1	Research Note
2	Dimensional Analysis on Forest Fuel Bed Fire Spread ¹
3	Jiann C. Yang
4	Fire Research Division, Engineering Laboratory
5	National Institute of Standards and Technology
6	Gaithersburg, Maryland 20899 U.S.A.
7	E-mail: jiann.yang@nist.gov
8	

¹ Official contribution of the National Institute of Standards and Technology (NIST) not subject to copyright in the United States.

Abstract

A dimensional analysis was performed to correlate the fuel bed fire rate of 10 spread data previously reported in the literature. Under wind condition, six 11 pertinent dimensionless groups were identified, namely dimensionless fire 12 spread rate, dimensionless fuel particle size, fuel moisture content, 13 dimensionless fuel bed depth or dimensionless fuel loading density, 14 dimensionless wind speed, and angle of inclination of fuel bed. Under no-15 wind condition, five similar dimensionless groups resulted. Given the 16 uncertainties associated with some of the parameters used to estimate the 17 dimensionless groups, the dimensionless correlations using the resulting 18 dimensionless groups correlate the fire rates of spread reasonably well 19 20 under wind and no-wind conditions.

21

Key words: data correlation, dimensional analysis, flame spread, forestfuel, wildfires

25 Introduction

Research interest in fire spread along fuel beds has its origin in the study of 26 wildland fire behavior since the early 40s. In his pioneering work at the 27 U.S. Forest Service, Fons (1946) examined the effect of forest fuel size and 28 type, fuel bed compactness, fuel moisture content, wind velocity, and slope 29 30 on fire spread, provided a detailed analytical framework to understand the mechanism of fire spread, and lay down a sound foundation for subsequent 31 studies on fire spread along a forest fuel bed. Many studies from around 32 the world has since been conducted and appeared in the literature (Beaufait 33 1965; Rothermel and Anderson 1966; Anderson and Rothermel 1966; 34 Anderson 1969; Fang and Steward 1969; Pagni and Peterson 1973; Nelson 35 36 and Adkins 1986, 1988; Weise and Biging 1994; Dupuy 1995; Simeoni et al. 2001; Morandini et al. 2001; Viegas 2004a, 2004b; Weise et al. 2005, 37 2016; Zhou et al. 2005a, 2005b, 2005c, 2007; Morvan 2007, 2013, 2015; 38 Boboulos and Purvis 2009; Silvani and Morandini 2009; Anderson et al. 39 2010; Viegas et al. 2010; Dupuy et al. 2011; Dupuy and Maréchal 2011; 40 41 Pérez et al. 2011; Marino et al. 2012; Liu et al. 2014; Nelson 2015; Rossa et al. 2016; and Mulvaney et al. 2016). The reference list is by no means 42 exhaustive. Most of the studies fall into one of the two major categories: 43 (1) development of empirical fire spread models using experimental data 44 obtained in laboratories or in situ fuel complexes in the field under various 45

test conditions or (2) comparisons of predicted rate of spread using developed analytical or numerical models with experimental fire spread data. The availability of experimental data in the literature, especially the extensive tabulated data set with detailed experimental conditions in the recent work by Anderson et al. (2010) and the comprehensive database made available online by Weise et al. (2015), give impetus to the work presented here.

53

Dimensional analysis is a very powerful tool for correlating experimental 54 data in a compact and concise way. In addition, important dimensionless 55 groups could be identified and used for scaling to reduce experimental 56 57 efforts. The importance and the application of the theory of similitude to fire spread were mentioned in passing in Fons (1946). Pagni and Peterson 58 (1973) obtained a nondimensional flame spread rate as a function of 59 nondimensional fuel, flame, and ambient flow properties. Dimensional 60 analysis on the spread of wind-driven fires was carried out by Nelson and 61 62 Adkins (1988) to correlate the experimental data from their own studies and previous investigators with some success. Pérez et al. (2011) used 63 dimensional analysis in an attempt to develop scaling laws for the effect of 64 wind and slope on the fuel bed fire spread rate. The approach presented in 65 this paper differs from Nelson and Adkins (1988) and Pérez et al. (2011) in 66

that the primary variables used in the dimensional analysis to correlate fire spread rate are the initial conditions, thermo-physical properties of the fuel, configuration of the fuel bed, and the wind conditions. Flame properties were not included as primary variables in this analysis because they could, in principle, be correlated with fuel properties, fuel bed structure, and ambient conditions.

73

74 Dimensional Analysis

Fig. 1 is an illustration of the configuration considered in the analysis. Based on the experimental results reported in the literature, the fuel bed properties and geometry and ambient conditions were found to affect the rate of spread (ROS). To use dimensional analysis to correlate the data, we start by expressing the rate of spread as a function of the following relevant parameters, which characterize the fuel bed and ambient conditions.

81

82 (1)
$$S_p = f_1(A_b, \delta_b, \alpha_s, \beta_s, M_s, M_w, U, \theta)$$

83

Note that dimensional analysis does not provide specific nature of the function. The fuel volume-to-surface ratio β_s could be considered as a characteristic dimension of the fuel particle. Following the systematic matrix operation described in Langhaar (1951) on the dimensional matrix obtained from eq. 1, an application of the Buckingham π -theorem (Buckingham 1914) results in the following dimensionless groups.

90

91 (2)
$$\pi_1 = \frac{S_p \sqrt{A_b}}{\alpha_s}$$

- 92 (3) $\pi_2 = \frac{\beta_s}{\sqrt{A_b}}$
- 93 (4) $\pi_3 = \frac{M_w}{M_s}$

94 (5)
$$\pi_4 = \frac{\delta_b}{\sqrt{A_b}}$$

95 (6)
$$\pi_5 = \frac{U\sqrt{A_b}}{\alpha_s}$$

96 (7)
$$\pi_6 = \cos \theta$$

97

The variable θ needs not appear in the formal matrix operation because it is considered dimensionless (Langhaar, 1951), and we express π_6 in terms of $\cos\theta$ instead of simply θ to avoid the trivial (unrealistic) situation of $S_p \equiv 0$ when $\theta = 0$ (i.e., horizontal fuel bed) in the functional forms for the dimensionless correlations given below. The dimensionless group π_3 is, by definition, the moisture content (MC) (wet basis) of the fuel. The 104 dimensionless group π_4 is a dimensionless fuel bed depth. The 105 dimensionless group π_4 can also be expressed in terms of F_{ld} , $\sqrt{A_b}$, ρ_s ,

106 and
$$\varepsilon_b$$
. Since $V_b = A_b \delta_b$, $F_{ld} \equiv \frac{M_s}{A_b}$, and $\varepsilon_b \equiv 1 - \frac{V_s}{V_b}$,

107

108 (8)
$$\pi_4 = \frac{\delta_b}{\sqrt{A_b}} = \frac{V_b}{A_b\sqrt{A_b}} = \frac{F_{ld}V_b}{M_s\sqrt{A_b}} = \frac{F_{ld}V_s}{V_s\rho_s\sqrt{A_b}}\frac{V_b}{V_s} = \frac{F_{ld}}{\rho_s\sqrt{A_b}}\frac{1}{(1-\varepsilon_b)}$$

109

The dimensionless group π_4 can also be considered as a dimensionless fuel loading density. The functional form in eq. 1 can now be expressed in terms of the six dimensionless groups as $\pi_1 = g(\pi_2, \pi_3, \pi_4, \pi_5, \pi_6)$. If we assume the following functional form for the dimensionless correlation,

115 (9)
$$\pi_1 = a_1 [\pi_2]^{b_1} [\pi_3]^{b_2} [\pi_4]^{b_3} [\pi_5]^{b_4} [\pi_6]^{b_5}$$

116 or

117 (10)

118
$$\frac{S_{p}\sqrt{A_{b}}}{\alpha_{s}} = a_{1} \left[\frac{\beta_{s}}{\sqrt{A_{b}}}\right]^{b_{1}} \left[\frac{M_{w}}{M_{s}}\right]^{b_{2}} \left[\frac{F_{ld}}{\rho_{s}\sqrt{A_{b}}}\frac{1}{(1-\varepsilon_{b})}\right]^{b_{3}} \left[\frac{U\sqrt{A_{b}}}{\alpha_{s}}\right]^{b_{4}} [\cos\theta]^{b_{5}}$$

123 coefficients a₁, and b₁ to b₅ can be determined by performing a multiple
124 linear regression analysis on the logarithmic form of eq. 10.
125 Under no-wind condition, we could similarly express the fire spread rate as
126

Although other convenient functional form can be assumed to correlate the

dimensionless variables, the form expressed in eq. 9 is less complex and

amenable to simple statistical analysis. Using the experimental data, the

127 (11)
$$S_p = f_2(A_b, \delta_b, \alpha_s, \beta_s, M_s, M_w, \theta)$$

128

120

121

122

Following similar procedure, the resulting dimensionless groups are π_1 , π_2 , π_3 , π_4 , and π_6 . We will also use a functional form similar to eq. 10 to correlate the experimental data reported in the literature.

132

133 (12)
$$\frac{S_p \sqrt{A_b}}{\alpha_s} = a_2 \left[\frac{\beta_s}{\sqrt{A_b}} \right]^{c_1} \left[\frac{M_w}{M_s} \right]^{c_2} \left[\frac{F_{ld}}{\rho_s \sqrt{A_b}} \frac{1}{(1 - \varepsilon_b)} \right]^{c_3} \left[\cos \theta \right]^{c_4}$$

134

135 **Results and discussion**

Table 1 summarizes the data sources used to correlate the fire spread rate
using eq. 10 or eq. 12 depending on the wind conditions. The data sources
cover wide ranges of homogenous fuel species, fuel loading densities, fuel

moisture contents and wind speeds. The sources were selected solely based 139 140 on the completeness of the experimental data provided in the published works by the author(s) of the sources so that all dimensionless groups for 141 the dimensionless correlations could be estimated readily from the available 142 data. However, there is one caveat. Since the thermal diffusivities of most 143 144 of the fuels used in the studies were not given or known, a nominal thermal diffusivity α_{s} of 1.6×10^{-7} m²/s (Glass et al. 2010) was used for all the fuel 145 species in the calculations of the dimensionless groups π_1 and π_5 . 146

147

Whenever the experimental data were not given in tabulated forms in the 148 papers by the author(s) of the data sources, the data were extracted from the 149 figures using the following procedure. The figures were first digitally 150 scanned and followed by measuring the relative coordinates of the data 151 points on the scanned figures with respect to the origin of each figure using 152 a computer-aided design (CAD) software. The actual coordinates of the 153 154 data point were derived using a scale factor based on the distance of the two adjacent tick marks on the two axes of the scanned figure and their 155 corresponding values associated with the tick marks. 156

Note that moisture content expressed in fraction (not in percent) was used in the dimensionless variable, π_3 and was based on moist fuel (wet-basis). If the MC data reported in the literature were based on dry fuel, the wetbasis moisture content (in fraction) can be easily obtained using the following equation.

163

164 (13)
$$MC_{wet-basis} = \frac{MC_{dry-basis}}{1 + MC_{dry-basis}}$$

165

166 If δ_b was given, eq. 5 was used to calculate the dimensionless group π_4 , 167 whereas eq. 8 was used if F_{ld} , ρ_s , and ε_b were available.

168

Field data from Table 6 of Rothermel and Anderson (1966) and from Table 2 of Nelson and Adkins (1986), laboratory data of needles plus palmetto fronds from Table 1 of Nelson and Adkins (1986) and scrub oak data from Weise et al. (2015) were excluded in the correlations because the information needed to estimate some of the dimensionless groups was not available or given.

175

In some cases, as discussed below, informed and educated estimates wereused for the parameters in the dimensionless groups. When only a range of

the experimental parameter values associated with the test series was given, the average value was used, as a nominal value for that particular parameter, to calculate the dimensionless variables for that test series.

181

In the work of Anderson (1969), the fuel loading density and fuel bed surface area were not specified in each fire spread test, and only a range of values was given; $(0.612 \text{ kg/m}^2 - 1.223 \text{ kg/m}^2)$ for fuel loading density and $(0.0465 \text{ m}^2 - 0.0929 \text{ m}^2)$ for fuel bed surface area. A nominal fuel loading density of 0.917 kg/m² and fuel bed surface area of 0.0697 m² were used to correlate the experimental data from Anderson (1969).

188

Since β_s was not given in Nelson and Adkins (1986) for needles of pinus elliottii engelm, the needles were assumed to have a nominal length of 0.2 m and 2 mm in diameter. These values were used to estimate β_s in π_2 .

192

In the experimental studies of Dupuy (1995), tests were separately conducted in 1991 and 1993, and the results were given without reference to which year a specific test series was conducted; the fuel bed area was varied from 1 m² to 1.5 m² and moisture content from 1 % to 3 % in 1991 and 1.5 % to 3.5 % in 1993. Respective nominal values of 1.25 m^2 for fuel load density and 2.25 % for moisture content were assumed and used to correlate the data. In addition, only the median fire spread rates were used
in the correlations because individual data points were aggregated together
in the plot making them very difficult to extract.

202

203 Under no-wind condition, the fuel bed length varied from 4 m to 7.5 m was

stated in Anderson et al. (2010) and was not identified for each test. A

nominal value of 5.75 m was used to calculate A_b for all the no-wind test

206 data taken from Anderson et al. (2010).

207

If the void fractions of the fuel beds were not given, they were estimatedusing the following equation.

210

211 (14) $\varepsilon_b \equiv 1 - \frac{1}{\sigma \lambda + 1}$

212

213 where $\sigma \equiv 1/\beta_s$ and λ is defined in Anderson (1969) as

214

215 (15)
$$\lambda \equiv \frac{V_b - V_s}{\sigma V_s}$$

Eq. 14 can be easily derived using the definition of ε_b and eq. 15. If fuel bed porosity is not available, it can be estimated using the following formulae from Anderson (1969) with known σ , ρ_s and $\rho_b \equiv M_s / V_b$.

220

221 (16)
$$\sigma\lambda + 1 = \frac{\rho_s}{\rho_b}$$

222

The ratio ρ_s / ρ_b is sometimes termed the packing ratio of the fuel bed in the literature. Substituting eq. 16 into eq. 14, the void fraction can be expressed in terms of ρ_s / ρ_b .

226

227 (17)
$$\varepsilon_b \equiv 1 - \frac{\rho_b}{\rho_s}$$

228

The dimensional groups in eq. 10 and eq. 12 were calculated using the data sources listed in Table 1. Taking logarithm of both sides of the two equations and performing multiple linear regression analysis using the leastsquares method result in the following dimensionless correlations for wind and no-wind conditions. 234 (18)

235
$$\frac{S_{p}\sqrt{A_{b}}}{\alpha_{s}} = 1.03 \times 10^{-7} \left[\frac{\beta_{s}}{\sqrt{A_{b}}}\right]^{-0.41} \left[\frac{M_{w}}{M_{s}}\right]^{-0.82} \left[\frac{\delta_{b}}{\sqrt{A_{b}}}\right]^{-0.29} \left[\frac{U\sqrt{A_{b}}}{\alpha_{s}}\right]^{1.32} \left[\cos\theta\right]^{-13.98}$$

236 (19)
$$\frac{S_p \sqrt{A_b}}{\alpha_s} = 0.169 \left[\frac{\beta_s}{\sqrt{A_b}} \right]^{-1.27} \left[\frac{M_w}{M_s} \right]^{-0.068} \left[\frac{\delta_b}{\sqrt{A_b}} \right]^{-0.158} [\cos \theta]^{-6.249}$$

237

Table 2 shows the regression coefficients for the correlations and their 238 respective standard errors under wind and no-wind conditions. Figs. 2 and 239 240 3 show the dimensionless correlations using eq. 18 and eq. 19 respectively. A total of 334 data points for wind condition and 319 data for no-wind 241 conditions were extracted from the sources in Table 1 and used in the 242 regression analysis. The coefficient of correlations (R^2) for eq. 18 and eq. 243 19 are 0.83 and 0.66, respectively. The correlation under wind condition 244 correlate the experimental data slightly better than the no-wind condition; 245 246 this could be due to the fact that less data information needed to evaluate the dimensionless groups was given or available in the studies with no-wind 247 and more nominal values and informed estimates had to be used for some 248 of the parameters in the calculations. 249

251 Conclusions

A dimensional analysis was performed to correlate the fuel bed fire rate of 252 spread data previously reported in the literature. Under wind condition, six 253 pertinent dimensionless groups were identified, namely dimensionless fire 254 rate of spread, dimensionless fuel particle size, fuel moisture content, 255 256 dimensionless fuel bed depth or dimensionless fuel loading density, dimensionless wind speed, and angle of inclination of fuel bed. Under no-257 wind condition, five similar dimensionless groups resulted. The 258 dimensionless correlations using the resulting dimensionless groups 259 correlate the fire rates of spread reasonably well in light of the wide range 260 of uncertainties associated with some of the parameters used for the 261 262 calculations.

263

264 **References**

Anderson, H.E., and Rothermel, R.C. 1966. Influence of moisture and
wind upon the characteristics of free-burning fires. *In* Proceedings of
the 10th Symp. (Int.) Combust. The Combustion Institute, Pittsburgh,
PA. pp. 1009-1019.
Anderson, H.E. 1969. Heat transfer and fire spread. USDA For. Serv.,

270 Intermountain For. Range Exp. Stn. Res. Pap. INT-69.

271	Anderson, W.R., Catchpole, E.A., and Butler, B.W. 2010. Convective heat
272	transfer in fire spread through fine fuel beds. Int. J. Wildland Fire
273	19 (3): 284-298. <u>http://dx.doi.org/10.1071/WF09021</u> .
274	Beaufait, W.R. 1965. Characteristics of backfires and headfires in a pine
275	needle fuel bed. USDA For. Serv., Intermountain For. Range Exp. Stn.
276	Res. Note INT-39.
277	Boboulos, M., and Purvis, M.R.I. 2009. Wind and slope effect on ROS
278	during the fire propagation in East-Mediterranean pine forest litter.
279	Fire Saf. J. 44 (5): 764-769.
280	http://dx.doi.org/10.1016/j.firesaf.2009.03.006.
281	Buckingham, E. (1914) On physically similar systems; illustrations of the
282	use of dimensional equations. Phys. Rev. 4(4): 45–376.
283	http://doi:10.1103/PhysRev.4.345.
284	Dupuy, J.L. 1995. Slope and fuel load effects on fire behavior: Laboratory
285	experiments in pine needles fuel beds. Int. J. Wildland Fire 5(3): 153-
286	164. <u>http://dx.doi.org/10.1071/WF9950153</u> .
287	Dupuy, J.L., Maréchal, J., Portier, D., and Valette, J.C. 2011. The effects
288	of slope and fuel bed width on laboratory fire behavior. Int. J.
289	Wildland Fire 20 (2): 272-288. <u>http://dx.doi.org/10.1071/WF09075</u> .

290	Dupuy, J.L.	., and Maréchal,	J. 2011. Slop	be effect on	laboratory	fire spre	ad:
290	Dupuy, J.L.	, and marconar,	J. 2011. DIO		1aborator y	inc spr	L

- 291 contribution of radiation and convection to fuel bed preheating. Int. J.
- 292 Wildland Fires **20**(2): 289-307. <u>http://dx.doi.org/10.1071/WF09076</u>.
- ²⁹³ Fang, J.B., and Steward, F.R. 1969. Flame spread through randomly
- packed fuel particles. Combust. Flame **13**(4): 392-398.
- 295 <u>http://dx.doi.org/10.1016/0010-2180(69)90108-4</u>.
- 296 Fons, W.L. 1946. Analysis of fire spread in light forest fuels. J. Agric.
- 297 Res. **72**(13): 93-121.
- 298 Glass, S.V., and Zelinka, S.L. 2010 Moisture relations and physical
- 299 properties of wood. *In* Wood handbook: Wood as an engineering
- 300 material'. *Edited by* R.J. Ross. USDA For. Serv., For. Prod. Lab. Gen.
- 301 Tech. Rep. FPL-GTR-190. Madison, WI.
- 302 Langhaar, H.L. 1951. Dimensional analysis and theory of models. John
- 303 Wiley and Sons, New York.
- Liu, N., Wu, J., Chen, H., Xie, X., Zhang, L., Yao, B., Zhu, J., and
- 305 Shan, Y. 2014. Effect of slope on spread of a linear flame front over a
- ³⁰⁶ pine needle fuel bed: Experiments and modelling. Int. J. Wildland Fire
- 307 **23**(8): 1087-1096. <u>http://dx.doi.org/10.1071/WF12189</u>.
- 308 Marino, E., Dupuy, J.L., Pimont, F., Guijarro, M., Hernando, C., and Linn,
- 309 R. 2012. Fuel bulk density and fuel moisture content effects on fire
- 310 rate of spread: A comparison between FIRETEC model predictions

- and experimental results in shrub fuels. J. Fire Sci. **30**(4): 277-299.
- 312 <u>http://doi:10.1177/0734904111434286</u>.
- 313 Morandini, F., Santoni, P.A., and Balbi, J.H. 2001. The contribution of
- 314 radiant heat transfer to laboratory-scale fire spread under the
- influences of wind and slope. Fire Saf. J. **36**(6): 519-543.
- 316 http://dx.doi.org/10.1016/S0379-7112(00)00064-3.
- 317 Morvan, D. 2007. A numerical study of flame geometry and potential for
- crown fire initiation for a wildfire propagating through shrub fuel. Int.
- 319 J. Wildland Fire **16**(5): 511-518. <u>http://dx.doi.org/10.1071/WF06010</u>.
- Morvan, D. 2013. Numerical study of the effect of fuel moisture content
- 321 (FMC) upon the propagation of a surface fire on a flat terrain. Fire Saf.
- 322 J. **58**, 121–131. <u>http://dx.doi.org/10.1016/j.firesaf.2013.01.010</u>.
- Morvan, D. 2015. Numerical study of the behaviour of a surface fire
- 324 propagating through a firebreak built in a Mediterranean shrub layer.
- 325 Fire Saf. J. **71**, 34–48. <u>http://dx.doi.org/10.1016/j.firesaf.2014.11.012</u>.
- Mulvaney, J.L., Sullivan, A.L., Cary, G.J., and Bishop, G.R. 2016.
- 327 Repeatability of free-burning fire experiments using heterogeneous
- forest fuel beds in a combustion wind tunnel. Int. J. Wildland Fire
- 329 **25**(4): 445-455. <u>http://dx.doi.org/10.1071/WF15068</u>.

330	Nelson,	R.M.,	Jr.,	and	Adkins,	C.W.	1986.	Flame	characteristics	of	wind
		,									

- driven surface fires. Can. J. For. Res. **16**(6): 1293-1300.
- 332 <u>http://doi:10.1139/x86-229</u>.
- 333 Nelson, R.M., Jr., and Adkins, C.W. 1988. A dimensionless correlation for
- the spread of wind-driven fires. Can. J. For. Res. **18**(4): 391-397.
- 335 <u>http://doi:10.1139/x88-058</u>.
- Nelson, R.M., Jr. 2015. Re-analysis of wind and slope effects on flame
- 337 characteristics of Mediterranean shrub fires. Int. J. Wildland Fire
- 338 **24**(7): 1001-1007. <u>http://dx.doi.org/10.1071/WF14155</u>.
- 339 Pagni, P.J., and Peterson, T.G. 1973. Flame spread through porous fuels. In
- Proceedings of the 14th Symp. (Int.) Combust. The Combustion
 Institute, Pittsburgh, PA. pp. 1099–1106.
- 342 Pérez, Y., Pastor, E., Àgueda, A., and Planas, E. 2011. Effect of wind and
- 343 slope when scaling the forest fires rate of spread of laboratory
- 344 experiments. Fire Technol. **47**(2): 475-489. <u>http://doi:10.1007/s10694-</u>
- 345 <u>010-0168-7</u>.
- Rossa, C.G., Veloso, R., and Fernandes, P.M. 2016. A laboratory-based
- 347 quantification of the effect of live fuel moisture content on fire spread
- 348 rate. Int. J. Wildland Fire **25**(5): 569-573.
- 349 <u>http://dx.doi.org/10.1071/WF15114</u>.

350	Rothermel, R.C., and Anderson, H.E. 1966. Fire spread characteristics
351	determined in the laboratory. USDA For. Serv., Intermountain For.
352	Range Exp. Stn. Res. Pap. INT-30.
353	Silvani, X., and Morandini, F. 2009. Fire spread experiments in the field:
354	Temperature and heat fluxes measurements. Fire Saf. J. 44(2): 279-
355	285. http://dx.doi.org/10.1016/j.firesaf.2008.06.004.
356	Simeoni, A., Santoni, P.A., Larini, M., and Balbi, J.H. 2001. On the wind
357	advection influence on the fire spread across a fuel bed: Modelling by
358	a semi-physical approach and testing with experiments. Fire Saf. J.
359	36 (5): 491-513. <u>http://dx.doi.org/10.1016/S0379-7112(00)00063-1</u> .
360	Viegas, D.X. 2004a. On the existence of a steady state regime for slope
361	and wind driven fires. Int. J. Wildland Fire 13(1): 101-117.
362	http://dx.doi.org/10.1071/WF03008.
363	Viegas, D.X. 2004b. Slope and wind effects on fire propagation. Int. J.
364	Wildland Fire 13 (2): 143-156. <u>http://dx.doi.org/10.1071/WF03046</u> .
365	Viegas, D.X., Almeida, M., Miranda, A., and Ribeiro, L.M. 2010. Linear
366	model for spread rate and mass loss rate for mixed-size fuel beds. Int.

- 367 J. Wildland Fire **19**(5): 531-540. <u>http://dx.doi.org/10.1071/WF09007</u>.
- Weise, D.R., and Biging, G. 1994. Effects of wind velocity and slope on
- 369 fire behavior. *In* Fire Safety Science Proceedings of the 4th
- International Symposium, Ottawa, Canada, 13-17 July 1994.

- 371 International Association of Fire Safety Science, London, UK. pp.
- 372 1041–1051. <u>http://dx.doi.org/10.3801/IAFSS.FSS.4-1041</u>.
- Weise, D.R., Zhou, X., Sun, L., and Mahalingam, S. 2005. Fire spread in
- 374 chaparral 'Go or no-go?'. Int. J. Wildland Fire **14**(1): 99-106.
- 375 <u>http://dx.doi.org/10.1071/WF04049</u>.
- Weise, D.R., Zhou, X., Mahalingam, S., and Chong, J. 2015. Marginal fire
- 377 spread in live fuel beds Horizontal fuels. USDA For. Serv. Res. Data
- Arch., archived data and computer code RDS-2015–0007. Fort
- 379 Collins, CO. <u>http://doi:10.2737/RDS-2015-0007</u>.
- 380 Weise, D.R., Koo, E., Zhou, X., Mahalingam, S., Morandini, F., and
- 381 Balbi, J.H. 2016. Fire spread in chaparral A comparison of
- laboratory data and model predictions in burning live fuels. Int. J.
- 383 Wildland Fire **25**(9): 980-994. <u>http://dx.doi.org/10.1071/WF15177</u>.
- Zhou, X., Mahalingam, S., and Weise, D. 2005. Experimental modeling of
- the effect of terrain slope on marginal burning. *In* Fire Safety Science
- Proceedings of the 8th International Symposium, Beijng, China, 18-
- 387 23 September 2005. International Association of Fire Safety Science,
- 388 London, UK. pp. 863–874. <u>http://doi:10.3801/IAFSS.FSS.8-863</u>.
- Zhou, X., Mahalingam, S., and Weise, D. 2005. Modeling of marginal
- burning state of fire spread in live chaparral shrub fuel bed. Combust.

- 391 Flame **143**(3): 183–198.
- 392 <u>http://dx.doi.org/10.1016/j.combustflame.2005.05.013</u>.
- Zhou, X., Weise, D., and Mahalingam, S. 2005. Experimental
- 394 measurements and numerical modeling of marginal burning in live
- chaparral fuel beds. Proc. Combust. Inst. **30**: 2287–2294.
- 396 <u>http://doi:10.1016/J.PROCI.2004.08.022</u>.
- 397 Zhou, X., Mahalingam, S., and Weise, D. 2007. Experimental study and
- ³⁹⁸ large eddy simulation of effect of terrain slope on marginal burning in
- 399 shrub fuel beds. Proc. Combust. Inst. **31**: 2547–2555.
- 400 <u>http://doi:10.1016/J.PROCI.2006.07.222</u>.

402	List of symbols	
403	<i>a</i> ₁ , <i>a</i> ₂	regression coefficients
404	b_1, b_2, b_3, b_4, b_5	regression coefficients
405	<i>c</i> ₁ , <i>c</i> ₂ , <i>c</i> ₃ , <i>c</i> ₄	regression coefficients
406	A_b	fuel bed surface area (m ²)
407	F_{ld}	fuel loading density (kg/m ²)
408	M_{s}	moist fuel mass (kg)
409	M_{w}	water content of fuel (kg)
410	S_{p}	fire spread rate (m/s)
411	U	wind speed (m/s)
412	V_b	fuel bed volume (m ³)
413	V_s	fuel volume (m ³)
414	α_s	fuel thermal diffusivity (m ² /s)
415	β_s	fuel volume-to-surface ratio (m)
416	δ_b	fuel bed thickness (m)
417	ε_{b}	fuel bed void fraction
418	θ	inclined angle of fuel bed (°)
419	λ	fuel bed porosity (m)
420	$ ho_b$	fuel bed bulk density (kg/m ³)

421	ρ _s	fuel density (kg/m ³)
422	σ	fuel surface-to-volume ratio (m ⁻¹)
423		

Data sources	Wind	No wind	Slope
Tables 1-2 (Fons 1946)	\checkmark		
Figure 1 (Beaufait 1965)	\checkmark	\checkmark	
Tables 2-5 (Rothermel and Anderson 1966)	\checkmark	\checkmark	
Tables 1-2 (Anderson 1969)		\checkmark	
Figures 2-5 (Fang and Steward 1969)		\checkmark	
Table 1 (Nelson and Adkins 1986)	\checkmark		
Figure 2 (Dupuy 1995)		\checkmark	✓a
Figures 6, 8, and 9 (Simeoni et al. 2001)	\checkmark	\checkmark	✓b
Appendix table (Anderson et al. 2010)	\checkmark	\checkmark	
Tables 1 and 3 (Dupuy and Maréchal 2011)		\checkmark	√ ^c
Weise et al. (2015)	\checkmark	\checkmark	✓d

 Table 1. Data sources used for the dimensionless correlations.

^a $-30^{\circ} \le \theta \le 20^{\circ}$; ^b $0^{\circ} \le \theta \le 10^{\circ}$; ^c $0^{\circ} \le \theta \le 30^{\circ}$; ^d $-30^{\circ} \le \theta \le 35^{\circ}$

b_1	b_2	b_3	b_4	b_5	c_1	c_2	<i>c</i> ₃	c_4
-0.41	-0.82	-0.29	1.32	-13.98	-1.27	-0.07	-0.16	-6.25
(0.045)	(0.052)	(0.048)	(0.046)	(5.344)	(0.074)	(0.057)	(0.058)	(0.713)

 Table2. Regression coefficients and their standard errors

Figure captions

Fig. 1. A schematic showing the pertinent parameters used in the dimensional analysis.

Fig. 2. Dimensionless correlation for fuel bed fire spread under wind condition (334 data points).

Fig. 3. Dimensionless correlation for fuel bed fire spread under no-wind condition (319 data points).



Fig. 1. A schematic showing the pertinent parameters used in the dimensional analysis.



Fig. 2. Dimensionless correlation for fuel bed fire spread under wind

condition (334 data points).



Fig. 3. Dimensionless correlation for fuel bed fire spread under no-

wind condition (319 data points).