'_h/μ_p</td>
<td>-0.761 766 5603(92)</td>
<td>1.2 × 10^{-8}</td>
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</tr>
<tr>
<td>Shielded helion to shielded proton magnetic-moment ratio (gas/H_2O, spheres, 25 °C)</td>
<td>μ'_h/μ'_p</td>
<td>-0.761 786 1313(33)</td>
<td>4.3 × 10^{-9}</td>
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</table>
Table XXXIII. The CODATA recommended values of the fundamental constants of physics and chemistry based on the 2014 adjustment—Continued

<table>
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<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Numerical value</th>
<th>Unit</th>
<th>Relative std. uncert. u₀</th>
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<tr>
<td>Shielded helion gyromagnetic ratio</td>
<td>γ₀'</td>
<td>2.037 894 585(27) × 10⁶</td>
<td>s⁻¹ T⁻¹</td>
<td>1.3 × 10⁻⁴</td>
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<tr>
<td></td>
<td>γ₀'/2π</td>
<td>32.434 099 66(43)</td>
<td>MHz T⁻¹</td>
<td>1.3 × 10⁻⁴</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alpha particle mass</td>
<td>m₀</td>
<td>6.644 657 230(82) × 10⁻²⁷</td>
<td>kg</td>
<td>1.2 × 10⁻⁸</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.001 506 179 127(63)</td>
<td>u</td>
<td>1.6 × 10⁻¹¹</td>
</tr>
<tr>
<td>Alpha particle to electron mass ratio</td>
<td>m₀/mₑ</td>
<td>7294.299 541 36(24)</td>
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<td>3.3 × 10⁻¹¹</td>
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<tr>
<td>Alpha particle to proton mass ratio</td>
<td>m₀/mₚ</td>
<td>3.972 599 689 07(36)</td>
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<td>9.2 × 10⁻¹¹</td>
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<tr>
<td>Alpha particle molar mass N₀m₀</td>
<td>M(α, M₀)</td>
<td>4.001 506 179 127(63) × 10⁻³</td>
<td>kg mol⁻¹</td>
<td>1.6 × 10⁻¹¹</td>
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<tr>
<td>Avogadro constant</td>
<td>NₐL</td>
<td>6.022 140 857(74) × 10²³</td>
<td>mol⁻¹</td>
<td>1.2 × 10⁻⁸</td>
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<tr>
<td>Atomic mass constant</td>
<td>m₀</td>
<td>1.660 539 040(20) × 10⁻²⁷</td>
<td>kg</td>
<td>1.2 × 10⁻⁸</td>
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<tr>
<td>Faraday constant</td>
<td>F</td>
<td>96.485 332 89(59)</td>
<td>C mol⁻¹</td>
<td>6.2 × 10⁻⁹</td>
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<tr>
<td>Molar Planck constant</td>
<td>Nₐh</td>
<td>3.990 312 7110(18) × 10⁻¹⁰</td>
<td>J s⁻¹ mol⁻¹</td>
<td>4.5 × 10⁻¹⁰</td>
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<tr>
<td>Molar gas constant</td>
<td>R</td>
<td>8.314 598(48)</td>
<td>J mol⁻¹ K⁻¹</td>
<td>5.7 × 10⁻⁷</td>
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<tr>
<td>Boltzmann constant R/Nₐ</td>
<td>k</td>
<td>1.380 648 52(79) × 10⁻²¹</td>
<td>J K⁻¹</td>
<td>5.7 × 10⁻⁷</td>
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<tr>
<td>Molar volume of ideal gas RT/p</td>
<td>Vₚ</td>
<td>22.710 947(13) × 10⁻³</td>
<td>m³ mol⁻¹</td>
<td>5.7 × 10⁻⁷</td>
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<tr>
<td>Loschmidt constant Nₐ/Vₚ</td>
<td>n₀</td>
<td>2.651 646(15) × 10⁻²⁵</td>
<td>m⁻³</td>
<td>5.7 × 10⁻⁷</td>
</tr>
<tr>
<td>Molar volume of ideal gas RT/p</td>
<td>Vₚ</td>
<td>22.413 962(13) × 10⁻³</td>
<td>m³ mol⁻¹</td>
<td>5.7 × 10⁻⁷</td>
</tr>
<tr>
<td>Loschmidt constant Nₐ/Vₚ</td>
<td>n₀</td>
<td>2.686 721(15) × 10⁻²⁵</td>
<td>m⁻³</td>
<td>5.7 × 10⁻⁷</td>
</tr>
<tr>
<td>Sackur-Tetrode (absolute entropy) constant</td>
<td>S₀/R</td>
<td>−1.151 7084(14)</td>
<td>J K⁻¹</td>
<td>1.2 × 10⁻⁶</td>
</tr>
<tr>
<td>Stefan-Boltzmann constant</td>
<td>α</td>
<td>5.670 367(13) × 10⁻⁸</td>
<td>W m⁻² K⁻⁴</td>
<td>2.3 × 10⁻⁶</td>
</tr>
<tr>
<td>First radiation constant 2hc²</td>
<td>c₁</td>
<td>3.741 771 790(46) × 10⁻¹⁶</td>
<td>W m²</td>
<td>1.2 × 10⁻⁸</td>
</tr>
<tr>
<td>First radiation constant spectral radiance 2hc²</td>
<td>c₁L</td>
<td>1.191 042 953(15) × 10⁻¹⁶</td>
<td>W m² sr⁻¹</td>
<td>1.2 × 10⁻⁸</td>
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<tr>
<td>Second radiation constant hc/k</td>
<td>c₂</td>
<td>1.438 777 36(83) × 10⁻²</td>
<td>m K</td>
<td>5.7 × 10⁻⁷</td>
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<tr>
<td>Wien displacement law constants</td>
<td>h</td>
<td>2.897 772(17) × 10⁻³</td>
<td>m K</td>
<td>5.7 × 10⁻⁷</td>
</tr>
<tr>
<td></td>
<td>b'</td>
<td>5.878 923(34) × 10⁻¹⁰</td>
<td>Hz K⁻¹</td>
<td>5.7 × 10⁻⁷</td>
</tr>
<tr>
<td></td>
<td>b'</td>
<td>5.878 923(34) × 10⁻¹⁰</td>
<td>Hz K⁻¹</td>
<td>5.7 × 10⁻⁷</td>
</tr>
</tbody>
</table>

*See Table XXXIV for the conventional value adopted internationally for realizing representations of the volt using the Josephson effect.
*See Table XXXIV for the conventional value adopted internationally for realizing representations of the ohm using the quantum-Hall effect.

*Value recommended by the Particle Data Group (Olive et al., 2014).

*Based on the ratio of the masses of the W and Z bosons m₁/μ₁ recommended by the Particle Data Group (Olive et al., 2014). The value for sin²θw they recommend, which is based on a particular variant of the modified minimal subtraction (MS) scheme, is sin²θw(MZ) = 0.231 26(5).

*This and all other values involving m are based on the value of m₁c² in MeV recommended by the Particle Data Group (Olive et al., 2014).

*The numerical value of F to be used in coulometric chemical measurements is 96.485 3253(12) [1.2 × 10⁻⁸] when the relevant current is measured in terms of representations of the volt and ohm based on the Josephson and quantum-Hall effects and the internationally adopted conventional values of the Josephson and von Klitzing constants K₁,₀ and K₀,₀ given in Table XXXV.

*The entropy of an ideal monoatomic gas of relative atomic mass Aᵣ is given by S = S₀ + ζ R lnAᵣ − R ln(p/p₀) + ζ R ln(T/K).

meeting in Paris in the fall of 2018 a resolution that will revise the SI. In the “new SI,” as it has come to be called to distinguish it from the present SI, the definitions of the kilogram, ampere, kelvin, and mole are linked to exact values of the Planck constant $h$, elementary charge $e$, Boltzmann constant $k$, and Avogadro constant $N_A$, in much the same way as the present definition of the meter is linked to an exact value of the speed of light in vacuum $c$. CODATA, through its Task Group on Fundamental Constants, is to provide the values of $h$, $e$, $k$, and $N_A$ for the new definitions by carrying out a special least-squares adjustment during the summer of 2017. Details of the proposed new-SI may be found on the BIPM website at bipm.org/en/measurement-units/new-si/ [see also Milton, Davis, and Fletcher (2014)].

A. Comparison of 2014 and 2010 CODATA recommended values

Table XL compares the 2014 and 2010 recommended values of a representative group of constants. The regularities observed

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**Table XXXIV.** The variances, covariances, and correlation coefficients of the values of a selected group of constants based on the 2014 CODATA adjustment. The numbers in bold above the main diagonal are $10^{16}$ times the numerical values of the relative covariances; the numbers in bold on the main diagonal are $10^{16}$ times the numerical values of the relative variances; and the numbers in italics below the main diagonal are the correlation coefficients. 

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$h$</th>
<th>$e$</th>
<th>$m_e$</th>
<th>$N_A$</th>
<th>$m_e/m_u$</th>
<th>$F$</th>
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<td>0.0005</td>
<td>0.0005</td>
<td>0.0005</td>
<td>-0.0005</td>
<td>0.0005</td>
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<td>0.0176</td>
<td>1.5096</td>
<td>0.7550</td>
<td>1.5088</td>
<td>-1.5086</td>
<td>-0.0010</td>
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<tr>
<td>0.0361</td>
<td>0.9998</td>
<td>0.3778</td>
<td>0.7540</td>
<td>-0.7540</td>
<td>-0.0010</td>
<td>-0.3763</td>
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<tr>
<td>-0.0193</td>
<td>0.9993</td>
<td>0.9985</td>
<td>1.5097</td>
<td>-1.5097</td>
<td>0.0011</td>
<td>-0.7556</td>
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<tr>
<td>0.0193</td>
<td>-0.9993</td>
<td>-0.9985</td>
<td>-1.0000</td>
<td>1.5097</td>
<td>-0.0011</td>
<td>0.7557</td>
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<tr>
<td>-0.0202</td>
<td>-0.0004</td>
<td>-0.0007</td>
<td>0.0004</td>
<td>-0.0004</td>
<td>4.9471</td>
<td>-0.0021</td>
</tr>
<tr>
<td>0.0745</td>
<td>-0.9957</td>
<td>-0.9939</td>
<td>-0.9985</td>
<td>0.9985</td>
<td>-0.0015</td>
<td>0.3794</td>
</tr>
</tbody>
</table>

---

**Table XXXV.** Internationally adopted values of various quantities

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Numerical value</th>
<th>Unit</th>
<th>Relative std. uncert. $u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative atomic mass$^a$ of $^{12}$C</td>
<td>$A_1$($^{12}$C)</td>
<td>12</td>
<td>Exact</td>
<td></td>
</tr>
<tr>
<td>Molar mass constant</td>
<td>$M_u$</td>
<td>$1 \times 10^{-3}$</td>
<td>kg mol$^{-1}$</td>
<td>Exact</td>
</tr>
<tr>
<td>Molar mass of $^{12}$C</td>
<td>$M$($^{12}$C)</td>
<td>$12 \times 10^{-3}$</td>
<td>kg mol$^{-1}$</td>
<td>Exact</td>
</tr>
<tr>
<td>Conventional value of Josephson constant</td>
<td>$K_{J,\text{SI}}$</td>
<td>483.597.9</td>
<td>GHz V$^{-1}$</td>
<td>Exact</td>
</tr>
<tr>
<td>Conventional value of von Klitzing constant</td>
<td>$K_{K,\text{SI}}$</td>
<td>25 812.807</td>
<td>Ω</td>
<td>Exact</td>
</tr>
<tr>
<td>Standard-state pressure</td>
<td>$P_0$</td>
<td>100</td>
<td>kPa</td>
<td>Exact</td>
</tr>
<tr>
<td>Standard atmosphere</td>
<td>$P_0$</td>
<td>101.325</td>
<td>kPa</td>
<td>Exact</td>
</tr>
</tbody>
</table>

---

**Table XXXVI.** Values of some x-ray-related quantities based on the 2014 CODATA adjustment of the values of the constants

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Numerical value</th>
<th>Unit</th>
<th>Relative std. uncert. $u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cu x unit: $\lambda$(CuK$_\alpha$)/1 537.400</td>
<td>$x_{\text{Cu}(\text{CuK}_\alpha)}$</td>
<td>$1.002 076 97(28) \times 10^{-13}$</td>
<td>m</td>
<td>2.8 $\times 10^{-7}$</td>
</tr>
<tr>
<td>Mo x unit: $\lambda$(MoK$_\alpha$)/707.831</td>
<td>$x_{\text{Mo}(\text{MoK}_\alpha)}$</td>
<td>$1.000 099 52(53) \times 10^{-13}$</td>
<td>m</td>
<td>5.3 $\times 10^{-7}$</td>
</tr>
<tr>
<td>Ångström star: $\lambda$(W$_\text{K\alpha}$)/0.2090100</td>
<td>$\lambda$</td>
<td>$1.000 014 95(90) \times 10^{-10}$</td>
<td>m</td>
<td>9.0 $\times 10^{-7}$</td>
</tr>
<tr>
<td>Lattice parameter$^a$ of Si (in vacuum, 22.5 °C)</td>
<td>$a$</td>
<td>543.102 0504(89) $\times 10^{-12}$</td>
<td>m</td>
<td>1.6 $\times 10^{-8}$</td>
</tr>
<tr>
<td>(220) lattice spacing of Si $a/\sqrt{2}$ (in vacuum, 22.5 °C)</td>
<td>$d_{220}$</td>
<td>$192.015 5714(32) \times 10^{-12}$</td>
<td>m</td>
<td>1.6 $\times 10^{-8}$</td>
</tr>
<tr>
<td>Molar volume of Si $M$(Si)/$\rho$(Si) = $N_Aa^3/8$ (in vacuum, 22.5 °C)</td>
<td>$V_m$(Si)</td>
<td>$12.058 832 14(61) \times 10^{-6}$</td>
<td>m$^3$ mol$^{-1}$</td>
<td>5.1 $\times 10^{-8}$</td>
</tr>
</tbody>
</table>

---

$^a$The relative atomic mass $A_i$ of particle $X$ with mass $m(X)$ is defined by $A_i(X)=m(X)/m_u$, where $m_u=m(^{12}$C$)/12=M_u/N_A=1$ $u$ is the atomic mass constant, $M_u$ is the molar mass constant, $N_A$ is the Avogadro constant, and $u$ is the unified atomic mass unit. Thus the mass of particle $X$ is $m(X)=A_i(X)u$ and the molar mass of $X$ is $M(X)=A_i(X)M_u$.

$^b$This is the value adopted internationally for realizing representations of the volt using the Josephson effect.

$^c$This is the value adopted internationally for realizing representations of the ohm using the quantum-Hall effect.
in the numbers in columns 2 to 4 arise because the values of many constants are obtained from expressions proportional to the fine-structure constant $\alpha$, Planck constant $\hbar$, or molar gas constant $R$ raised to various powers. For example, the first six quantities are obtained from expressions proportional to $a^4\hbar^2$, where $|a| = 1, 2, 3,$ or 6. The next 15 quantities, from $h$ to the magnetic moment of the proton $\mu_p$, are calculated from expressions containing the factor $h^3$, where $|a| = 1$ or 1/2. And the five quantities from $R$ to the Stefan-Boltzmann constant $\sigma$ are proportional to $R^3$, where $|a| = 1$ or 4.

Additional comments on some of the entries in Table XL are as follows.

(i) The shift and uncertainty reduction of the 2014 recommended value of $\alpha$ is mainly due to the availability for the first time of a numerically calculated result for the tenth-order coefficient $A_{10}^{(10)}$ (12 672 Feynman diagrams) in the theoretical expression for $\alpha_0$; see Sec. V.A.1. The value used in 2010, based on a procedure described in CODATA-98, is 0.0(4.6) compared with the newly available value 7.79(34) used in 2014.

(ii) In the 2010 adjustment inconsistencies among watt-balance measurements of $h$ and the value inferred from an x-ray-crystal-density (XRCMD) measurement of $N_A$ using highly enriched silicon led the Task Group to expand the uncertainties assigned to these data by a multiplicative factor, or expansion factor, of 2. These inconsistencies have since been resolved and further,
Table XXXVIII. The values of some energy equivalents derived from the relations \( E = mc^2 = h c / \lambda = h v = k T \), and based on the 2014 CODATA adjustment of the values of the constants; \( 1 \text{ eV} = (e/C) J \), \( 1 u = m_\text{u} = \frac{1}{12} m(^{12}\text{C}) = 10^{-3} \text{ kg mol}^{-1}/N_\text{a} \), and \( E_0 = 2K_0 h c = \alpha^2 m_\text{e} c^2 \) is the Hartree energy (hartree)

<table>
<thead>
<tr>
<th>Relevant unit</th>
<th>J</th>
<th>kg</th>
<th>m(^{-1})</th>
<th>Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 J (1 J) = 1 J</td>
<td>(1 J)c(^2) = 1.112650056 \ldots \times 10^{-17} \text{ kg}</td>
<td>(1 J)h c/\lambda = 5.034116551(62) \times 10^{13} \text{ m}(^{-1})</td>
<td>(1 J)/h = 1.509190205(19) \times 10^{15} \text{ Hz}</td>
<td></td>
</tr>
<tr>
<td>1 kg (1 kg)c(^2) = 8.987551787 \ldots \times 10^{-6} J</td>
<td>(1 kg)c(^2)/h = 4.524438411(56) \times 10^{16} \text{ m}(^{-1})</td>
<td>(1 kg)c(^2)/h = 1.356392512(17) \times 10^{18} \text{ Hz}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 m(^{-1}) (1 m(^{-1}))h c = 1.986448824(24) \times 10^{-25} J</td>
<td>(1 m(^{-1}))h/c = 2.210219057(27) \times 10^{-32} \text{ kg}</td>
<td>(1 m(^{-1}))c = 3.335640951 \ldots \times 10^{-4} \text{ m}(^{-1})</td>
<td>(1 m(^{-1}))c = 299 792 458 Hz</td>
<td></td>
</tr>
<tr>
<td>1 Hz (1 Hz)/h = 6.626070040(81) \times 10^{-34} J</td>
<td>(1 Hz)c/\lambda = 7.372497201(91) \times 10^{-55} \text{ kg}</td>
<td>(1 Hz)c = 1.1 \times 10^{-1} \text{ Hz}</td>
<td>(1 Hz)c = 1 Hz</td>
<td></td>
</tr>
<tr>
<td>1 K (1 K)c(^2) = 1.53617865(88) \times 10^{-26} J</td>
<td>(1 K)c(^2)/h = 69.503457(40) \text{ m}(^{-1})</td>
<td>(1 K)c(^2)/h = 2.0836612(12) \times 10^{16} \text{ Hz}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 eV (1 eV)c(^2) = 1.782661907(11) \times 10^{-20} \text{ kg}</td>
<td>(1 eV)c(^2)/h = 8.065544005(50) \times 10^{-3} \text{ m}(^{-1})</td>
<td>(1 eV)c(^2)/h = 2.417989626(15) \times 10^{14} \text{ Hz}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 u (1 u)c(^2) = 1.660539040(20) \times 10^{-27} \text{ kg}</td>
<td>(1 u)c(^2)/h = 7.513006166(34) \times 10^{-14} \text{ m}(^{-1})</td>
<td>(1 u)c(^2)/h = 2.252342706(10) \times 10^{10} \text{ Hz}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 J(\text{E}_0) = 4.359744650(54) \times 10^{-18} J</td>
<td>(1 J(\text{E}_0))c(^2) = 4.850870129(60) \times 10^{-15} \text{ kg}</td>
<td>(1 J(\text{E}_0))/h = 2.194746313702(13) \times 10^{-7} \text{ m}(^{-1})</td>
<td>(1 J(\text{E}_0))/h = 6.579683920711(39) \times 10^{15} \text{ Hz}</td>
<td></td>
</tr>
</tbody>
</table>

Table XXXIX. The values of some energy equivalents derived from the relations \( E = mc^2 = h c / \lambda = h v = k T \) and based on the 2014 CODATA adjustment of the values of the constants; \( 1 \text{ eV} = (e/C) J \), \( 1 u = m_\text{u} = \frac{1}{12} m(^{12}\text{C}) = 10^{-3} \text{ kg mol}^{-1}/N_\text{a} \), and \( E_0 = 2K_0 h c = \alpha^2 m_\text{e} c^2 \) is the Hartree energy (hartree)

<table>
<thead>
<tr>
<th>Relevant unit</th>
<th>K</th>
<th>eV</th>
<th>u</th>
<th>( E_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 J (1 J)/h = 7.2429731(42) \times 10^{22} \text{ K}</td>
<td>(1 J)c(^2) = 6.700535363(82) \times 10^{16} \text{ u}</td>
<td>(1 J)c(^2) = 2.293712317(28) \times 10^{17} \text{ E}_0</td>
<td>(1 J)c(^2) = 6.6173303(50) \times 10^{-5} \text{ eV}</td>
<td></td>
</tr>
<tr>
<td>1 kg (1 kg)c(^2)/h = 6.5096595(37) \times 10^{9} \text{ K}</td>
<td>(1 kg)c(^2) = 6.022140857(74) \times 10^{16} \text{ u}</td>
<td>(1 kg)c(^2) = 2.061485823(25) \times 10^{18} \text{ E}_0</td>
<td>(1 kg)c(^2) = 4.5563352527627(27) \times 10^{-8} \text{ eV}</td>
<td></td>
</tr>
<tr>
<td>1 m(^{-1}) (1 m(^{-1}))h/c = 1.43877736(83) \times 10^{-5} \text{ K}</td>
<td>(1 m(^{-1}))h/c = 1.331025049(00) \times 10^{-15} \text{ u}</td>
<td>(1 m(^{-1}))c(^2)/h = 4.439821661(20) \times 10^{-24} \text{ u}</td>
<td>(1 m(^{-1}))c(^2)/h = 3.674932248(23) \times 10^{-2} \text{ E}_0</td>
<td></td>
</tr>
<tr>
<td>1 Hz (1 Hz)c/\lambda = 4.135667662(25) \times 10^{-15} \text{ eV}</td>
<td>(1 Hz)c/\lambda = 9.2510842(53) \times 10^{-13} \text{ u}</td>
<td>(1 Hz)c/\lambda = 1.5198297860889(90) \times 10^{-16} \text{ E}_0</td>
<td>(1 Hz)c/\lambda = 3.166810518(18) \times 10^{-6} \text{ E}_0</td>
<td></td>
</tr>
<tr>
<td>1 K (1 K)c = 1 K</td>
<td>(1 K)c(^2) = 8.6173303(50) \times 10^{-5} \text{ eV}</td>
<td>(1 K)c(^2) = 2.9212623197(13) \times 10^{-9} \text{ u}</td>
<td>(1 K)c(^2) = 3.4231776602(16) \times 10^{7} \text{ E}_0</td>
<td></td>
</tr>
<tr>
<td>1 eV (1 eV)c = 1.1604522167(67) \times 10^{4} \text{ K}</td>
<td>(1 eV)c(^2) = 1.0735441105(66) \times 10^{-9} \text{ u}</td>
<td>(1 eV)c(^2) = 3.674932248(23) \times 10^{-2} \text{ E}_0</td>
<td>(1 eV)c(^2) = 3.4231776602(16) \times 10^{7} \text{ E}_0</td>
<td></td>
</tr>
<tr>
<td>1 u (1 u)c(^2)/h = 1.08095438(62) \times 10^{-33} \text{ K}</td>
<td>(1 u)c(^2) = 931.4940954(57) \times 10^{-6} \text{ eV}</td>
<td>(1 u)c(^2) = 2.9212623197(13) \times 10^{-9} \text{ u}</td>
<td>(1 u)c(^2) = 3.4231776602(16) \times 10^{7} \text{ E}_0</td>
<td></td>
</tr>
<tr>
<td>1 J(\text{E}_0) = 3.1577513(18) \times 10^{9} \text{ K}</td>
<td>(1 J(\text{E}_0))c = 27.21138602(17) \text{ eV}</td>
<td>(1 J(\text{E}_0))c = 2.9212623197(13) \times 10^{-9} \text{ u}</td>
<td>(1 J(\text{E}_0))c = 3.4231776602(16) \times 10^{7} \text{ E}_0</td>
<td></td>
</tr>
</tbody>
</table>
an improved watt-balance result for $h$ with a relative uncertainty of $1.8 \times 10^{-8}$ and an improved XRCDAvogadro constant inferred value of $h$ with a relative uncertainty of $2.0 \times 10^{-8}$ have become available for the 2014 adjustment. Because the data are now sufficiently consistent that it is no longer necessary to increase uncertainties as in 2010, the relative uncertainty of the 2014 recommended value of $h$ is $1.2 \times 10^{-8}$ compared to $4.4 \times 10^{-8}$ for the 2010 value.

(iii) The 2010 recommended value of $R$ is based on six acoustic-gas-thermometry (AGT) results with relative uncertainties in the range $1.2 \times 10^{-6}$ to $8.4 \times 10^{-6}$ while the 2014 recommended value is based mainly on seven AGT results with relative uncertainties in the range $0.90 \times 10^{-6}$ to $3.7 \times 10^{-6}$. Of these seven, three are new results, two of which have uncertainties of $0.90 \times 10^{-6}$ and $1.0 \times 10^{-6}$, respectively. Also contributing to the determination of the 2014 recommended value of $R$ is a Johnson noise thermometry measurement of $k/h$ and a dielectric-constant gas thermometry measurement of $A_e/R$ with relative uncertainties of $3.9 \times 10^{-6}$ and $4.0 \times 10^{-6}$, respectively. It is the significant advances made in AGT in the past four years that have led to the reduction of the uncertainty of the recommended value of $R$ by nearly 40%. The consistency of the new and previous AGT results have led to the comparatively small shift of the 2014 value from that of 2010.

(iv) Other constants in Table XL, whose changes are worth noting are $G$, $m_e/m_p$, $A_e(e)$, $A_e(t)$, $A_e(h)$, $\mu_p/\mu_B$, $\mu_p/\mu_N$, and $\nu_e/\nu_P$. The 2010 recommended value of $G$ with relative uncertainty $12 \times 10^{-5}$ is the weighted mean of 11 results whose $a$ priori uncertainties were increased by an expansion factor of 14 so that the smallest and largest result differed from the recommended value by about twice the latter’s uncertainty. For the 2014 adjustment the Task Group decided to follow its usual practice and to choose an expansion factor that reduces the normalized residual of each datum to less than 2. Thus in 2014 the expansion factor is 6.3 and the weighted mean of the 14 available values, which is the 2014 recommended value, has a relative uncertainty of $4.7 \times 10^{-5}$. The shift of the 2014 value from the 2010 value is due to the three new results that became available for the 2014 adjustment.

The reduction in uncertainty of $m_e/m_p$ is a consequence of the large reduction in uncertainty of $A_e(e)$, which resulted from measurement of the ratio of the electron spin-flip frequency of the $^{12}$C$^{5+}$ ion to the cyclotron frequency of the ion in the same magnetic flux density, and the same ratio for $^{28}$Si$^{13+}$, with the extraordinarily small relative uncertainties of $2.8 \times 10^{-11}$ and $4.8 \times 10^{-11}$, respectively. The significant reduction in uncertainty of $A_e(t)$ and $A_e(h)$ is the result of the measurement of two pairs of cyclotron frequency ratios carried out in two different laboratories, the first being that of $d$ and $h$ to $^{12}$C$^{6+}$ with relative uncertainties of $2.0 \times 10^{-11}$ and $1.4 \times 10^{-11}$; and the second is that of HD$^+$ to $^3$He$^+$ and to $t$ with relative uncertainties for each of $4.8 \times 10^{-11}$. However, because of the significant inconsistency of the values of $A_e(^3$He) implied by the second ratio of the first pair and the first ratio of the second pair, the Task Group applied a multiplicative factor of 2.8 to the uncertainties of these two ratios in order to reduce the residual of each to less than 2 in the final adjustment on which
the 2014 recommended values are based. Finally, the reduction in the uncertainties of $\mu_p/\mu_N$, $\mu_p/\mu_S$, and $\mu_e/\mu_p$ are a consequence of a new, directly measured value of $\mu_p/\mu_N$ with a relative uncertainty of $3.3 \times 10^{-9}$.

B. Some implications of the 2014 CODATA recommended values and adjustment for metrology and physics

1. Conventional electrical units

The conventional values $K_{1,90} = 483.597.9$ GHz/V and $R_{K,90} = 25812.807$ Ω adopted in 1990 for the Josephson and von Klitzing constants established conventional units of voltage and resistance, $V_{90}$ and $\Omega_{90}$, given by $V_{90} = (K_{3,90}/K_{1})$ V and $\Omega_{90} = (R_{K}/R_{K,90})$ Ω. Other conventional electric units follow from $V_{90}$ and $\Omega_{90}$, for example, $A_{90} = V_{90}/\Omega_{90}$, $C_{90} = A_{90}$ s, $W_{90} = A_{90}V_{90}$, $F_{90} = C_{90}/V_{90}$, and $H_{90} = \Omega_{90}$ s, which are the conventional units of current, charge, power, capacitance, and inductance, respectively, (Taylor and Mohr, 2001). The 2014 adjustment gives for the relations between $K_1$ and $K_{1,90}$, and for $R_K$ and $R_{K,90}$,

$$K_1 = K_{1,90}[1 - 9.83(61) \times 10^{-8}],$$

$$R_K = R_{K,90}[1 + 1.765(23) \times 10^{-8}],$$

which lead to

$$V_{90} = [1 + 9.83(61) \times 10^{-8}] \text{ V},$$

$$\Omega_{90} = [1 + 1.765(23) \times 10^{-8}] \text{ Ω},$$

$$A_{90} = [1 + 8.06(61) \times 10^{-8}] \text{ A},$$

$$C_{90} = [1 + 8.06(61) \times 10^{-8}] \text{ C},$$

$$W_{90} = [1 + 17.9(1.2) \times 10^{-8}] \text{ W},$$

$$F_{90} = [1 - 1.765(23) \times 10^{-8}] \text{ F},$$

$$H_{90} = [1 + 1.765(23) \times 10^{-8}] \text{ H}.$$  

Equations (275) and (276) show that $V_{90}$ exceeds $V$ and $\Omega_{90}$ exceeds $\Omega$, which means that measured voltages and resistances traceable to the Josephson effect and $K_{1,90}$ and the quantum-Hall effect and $R_{K,90}$, respectively, are too small relative to the SI. However, the differences are well within the 40×10^{-8} uncertainty assigned to $V_{90}$ and the 10×10^{-8} uncertainty assigned to $\Omega_{90}/\Omega$ by the Consultative Committee for Electricity and Magnetism (CCEM) of the International Committee to Weights and Measures (CIPM) (Quinn, 1989; Quinn, 2001).

2. Josephson and quantum-Hall effects

Watt-balance and XRCD-$n_A$ advances have led to tests of the exactness of the quantum-Hall and Josephson effect relations $R_K = h/e^2$ and $K_{fi} = 2e/h$ that are less clouded by inconsistencies of the data. Indeed, based on the 2014 data as used in adjustment 2 summarized in Table XXXVIII, Sec. XIII.B.2, the possible corrections $\varepsilon K$ and $\varepsilon_{fi}$ to the quantum-Hall and Josephson effect relations are $2.2(1.8) \times 10^{-8}$ and $-0.9(1.5) \times 10^{-8}$, respectively; see Table XXXX, Sec. XIII.B.3. Thus the exactness of these relations is experimentally confirmed within about 2 parts in 10^8. However, as Table XXXI indicates, some of the initial 2014 input data are not as supportive of the exactness of the relations, most notably the NIST-89 result for $\Gamma_{n=90}$; on the other hand, its normalized residual in the adjustment that produced the above values of $\varepsilon K$ and $\varepsilon_{fi}$ is 2.3 and because of its low-weight it is omitted from the 2006 and 2010 final adjustments as well as the 2014 final adjustment.

3. The new SI

The impact of the new data that have become available for the 2014 adjustment on the establishment of the new SI by the CGPM is discussed in detail in the Introduction section of this report (see Sec. I.B.1) and need not be repeated here. Sufficient to say that the uncertainties of the 2014 recommended values of the four new defining constants $h$, $e$, $k$, and $N_A$, which in parts in 10^8 are 1.2, 0.61, 57, and 1.2, are already sufficiently small to meet the requirements deemed necessary by the CIPM and CGPM for the adoption of the new SI.

4. Proton radius

The severe disagreement of the proton rms charge radius $r_p$ determined from the Lamb shift in muonic hydrogen with values determined from H and D transition frequencies and electron-proton scattering experiments present in the 2010 adjustment remain present in the 2014 adjustment. Although the uncertainty of the muonic hydrogen value is significantly smaller than the uncertainties of these other values, its negative impact on the internal consistency of the theoretically predicted and experimentally measured frequencies, as well as on the value of the Rydberg constant, was considered by the Task Group to be still so severe that the only recourse was to once again exclude it from the final adjustment.

5. Muon magnetic-moment anomaly

The long-standing significant difference between the theoretically predicted, standard-model value of $a_\mu$ and the experimentally determined value that led to the exclusion of the theoretical expression for $a_\mu$ from the 2010 adjustment remains; the difference is still at about the 3σ level depending on the way the all-important lowest-order hadronic vacuum polarization and hadronic light-by-light contributions are evaluated. Because of the continuing difficulty of reliably calculating these terms, the Task Group decided to omit the theory from the 2014 adjustment as in 2010. The 2014 recommended values of $a_\mu$ and those of other constants that depend on it are, therefore, based on experiment.

6. Electron magnetic-moment anomaly, fine-structure constant, and QED

A useful test of the QED theory of $a_\mu$ is to compare two values of $a$: The first, with relative uncertainty $2.4 \times 10^{-10}$, is that obtained by equating the experimental value of $a_\mu$ with its QED theoretical expression; the second, which is only weakly

dependent on QED theory and with relative uncertainty $6.2 \times 10^{-10}$, is that obtained from the measurement of $h/m^{(87)\text{Rb}}$ using atom interferometry. These two values (see the first and second rows in Table XX, Sec. XIII.A) differ by 1.8 times the uncertainty of their difference, or 1.8$r$. Although this is acceptable agreement and supports the QED theory of $a_e$, the result of the same comparison in CODATA-10 based on the same experimental values of $a_e$ and $h/m^{(87)\text{Rb}}$ is 0.4$r$. The two main reasons for this rather significant change are the new and somewhat surprisingly large value of the $A_{10}^{(10)}$ coefficient in the theoretical expression for $a_e$, which decreased the $a_e$ value of $r$; and the decreased but highly accurate new value of $A_e(e)$ which increased the $h/m^{(87)\text{Rb}}$ value of $r$.

C. Suggestions for future work

As discussed, to deal with data inconsistencies the Task Group decided to (i) omit from the 2014 adjustment the value of $r_p$ obtained from measurements of the Lamb shift in muonic hydrogen; (ii) omit the theory of the muonic magnetic-moment anomaly $a_e$; (iii) increase the initial uncertainties of the cyclotron frequency ratios $\omega_e(h)/\omega_e(^{12}\text{C}^6\pi)$ and $\omega_e(^{3}\text{He}^+)/\omega_e(^{4}\text{He}^+)$ relevant to the determination of $A_e(h)$ by an expansion factor of 2.8; and (iv) increase the initial uncertainties of the 14 values of $G$ by an expansion factor of 6.3. Issues (i), (ii), and (iv) have been with us for some time and suggestions for their resolution were given in CODATA-10. Updated versions follow together with a suggestion regarding (iii).

As also discussed, the data now available provide values of the defining constants $h$, $e$, $k$, and $N_A$ of the new SI with uncertainties sufficiently small for its adoption by the 26th CGPM in the fall of 2018 as planned. The final values are to be based on a special adjustment carried out by the Task Group during the summer of 2017. Because of its importance, the CIPM has decided that all the data to be used in that adjustment must have been published in an archival journal or be available in a preprint accepted for publication by 1 July 2017. Nevertheless, we include suggestions for work that will improve the robustness of the currently available data that determine these important constants.

(i) Work currently underway could solve the “proton radius puzzle” and should continue to be pursued as vigorously as possible. This includes the measurement of hydrogen transition frequencies, the analysis of $\mu - p$ and $\mu - d$ data, and possible new linear-shift measurements in $\mu - h$ and $\mu - a$. New scattering data from experiments such as MUSE (Downie, 2014) and PRad (Gasparian, 2014) and improved methods to extract $r_p$ from such data as well as verification of the theory of $H$, $D$, and muonic hydrogenlike energy levels could also help.

(ii) Because the disagreement between $a_u$ theory and experiment remains even after many years of effort devoted to improving the theory and the experimental results on which the theory relies, the solution to the problem may have to wait until the completion over

the next 5 to 10 years of the two new experiments underway to remeasure $a_u$ (Mibe, 2011; Logashenko et al., 2015). Improved measurements of cross sections for the scattering of $e^+ e^-$ into hadrons and better data on the decay of the $\tau$ into hadrons could also be useful.

(iii) The two cyclotron frequency ratios in question were obtained from experiments at the University of Washington and at Florida State University (FSU). A careful study of the University of Washington apparatus that might uncover an overlooked systematic effect is not possible because it is no longer available. However, the research program at FSU continues and the researchers are encouraged to search for a possible explanation of the disagreement. An experiment under way to measure the $\tilde{Q}$ value of tritium at the Max-Planck-Institut für Kernphysik, Heidelberg, Germany, could resolve the problem (Streubel et al., 2014).

(iv) One or more measurements of $G$ with an uncertainty of 1 part in $10^5$ using new and innovative approaches might finally resolve some of the problems that have plagued the reliable determination of $G$ over the past three decades. The possibility of transferring the apparatus used by Quinn et al. (2014) and by Parks and Faller (2014) to other laboratories to be used there by new researchers in the hope of uncovering overlooked systematic effects could be helpful as well.

(v) Watt-balance and watt-balance-like measurements of $h$ and the XRCR measurement of $N_A$ currently under way should continue to be vigorously pursued with the goal of achieving uncertainties of no more than a few parts in $10^8$. This also applies to experiments to determine $R$, $k/h$, and $A_e/R$ with an uncertainty goal of no more than a few parts in $10^9$. An independent calculation of the $A_{10}^{(8)}$ and $A_{10}^{(10)}$ coefficients in the theoretical expression for $a_e$ would increase confidence in the value of $a$ from $a_e$. For this same reason, results from experiments currently under way to determine $h/m(X)$ with an uncertainty small enough to provide a value of $a$ with an uncertainty of 5 parts in $10^{10}$ or less would be valuable.

List of Symbols and Abbreviations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>ASD</td>
<td>NIST Atomic Spectra Database (online)</td>
</tr>
<tr>
<td>AMDC</td>
<td>Atomic Mass Data Center, transferred in 2013 to Institute of Modern Physics (IMP), Chinese Academy of Sciences, Lanzhou, PRC, from Centre de Spectrométrie Nucléaire et de Spectrométrie de Masse (CSNSM), Orsay, France</td>
</tr>
<tr>
<td>AME</td>
<td>Atomic mass evaluation from the AMDC (completed in year specified)</td>
</tr>
<tr>
<td>$A_{e}(X)$</td>
<td>relative atomic mass of $X$: $A_{e}(X) = m(X)/m_e$</td>
</tr>
<tr>
<td>$A_{90}$</td>
<td>conventional unit of electric current: $A_{90} = V_{90}/\Omega_{90}$</td>
</tr>
<tr>
<td>$\AA^*$</td>
<td>ångström star: $\lambda(WK\alpha_{1}) = 0.2090100 \AA^*$</td>
</tr>
</tbody>
</table>
\(a_e\)  electron magnetic-moment anomaly:  
\(a_e = \frac{(g_e - 1)}{2}\)

\(a_\mu\)  muon magnetic-moment anomaly:  
\(a_\mu = \frac{(g_\mu - 1)}{2}\)

BIPM  International Bureau of Weights and Measures,  Sèvres, France

BNL  Brookhaven National Laboratory, Upton, New York, USA

CERN  European Organization for Nuclear Research, Geneva, Switzerland

CGPM  General Conference on Weights and Measures

CIPM  International Committee for Weights and Measures

CODATA  Committee on Data for Science and Technology of the International Council for Science

\(c\)  speed of light in vacuum

\(d\)  deuteron (nucleus of deuterium D, or \(^2\)H)

\(d_{220}\)  \{220\} lattice spacing of an ideal crystal of naturally occurring silicon

\(d_{220}(X)\)  \{220\} lattice spacing of crystal X of naturally occurring silicon

\(e\)  symbol for either member of the electron-positron pair; when necessary, \(e^-\) or \(e^+\) is used to indicate the electron or positron

\(e\)  elementary charge: absolute value of the charge of the electron

\(F\)  Faraday constant:  
\(F = N_A e\)

FSU  Florida State University, Tallahassee, Florida, USA

FSUJ  Friedrich-Schiller University, Jena, Germany

\(F_{90}\)  \(F_{90} = (F/A_{90})\ A\)

\(G\)  Newtonian constant of gravitation

\(g\)  local acceleration due to gravity

\(g_d\)  deuteron \(g\)-factor:  
\(g_d = \frac{2 \mu_d}{\mu_N}\)

\(g_e\)  electron \(g\)-factor:  
\(g_e = \frac{2 \mu_e}{\mu_B}\)

\(g_p\)  proton \(g\)-factor:  
\(g_p = \frac{2 \mu_p}{\mu_N}\)

\(g_\mu\)  shielded proton \(g\)-factor:  
\(g_\mu = \frac{2 \mu_\mu}{\mu_N}\)

\(g_\pi\)  pion \(g\)-factor:  
\(g_\pi = \frac{2 \mu_\pi}{\mu_N}\)

\(g_X(Y)\)  \(g\)-factor of particle X in the ground (1S) state of hydrogenic atom Y

\(g_u\)  muon \(g\)-factor:  
\(g_u = \frac{2 \mu_u}{(e h)/(2 m_u)}\)

GSI  Gesellschaft für Schweironenforschung, Darmstadt, Germany

HD  HD molecule (bound state of hydrogen and deuterium atoms)

HT  HT molecule (bound state of hydrogen and tritium atoms)

HUST  Huazhong University of Science and Technology, Wuhan, PRC

HarvU  Harvard University, Cambridge, Massachusetts, USA

IAC  International Avogadro Coordination

\(h\)  helion (nucleus of \(^3\)He)

\(h\)  Planck constant

\(\hbar\)  reduced Planck constant; \(\hbar/2\pi\)

CODATA RECOMMENDED VALUES: 2014

\(\alpha\)  
\(\alpha = (\frac{g_e - 1}{2})/2\)

\(\alpha = \frac{(g_e - 1)}{2}\)

INRIM  Istituto Nazionale di Ricerca Metrologica, Torino, Italy

IRMM  Institute for Reference Materials and Measurements, Geel, Belgium

JILA  JILA, University of Colorado and NIST, Boulder, Colorado, USA

KRISS  Korea Research Institute of Standards and Science, Taedok Science Town, Republic of Korea

KR/VN  KRISS-VNIIM collaboration

K \(_{\text{J-90}}\)  Josephson constant:  
\(K_j = 2e/h\)

K \(_{\text{J-90}}\)  conventional value of the Josephson constant  
\(K_j, K_{\text{J-90}} = 483\ 597.9\ \text{GHz V}^{-1}\)

\(k\)  Boltzmann constant:  
\(k = R/N_A\)

LAMPF  Clinton P. Anderson Meson Physics Facility at Los Alamos National Laboratory, Los Alamos, New Mexico, USA

LSU  Louisiana State University, Baton Rouge, USA

LANL  Los Alamos National Laboratory, Los Alamos, New Mexico, USA

LENS  European Laboratory for Non-Linear Spectroscopy, University of Florence, Italy

LKB  Laboratoire Kastler-Brossel, Paris, France

LK/SY  LKB and SYRTE collaboration

LNE  Laboratoire national de métrologie et d’essais, Trappes, France

MIT  Massachusetts Institute of Technology, Cambridge, Massachusetts, USA

METAS  Federal Institute for Metrology, Bern-Wabern, Switzerland

MPIK  Max-Planck-Institut für Kernphysik, Heidelberg, Germany

MPQ  Max-Planck-Institut für Quantenoptik, Garching, Germany

MSL  Measurement Standards Laboratory, Lower Hutt, New Zealand

\(M(X)\)  molar mass of X:  
\(M(X) = A_u(X) m_u\)

\(M_u\)  muonium (\(u^-e^+\) atom)

\(M_d\)  molar mass constant:  
\(M_d = 10^{-3} \text{ kg mol}^{-1}\)

\(M_a\)  unified atomic mass constant:  
\(m_u = m(\text{^{12}C})/12\)

\(m_X, m(X)\)  mass of X (for the electron e, proton p, and other elementary particles, the first symbol is used, i.e., \(m_e, m_p, \) etc.)

\(N_A\)  Avogadro constant

NIM  National Institute of Metrology, Beijing, PRC

NIST  National Institute of Standards and Technology, Gaithersburg, Maryland and Boulder, Colorado, USA

NMI  National Metrology Institute, Lindfield, Australia

NMIJ  National Metrology Institute of Japan, Tsukuba, Japan

NPL  National Physical Laboratory, Teddington, UK

NRC  National Research Council of Canada, Measurement Science and Standards, Ottawa, Ontario, Canada

\(\pi\)  
\(\pi = \frac{(g_\pi - 1)}{2}\)

\(\pi = \frac{(g_\pi - 1)}{2}\)

\(\pi = \frac{(g_\pi - 1)}{2}\)
MOHR, NEWELL, AND TAYLOR


PTB Physikalisch-Technische Bundesanstalt,
Braunschweig and Berlin, Germany

p proton

QED quantum electrodynamics

$p(x^2|v)$ probability that an observed value of chi-square
for $v$ degrees of freedom would exceed $x^2$

$R$ molar gas constant

$\hat{R}$ ratio of muon anomaly difference frequency to
free proton NMR frequency

$R_B$ Birge ratio: $R_B = (x^2/v)^3$

$r_d$ bound-state rms charge radius of the deuteron

$R_K$ von Klitzing constant: $R_K = h/e^2$

$R_{K-90}$ conventional value of the von Klitzing constant

$r_p$ bound-state rms charge radius of the proton

$R_{\infty}$ Rydberg constant: $R_{\infty} = m_e c^2 / 2 h$

$r(x_i, x_j)$ correlation coefficient of estimated values $x_i$ and
$x_j$: $r(x_i, x_j) = u(x_i, x_j) / [u(x_i) u(x_j)]$

$S_k$ self-sensitivity coefficient

SI Système international d’unités (International
System of Units)

StPrsb various institutions in St. Petersburg, Russian
Federation

StanU Stanford University, Stanford, California, USA

SUREC Scottish Universities Environmental Research
Centre, University of Glasgow, Glasgow, Scotland

SYRTE Systèmes de référence Temps Espace, Paris,
France

$T$ thermodynamic temperature

TR&D Tribotech Research and Development Com-
p any, Moscow, Russian Federation

Type A uncertainty evaluation by the statistical analysis
of series of observations

Type B uncertainty evaluation by means other than the
statistical analysis of series of observations

t triton (nucleus of tritium $^3$H, or $^3$H)

UCI University of California, Irvine, Irvine, California, USA

UMZ Institut für Physik, Johannes Gutenberg
Universität Mainz (or simply the University of
Mainz), Mainz, Germany

USussex University of Sussex, Brighton, UK

UWash University of Washington, Seattle, Washington, USA

UWup University of Wuppertal, Wuppertal, Germany

UZur University of Zurich, Zurich, Switzerland

u unified atomic mass unit (also called the dalton,
Da): 1 u = $m_{^{12}C}/12$

$u_d$ standard uncertainty of the difference between
two values ($\sigma$ is sometimes used in place of $u_d$)

$u(x_i)$ standard uncertainty (i.e., estimated standard
deviation) of an estimated value $x_i$ of a quantity
$X_i$ (also simply $u_i$)

$u_r(x_i)$ relative standard uncertainty of an estimated
value $x_i$ of a quantity $X_i$: $u_r(x_i) = u(x_i) / |x_i|$, $x_i \neq 0$
(also simply $u_i$)

$u(x_i, x_j)$ covariance of estimated values $x_i$ and $x_j$

$u(x_i, x_j) = u(x_i, x_j) / (x_i x_j)$

$V_n(Si)$ molar volume of naturally occurring silicon

$V_{90}$ conventional unit of voltage based on the
Josephson effect and $R_{90}$: $V_{90} = (K_{J-90}/K_1) V$

WarsU University of Warsaw, Warsaw, Poland

W$_{90}$ conventional unit of power: $W_{90} = V_{90}^2 / \Omega_{90}$

XROI combined x-ray and optical interferometer

xu (Cu$\alpha_1$) Cu x unit: $\lambda$(Cu$\alpha_1$) = 1 537.400 xu(Cu$\alpha_1$)

xu (Mo$\alpha_1$) Mo x unit: $\lambda$(Mo$\alpha_1$) = 707.831 xu(Mo$\alpha_1$)

$X(X)$ amount-of-substance fraction of $X$

YaleU Yale University, New Haven, Connecticut, USA

fine-structure constant: $\alpha = e^2 / 4 \pi \epsilon_0 \hbar c \approx 1/137$

alpha particle (nucleus of $^4$He)

$\Gamma_{X-90}(lo)$ $\Gamma_{X-90}(lo) = (\gamma A_{\alpha_0})^{-1}$, $X = p$ or $h$

$\Gamma_{p-90}(hi)$ $\Gamma_{p-90}(hi) = (\gamma A_{\alpha_0}) A$

$\gamma_p$ proton gyromagnetic ratio: $\gamma_p = 2 \mu_p / h$

$\gamma_p$ shielded proton gyromagnetic ratio: $\gamma_p' = 2 \mu_p' / h$

$\gamma_e$ shielded helion gyromagnetic ratio: $\gamma_e' = 2 \mu_e' / h$

energy required to remove $n$ electrons from a neutral atom

$\Delta E_i (^{4}X^{+})$ electron ionization energies, $i = 0$ to $n - 1$

$\Delta V_{Ma}$ muonium ground-state hyperfine splitting

$\delta_{C}$ additive correction to the theoretical expression
for the electron ground-state $g$-factor in $^{12}$C$^{+}$

$\delta_{\alpha}$ additive correction to the theoretical expression
for the electron magnetic-moment anomaly $\alpha$

$\delta_{Ma}$ additive correction to the theoretical expression
for the ground-state hyperfine splitting of mu-
onium $\Delta V_{Ma}$

$\delta_{Si}$ additive correction to the theoretical expression
for the electron ground-state $g$-factor in $^{30}$Si$^{11+}$

$\delta_{\chi}(nL_{\gamma})$ additive correction to the theoretical expression
for an energy level of either hydrogen $H$ or deuterium $D$ with quantum numbers $n, L, J$

$\epsilon_j$ hypothetical correction to the Josephson effect
relation: $K_1 = (2e/h)(1 + \epsilon_j)$

$\epsilon_k$ hypothetical correction to the quantum-Hall-effect
relation: $R_K = (h/e^2)(1 + \epsilon_k)$

$\epsilon_e$ electric constant (vacuum electric permittivity): $\epsilon_0 = 1 / (\mu_0 c^2)$

$\pm$ symbol used to relate an input datum to its
observational equation

$\lambda(XK\alpha_1)$ wavelength of $K\alpha_1$ x-ray line of element $X$

$\mu$ symbol for either member of the muon-
anti-phonon pair; when necessary, $\mu^-$ or $\mu^+$ is used to
indicate the negative muon or positive muon

$\mu_B$ Bohr magneton: $\mu_B = e \hbar / 2 \mu_0$

$\mu_s$ nuclear magneton: $\mu_s = e \hbar / 2 m_p$

$\mu(X)$ magnetic moment of particle $X$ in atom or
molecule $Y$

$\mu_0$ magnetic constant (vacuum magnetic perme-
ability): $\mu_0 = 4 \pi \times 10^{-7}$ N/A$^2$

$\mu_X, \mu_Y$ magnetic moment, or shielded magnetic
moment, of particle $X$
\(\nu\) degrees of freedom of a particular adjustment
difference between muonium hyperfine splitting
Zeeman transition frequencies \(\nu_{34}\) and \(\nu_{12}\) at
a magnetic flux density \(B\) corresponding to the
free proton NMR frequency \(f_p\)
\(\sigma\) Stefan-Boltzmann constant:
\[\sigma = \frac{2 \pi^2 k^4}{15h^3 c^2}\]
\(\tau\) symbol for either member of the tau-antitau pair;
when necessary, \(\tau^-'\) or \(\tau^+\) is used to indicate the
negative tau or positive tau
\(\chi^2\)

\(\Omega_{90}\)
conventional “chi square”

\(\Omega_{90} = (R_K/R_{K-90})\Omega\)

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our many questions about their work. We wish to thank our
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and suggestions during the course of the 2014 adjustment
effort.

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