Detector-Independent Verification of Quantum Light

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We introduce a method for the verification of nonclassical light which is independent of the complex interaction between the generated light and the material of the detectors, which are in our work superconducting transition-edge sensors. This is achieved by an optical multiplexing scheme. The measured coincidence statistics is shown to be a mixture of multinomial distributions for any classical light field and any type of detector. This allows us to formulate bounds for the statistical properties of classical states. We apply our directly accessible method to heralded multi-photon states which are detected with a single multiplexing step only and two transition-edge sensors. The nonclassicality of the generated light is verified and characterized through the violation of the classical bounds without the need for characterizing the used detectors.

Introduction.— The generation and verification of nonclassical light is one of the main challenges for realizing optical quantum communication and computation [1–4]. The protocols and the needed resources are becoming more and more sophisticated. However, robust and easily applicable methods are required to verify quantum features in order to employ quantum states of light for real-world applications; see, e.g., [5].

The complexity of producing reliable sensors stems from the problem that new detectors need to be characterized initially. For this task, various techniques have been proposed, e.g., detector tomography [6–11]. However, such a calibration requires many resources, for example, computational/numerical efforts, reference measurements, etc. Only after this handling, the interaction between quantum light and the bulk material of the detector can be inferred and quantum features can be uncovered. The latter verification of quantumness also depends on the bare existence of criteria that are applicable to this measurement. Here, we will prove that detectors with a general response to incident light can be employed in an optical detection scheme, which is well-characterized, to identify nonclassical radiation fields based on simple nonclassicality conditions.

The concept of device-independent quantumness has recently gained a lot of importance, because it allows one to employ even untrusted devices; see, e.g., [12]. For instance, device-independent entanglement witnesses can be used without relying on properties of the measurement system [13, 14]. It has been further studied to treat protocols for communication and computation tasks [15, 16]. The concept of detector-independence has been also applied to state estimation and quantum metrology [17, 18] for the aim of gaining knowledge about a physical system which might be too complex for a full characterization.

An equivalently remarkable progress has been made in the field of well-characterized photon-number-resolving (PNR) detectors [19, 20]. A charge-coupled-device camera is one example of a system that can record many photons at a time. The correlation between different pixels can be used to infer quantum correlated light [21, 22]. Another example of a PNR device is a superconducting transition-edge sensor (TES) [23–25]. This detector requires a cryogenic environment and its operation is based on superconductivity. Hence, a proper detection model for this detector would require the quantum mechanical treatment of a solid-state bulk material which interacts with a quantized radiation field in the frame of low-temperature physics.

Along with the development of PNR detectors, multiplexing layouts define another approach to realize photon-number resolution [26–29]. The main idea is that an incident light field, which consists of many photons, is split into a number of spatial or temporal modes, which consist of a few photons only. These resulting beams are measured with single-photon detectors which do not have any photon-number-resolution capacity. They can only discriminate between the presence ("click") and absence of absorbed photons. Hence, the multiplexing is used to get some insight into the photon-statistics despite the limited capacity of the individual detectors. With resulting click-counting statistics, one can verify nonclassical properties of correlated light fields [30–34]. Recently, a multiplexing layout has been used in combination with TESs to characterize quantum light with a mean photon number of 50 and a resolution of up to 80 photons for each of the two correlated modes [35].

In this Letter, we formulate and apply a method to verify quantum light with arbitrary detectors. This technique is based on a well-defined multiplexing scheme and individual detectors which can discriminate different measurement outcomes. The resulting correlation measurement is always described as a mixture of multinomial distributions in classical optics. Based on this finding, we formulate nonclassicality conditions in terms of covariances whose violation directly certifies nonclassical light. We implement our approach for quantum light which is produced by heralding photon-number states from a parametric down-conversion (PDC) source. We show that a single multiplexing step is already sufficient to verify the nonclassicality of such states without the need of characterizing the used TESs.

Theory.— The detection scenario under study is shown in Fig. 1. Its detector-independence is achieved by the optical multiplexing layout whose optical elements, e.g., beam splitters, are much simpler and better characterized than the detectors. Our only requirement for the following detection model is that the measured statistics are the same at each detector. In contrast to the typical idea of multiplexing, e.g., in Ref. [35], we do not seek for a higher photon-number resolution, but we employ this scheme for obtaining a measurement model which does not depend on the properties of the individual detectors.

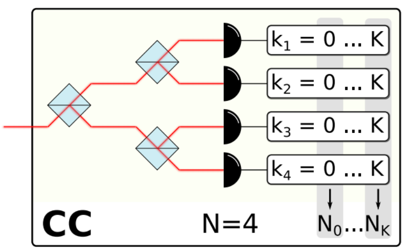

FIG. 1. (Color online) Multiplexed click-counting (CC) layout consisting of $N = 4$ individual detectors. An incident light field is split into N beams with identical intensities. Each of the N identical detectors returns a measurement outcome k_n. The number of detectors N_k with the same outcome $0 \leq k \leq K$ is recorded.

First, we consider a single coherent, classical light field. Suppose the detector can resolve the outcomes $k = 0, \ldots, K$—or, equivalently, $K + 1$ bins—which have a probability p_k. Hence, the probability to have a coincidence $(k_1, \ldots, k_N)$ from the N individual detectors is $p_{k_1} \cdots p_{k_N}$ after the 50/50 splittings in Fig. 1. Further on, the integer N_k represents the number of individual detectors that measure simultaneously the outcome k. This means we have N_0-times the outcome 0 together with N_1-times the outcome 1, etc. from the $N = N_0 + \cdots + N_K$ detectors. The probability to get $(N_0, \ldots, N_K)$ in such scenario is known to be described by a multinomial distribution [36],

$$c(N_0, \ldots, N_K) = \frac{N!}{N_0! \cdots N_K!} p_0^{N_0} \cdots p_K^{N_K}. \tag{1}$$

Note that we counter a deviation from the 50/50 splitting by including a corresponding systematic error.

For a different intensity, the probabilities p_k of the individual outcomes k might change. Hence, if we consider a statistical mixture of arbitrary intensities, we can generalize the distribution in Eq. (1) by averaging over a classical probability distribution P,

$$\begin{aligned} c(N_0, \ldots, N_K) &= \left\langle \frac{N!}{N_0! \cdots N_K!} p_0^{N_0} \cdots p_K^{N_K} \right\rangle \\ &= \int dP(p_0, \ldots, p_K) \frac{N!}{N_0! \cdots N_K!} p_0^{N_0} \cdots p_K^{N_K}. \end{aligned} \tag{2}$$

Because any light field in classical optics can be considered as an ensemble of coherent fields [38, 39], the measured statistics of the setup in Fig. 1 can be described as a mixture of multinomial distributions (2). This is not necessarily true for quantum light as we will demonstrate. The distribution (2) applies to arbitrary detectors and includes the case of on-off detectors ($K = 1$), which yields a binomial distribution and has been previously considered [37]. In addition, we determine the number of outcomes, $K + 1$, directly from our data.

Let us now formulate a criterion that allows for the identification of quantum correlations. The mean values of multinomial statistics obey $\overline{N_k} = N p_k$ [36]. Averaging over P yields

$$\overline{N_k} = N \langle p_k \rangle . \tag{3}$$

In the same way, we get for the second-order moments, $\overline{N_k N_{k'}} = N(N-1) p_k p_{k'} + \delta_{k,k'} N p_k$ [36] with $\delta_{k,k'} = 1$ for $k = k'$ and $\delta_{k,k'} = 0$ otherwise, an averaged expression

$$\overline{N_k N_{k'}} = N(N-1) \langle p_k p_{k'} \rangle + \delta_{k,k'} N \langle p_k \rangle . \tag{4}$$

Thus, we find the covariance from Eqs. (3) and (4),

$$\begin{aligned} \overline{\Delta N_k \Delta N_{k'}} =& N \langle p_k \rangle (\delta_{k,k'} - \langle p_{k'} \rangle) \\ &+ N(N-1) \langle \Delta p_k \Delta p_{k'} \rangle . \end{aligned} \tag{5}$$

Note that the multinomial distribution has the covariances $\overline{\Delta N_k \Delta N_{k'}} = N p_k (\delta_{k,k'} - p_{k'})$ [36]. Multiplying Eq. (5) with N and using Eq. (3), we can introduce the $(K+1) \times (K+1)$ matrix

$$\begin{aligned} M =& \left(N \overline{\Delta N_k \Delta N_{k'}} - \overline{N_k} (N \delta_{k,k'} - \overline{N_{k'}}) \right)_{k,k'=0,\ldots,K} \\ =& N^2 (N-1) \left(\langle \Delta p_k \Delta p_{k'} \rangle \right)_{k,k'=0,\ldots,K} . \end{aligned} \tag{6}$$

As the covariance matrix $(\langle \Delta p_k \Delta p_{k'} \rangle)_{k,k'}$ is nonnegative for any classical probability distribution P, we can conclude: We have a nonclassical light field if

$$0 \not\leq \left(N \overline{\Delta N_k \Delta N_{k'}} - \overline{N_k} (N \delta_{k,k'} - \overline{N_{k'}}) \right)_{k,k'=0,\ldots,K} , \tag{7}$$

i.e., the symmetric matrix M in Eq. (6) is not positive semidefinite. In other words, $M \ngeq 0$ means that the fluctuations of the parameters p_k in $(\langle \Delta p_k \Delta p_{k'} \rangle)_{k,k'}$ are below the classical threshold of zero. Based on condition (7), we will experimentally certify nonclassicality.

Experimental setup.— Our experimental implementation is outlined in Fig. 2(a). A PDC source produces correlated photons. Conditioned on the detection of k clicks from the heralding detector, we measure the click-counting statistics $c(N_0, \ldots, N_K)$, see also Eq. (2). Besides the standard optical elements, the key components of our experiment are (i) the PDC source and (ii) the three TESs used as our heralding detector and as our two individual detectors after the multiplexing step.

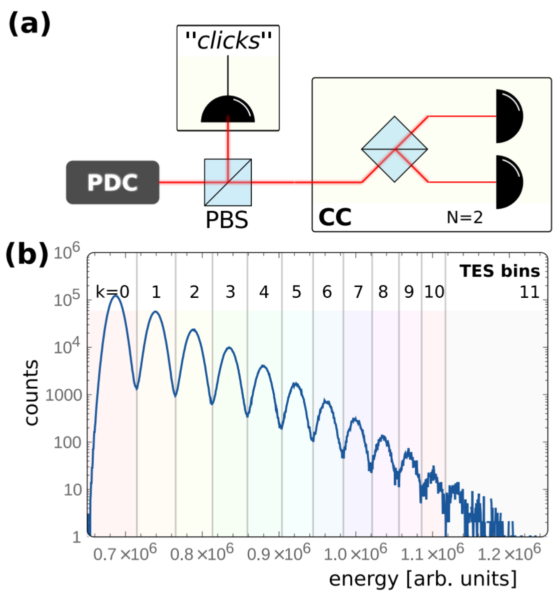

FIG. 2. (Color online) Panel (a) depicts an outline of the experiment. A PDC source produces correlated photon pairs which are separated with a polarizing beam splitter (PBS). A conditioning to a certain outcome (labeled as "click") of a single TES yields a certain number of photons in the other beam. The latter signal is measured with a multiplexing scheme that consists of $N = 2$ TESs [cf. Fig. 1]. Panel (b) shows the binning into $K + 1$ possible outcomes (bins). The energies that are counted with a TES (shown for the heralding detector) can be separated into 12 bins.

(i) PDC source. Our PDC source is a waveguide-written 8 mm-long periodically poled potassium titanyl phosphate crystal. We pump a type-II spontaneous PDC process with laser pulses at 775 nm and a full width at half maximum of 2 nm at a repetition rate of 75 kHz. The heralding idler mode (horizontal polarization) is centered at 1554 nm, while the signal mode (vertical polarization) is centered at 1546 nm. The output signal and idler pulses are spatially separated with a PBS. The pump beam is discarded using an edge filter. Subsequently, they are filtered by a 3 nm bandpass filters in order to filter out the broadband background which is typically generated in dielectric nonlinear waveguides [40].

(ii) TES detectors. We use superconducting TESs [23], provided by NIST, as our detectors. They consist of $25\,\mu\mathrm{m} \times 25\,\mu\mathrm{m} \times 20\,\mathrm{nm}$ slabs of tungsten inside an optical cavity designed to maximize absorption at the desired wavelengths. They are maintained at their transition temperature by Joule heating caused by a voltage bias, which is self-stabilized via an electro-thermal feedback effect [41]. When photons are absorbed, the increase in temperature causes a corresponding electrical signal which is picked up and amplified by a superconducting quantum interference device (SQUID) module and subsequently amplified at room temperature. This results in complex time-varying signals of about $5\,\mu$s duration which fall into clearly distinguishable bins [42]. Our TESs are operated within a dilution refrigerator with a base temperature of about 70 mK. They have an estimated detection efficiency of $0.98^{+0.02}_{-0.08}$ [42]. The electrical throughput is measured using a waveform digitizer and assign a bin (described below) to each output pulse [43]. We process incoming signals at a speed of up to 100 kHz.

The time-integral of the measured signal results in an energy whose counts are shown in Fig. 2(b) for the heralding TES. The energies are binned into $K+1$ different intervals. One typically fits such a signal with a number of Gaussian distributions to infer the photon statistics. As our technique of multinomial distributions (2) does not rely on a particular binning of the outcomes—as it is detector-independent—, we can make a much simpler division into disjoint energy intervals. Above a certain threshold energy, no further peaks can be significantly resolved—note the logarithmic scale in Fig. 2(b)—and those events are collected in the last bin. No measured event is discarded. Our heralding TES therefore allows for a resolution of $K + 1 = 12$ outcomes. Due to the splitting of the photons on the beam splitter in the multiplexing step, the other two TESs allow for a reduced distinction between $K + 1 = 8$ outcomes. Let us stress that we have not assumed any detection model for the TESs to perform this binning and that even the value K is inferred from the data themselves.

Results.— The nonclassicality in terms of the condition (7), $0 \not\leq M$, can be directly applied to the measured statistics $c(N_0, \ldots, N_K)$ by sampling its mean values, variances, and covariances [Eq. (6)]. In Fig. 3, we show the resulting nonclassicality of the heralded states. As the minimal eigenvalue of the matrix M has to be nonnegative for classical light, this eigenvalue is depicted in Fig. 3 for demonstrating the nonclassicality [32]. A systematic error is included that stems from the fact that the measured statistics of the two TESs in the multiplexing scheme are not perfectly identical.

To discuss our results, we compare our findings with a simple, idealized model. Our produced PDC state can be approximated by a two-mode squeezed-vacuum state which has a correlated photon statistics, $p(n, n') = (1 - \lambda)\lambda^n \delta_{n,n'}$, where $n(n')$ is the signal(idler) photon number and $r \geq 0$ ($\lambda = \tanh^2 r$) is the squeezing parameter which is a function of the pump power of the PDC process [44].

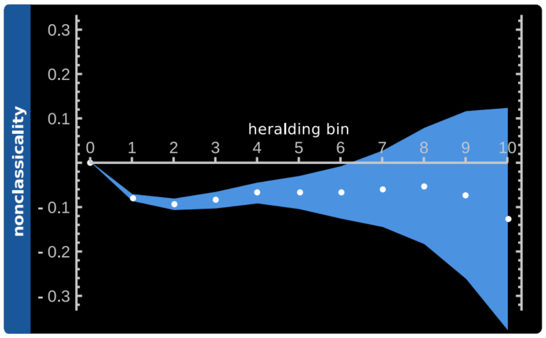

FIG. 3. (Color online) The minimal eigenvalue of the matrix M in Eq. (6) is shown including its error bars (shaded area) as a function of the generated states, which are defined by the bin of the heralding TES. A negative value is inconsistent with classical optics and, therefore, verifies quantum light.

Heralding with an ideal PNR detector, which can resolve any photon number with a finite efficiency $\tilde{\eta}$, we get a conditioned statistics of the form

$$\begin{aligned} p(n|k) =& \mathcal{N}_k \binom{n}{k} \tilde{\eta}^k (1-\tilde{\eta})^{n-k} (1-\lambda)\lambda^n, \\ \text{with } \mathcal{N}_k =& \frac{(1-\lambda)(\lambda\tilde{\eta})^k}{[1-\lambda(1-\tilde{\eta})]^{k+1}}, \end{aligned} \tag{8}$$

for the kth heralded state and $p(n|k) = 0$ for $n < k$ and $\lambda^0 = 1$. Here $\mathcal{N}_k$ is a normalization constant as well as the probability that the kth state is realized. The signal includes at least $n \geq k$ photons if k photoelectric counts have been recorded by the heralding detector.

In the ideal case, the heralding to the 0th bin yields a thermal state [Eq. (8)] and for small squeezing a vacuum state. That is a coherent state with a mean photon number of zero, $p(n|0) = \delta_{n,0}$ for $\lambda \to 0$. Hence, we expect that the measured statistics is close to a multinomial one and, therefore, $M \approx 0$. Our data are consistent with this consideration, cf. Fig. 3.

An ideal heralding to higher bin numbers gives a nonclassical Fock state with the corresponding photon number. The nonclassical character of the experimentally realized multi-photon states is certified in Fig. 3. The generation of k photon pairs in the PDC is less likely for higher photon numbers, $\mathcal{N}_k \propto \lambda^k$. Hence, this reduced count rate of events results in the increasing error in Fig. 3. The highest significance of nonclassicality is found for lower heralding bins.

Moreover, one gets higher mean photon numbers for increasing pump powers of the PDC process according to the model in Eq. (8). To demonstrate the impact on the nonclassicality, we also studied our criterion (7) as a function of the pump power in Fig. 4. The conditioning to zero clicks of the heralding TES is consistent with a classical signal. For higher heralding bins, we observe that the nonclassicality is larger for decreasing pump powers as the distribution in Eq. (8) becomes closer to a pure Fock state. We can also observe in Fig. 4 that the error is larger for smaller pump powers as fewer photon pairs are generated ($\mathcal{N}_k \propto \lambda^k$) within a definite measurement time. Note that the nonclassicality is examined in terms of the photon-number correlations. If our detector would allow for a phase resolution, we could observe the increase of squeezing with increasing pump power, which could be a future enhancement of the current setup.

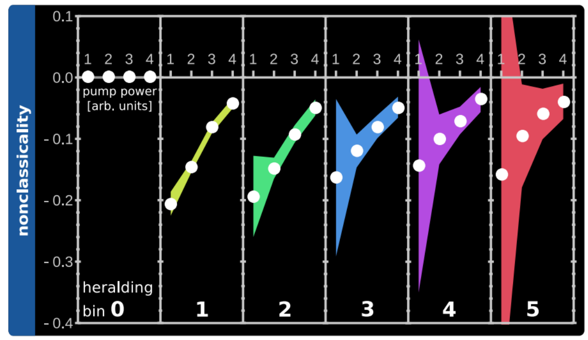

FIG. 4. (Color online) The minimal eigenvalue of M of the first six heralded states is shown as a function of the pump power. The nonclassicality (negative values) decreases with increasing power. However, the verification is more significant for higher pump powers.

Conclusions.— We have formulated and implemented a robust and easily accessible method that allows for the verification of nonclassical light with arbitrary detectors. Based on a multiplexing layout, we showed that a mixture of multinomial distributions describes the measured statistics in classical optics independently of the specific properties of the individual detectors. Subsequently, we could derive bounds to the covariance matrix whose violation is a clear signature of quantum light. Using a single multiplexing step and two superconducting transition-edge sensors, we successfully probed the nonclassicality of heralded multi-photon states. We also studied the dependence of the nonclassicality on the pump power of our spontaneous parametric-down-conversion light source and we could confirm the expected nonclassical properties of the generated states. Additional results and details on the performed analysis can be found in Ref. [45].

Our method is a straightforward technique that also applies to, e.g., temporal multiplexing or other types of individual detectors, e.g., multi-pixel cameras. Our nonclassicality analysis is only based on covariances between different outcomes which requires neither sophisticated data processing nor a lot of computational time. Hence, it presents a simple and yet reliable tool for characterizing quantum light for applications in quantum technologies.

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[1] E. Knill, R. Laflamme, and G. J. Milburn, A scheme for efficient quantum computation with linear optics, Nature (London) **409**, 46 (2001).

[2] P. Kok, W. J. Munro, K. Nemoto, T. C. Ralph, J. P. Dowling, and G. J. Milburn, Linear optical quantum computing with photonic qubits, Rev. Mod. Phys. **79**, 135 (2007).

[3] N. Gisin and R. Thew, Quantum communication, Nat. Photon. **1**, 165 (2007).

[4] J. H. Shapiro, The Quantum Theory of Optical Communications, IEEE J. Sel. Top. Quantum Electron. **15**, 1547 (2009).

[5] N. Thomas-Peter, B. J. Smith, A. Datta, L. Zhang, U. Dorner, and I. A. Walmsley, Real-World Quantum Sensors: Evaluating Resources for Precision Measurement, Phys. Rev. Lett. **107**, 113603 (2011).

[6] A. Luis and L. L. Sánchez-Soto, Complete Characterization of Arbitrary Quantum Measurement Processes, Phys. Rev. Lett. **83**, 3573 (1999).

[7] G. M. D'Ariano, L. Maccone, and P. Lo Presti, Quantum Calibration of Measurement Instrumentation, Phys. Rev. Lett. **93**, 250407 (2004).

[8] M. Lobino, D. Korystov, C. Kupchak, E. Figueroa, B. C. Sanders, and A. I. Lvovsky, Complete characterization of quantum-optical processes, Science **322**, 563 (2008).

[9] J. S. Lundeen, A. Feito, H. Coldenstrodt-Ronge, K. L. Pregnell, C. Silberhorn, T. C. Ralph, J. Eisert, M. B. Plenio, and I. A. Walmsley, Tomography of quantum detectors, Nat. Phys. **5**, 27 (2009).

[10] L. Zhang, A. Datta, H. B. Coldenstrodt-Ronge, X.-M. Jin, J. Eisert, M. B. Plenio, and I. A. Walmsley, Recursive quantum detector tomography, New J. Phys. **14**, 115005 (2012).

[11] G. Brida, L. Ciavarella, I. P. Degiovanni, M. Genovese, A. Migdall, M. G. Mingolla, M. G. A. Paris, F. Piacentini, and S. V. Polyakov, Ancilla-Assisted Calibration of a Measuring Apparatus, Phys. Rev. Lett. **108**, 253601 (2012).

[12] J. Kaniewski and S. Wehner, Device-independent two-party cryptography secure against sequential attacks, New J. Phys. **18**, 055004 (2016).

[13] C. Branciard, D. Rosset, Y.-C. Liang, and N. Gisin, Measurement-Device-Independent Entanglement Witnesses for All Entangled Quantum States, Phys. Rev. Lett. **110**, 060405 (2013).

[14] Q. Zhao, X. Yuan, and X. Ma, Efficient measurement-device-independent detection of multipartite entanglement structure, Phys. Rev. A **94**, 012343 (2016).

[15] C. C. W. Lim, B. Korzh, A. Martin, F. Bussieres, R. Thew, and H. Zbinden, Detector-Device-Independent Quantum Key Distribution, Appl. Phys. Lett. **105**, 221112 (2014).

[16] A. Gheorghiu, E. Kashefi, and P. Wallde, Robustness and device independence of verifiable blind quantum computing, New J. Phys. **17**, 083040 (2015).

[17] M. Cooper, M. Karpinski, and B. J. Smith, Quantum state estimation with unknown measurements, Nat. Commun. **5**, 4332 (2014).

[18] M. Altorio, M. G. Genoni, F. Somma, and M. Barbieri, Metrology with Unknown Detectors, Phys. Rev. Lett. **116**, 100802 (2016).

[19] C. Silberhorn, Detecting quantum light, Contemp. Phys. **48**, 143 (2007).

[20] R. H. Hadfield, Single-photon detectors for optical quantum information applications, Nat. Photon. **3**, 696 (2009).

[21] J.-L. Blanchet, F. Devaux, L. Furfaro, and E. Lantz, Measurement of Sub-Shot-Noise Correlations of Spatial Fluctuations in the Photon-Counting Regime, Phys. Rev. Lett. **101**, 233604 (2008).

[22] P.-A. Moreau, J. Mougin-Sisini, F. Devaux, and E. Lantz, Realization of the purely spatial Einstein-Podolsky-Rosen paradox in full-field images of spontaneous parametric down-conversion, Phys. Rev. A **86**, 010101(R) (2012).

[23] A. E. Lita, A. J. Miller, and S. W. Nam, Counting near-infrared single-photons with 95% efficiency, Opt. Express **16**, 3032 (2008).

[24] G. Brida, L. Ciavarella, I. P. Degiovanni, M. Genovese, L. Lolli, M. G. Mingolla, F. Piacentini, M. Rajteri, E. Taralli, and M. G. A. Paris, Quantum characterization of superconducting photon counters, New J. Phys. **14**, 085001 (2012).

[25] J. J. Renema, G. Frucci, Z. Zhou, F. Mattioli, A. Gaggero, R. Leoni, M. J. A. de Dood, A. Fiore, and M. P. van Exter, Modified detector tomography technique applied to a superconducting multiphoton nanodetector, Opt. Express **20**, 2806 (2012).

[26] H. Paul, P. Törmä, T. Kiss, and I. Jex, Photon Chopping: New Way to Measure the Quantum State of Light, Phys. Rev. Lett. **76**, 2464 (1996).

[27] P. Kok and S. L. Braunstein, Detection devices in entanglement-based optical state preparation, Phys. Rev. A **63**, 033812 (2001).

[28] D. Achilles, C. Silberhorn, C. Śliwa, K. Banaszek, and I. A. Walmsley, Fiber-assisted detection with photon number resolution, Opt. Lett. **28**, 2387 (2003).

[29] M. J. Fitch, B. C. Jacobs, T. B. Pittman, and J. D. Franson, Photon-number resolution using time-multiplexed single-photon detectors, Phys. Rev. A **68**, 043814 (2003).

[30] J. Sperling, W. Vogel, and G. S. Agarwal, Sub-Binomial Light, Phys. Rev. Lett. **109**, 093601 (2012).

[31] T. J. Bartley, G. Donati, X.-M. Jin, A. Datta, M. Barbieri, and I. A. Walmsley, Direct Observation of Sub-Binomial Light, Phys. Rev. Lett. **110**, 173602 (2013).

[32] J. Sperling, M. Bohmann, W. Vogel, G. Harder, B. Brecht, V. Ansari, and C. Silberhorn, Uncovering Quan-

tum Correlations with Time-Multiplexed Click Detection, Phys. Rev. Lett. **115**, 023601 (2015).

[33] R. Heilmann, J. Sperling, A. Perez-Leija, M. Gräfe, M. Heinrich, S. Nolte, W. Vogel, and A. Szameit, Harnessing click detectors for the genuine characterization of light states, Sci. Rep. **6**, 19489 (2016).

[34] J. Sperling, T. J. Bartley, G. Donati, M. Barbieri, X.-M. Jin, A. Datta, W. Vogel, and I. A. Walmsley, Quantum Correlations from the Conditional Statistics of Incomplete Data, Phys. Rev. Lett. **117**, 083601 (2016).

[35] G. Harder, T. J. Bartley, A. E. Lita, S. W. Nam, T. Gerrits, and C. Silberhorn, Single-Mode Parametric-Down-Conversion States with 50 Photons as a Source for Mesoscopic Quantum Optics, Phys. Rev. Lett. **116**, 143601 (2016).

[36] See Chapter 30 in C. Forbes, M. Evans, N. Hastings, and B. Peacock, *Statistical Distributions*, 4th ed. (Wiley & Sons, Hoboken, New Jersey, USA, 2011).

[37] J. Sperling, W. Vogel, and G. S. Agarwal, True photocounting statistics of multiple on-off detectors, Phys. Rev. A **85**, 023820 (2012).

[38] U. M. Titulaer and R. J. Glauber, Correlation functions for coherent fields, Phys. Rev. **140**, B676 (1965).

[39] L. Mandel, Non-classical states of the electromagnetic field, Phys. Scr. **T12**, 34 (1986).

[40] A. Eckstein, A. Christ, P. J. Mosley, and C. Silberhorn, Highly Efficient Single-Pass Source of Pulsed Single-Mode Twin Beams of Light, Phys. Rev. Lett. **106**, 013603 (2011).

[41] K. D. Irwin, An application of electrothermal feedback for high resolution cryogenic particle detection, Appl. Phys. Lett. **66**, 1998 (1995).

[42] P. C. Humphreys, B. J. Metcalf, T. Gerrits, T. Hiemstra, A. E. Lita, J. Nunn, S. W. Nam, A. Datta, W. S. Kolthammer, and I. A. Walmsley, Tomography of photon-number resolving continuous-output detectors, New J. Phys. **17**, 103044 (2015).

[43] T. Gerrits, *et al.*, On-chip, photon-number-resolving, telecommunication-band detectors for scalable photonic information processing, Phys. Rev. A **84**, 060301(R) (2011).

[44] See an analysis in G. S. Agarwal, *Quantum Optics* (Cambridge University Press, Cambridge, 2013), incl. Fig. 3.2.

[45] J. Sperling, *et al.*, Identification of nonclassical light with multiplexing layouts and arbitrary detectors (to be published).