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Spin decoherence in a two-qubit CPHASE gate: the critical role of tunneling noise

Peihao Huang^{1,2,3}, Neil M. Zimmerman² and Garnett W. Bryant^{1,2}

Rapid progress in semiconductor spin qubits has enabled experimental demonstrations of a two-qubit logic gate. Understanding spin decoherence in a two-qubit logic gate is necessary for optimal qubit operation. We study spin decoherence due to $1/f$ charge noise for two electrons in a double quantum dot used for a two-qubit controlled-phase gate. In contrast to the usual belief, spin decoherence can be dominated by the tunneling noise from $1/f$ charge noise instead of the detuning noise. Tunneling noise can dominate because the effect of tunneling noise on the spin qubit is first order in the charge admixture; while the effect of the detuning noise is only second order. The different orders of contributions result in different detuning dependence of the decoherence, which provides a way to identify the noise source. We find that decoherence in a recent two-qubit experiment was dominated by the tunneling noise from $1/f$ charge noise. The results illustrate the importance of considering tunneling noise to design optimal operation of spin qubits.

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INTRODUCTION

An electron spin confined in a semiconductor quantum dot (QD) is a promising candidate quantum bit (qubit) for quantum information processing because of its potential scalability and miniaturization.^{1,2} Tremendous progress has been made during the last decade.^{3–11} The spin qubit in a silicon QD has attracted wide interest because of its long coherence time and compatibility with Si electronics nanofabrication.^{12,13} With advances in fabricating QDs using accumulation mode,¹⁴ several groups have demonstrated two-qubit gates in silicon based on the exchange interaction.^{15–17} The recent achievement of strong coupling between spin qubits and microwave photons will also enable long-distance gate operations for spin qubits.^{18–20}

Two-qubit gate operation is an essential but challenging task for the building of a quantum computer. For spin qubits, there are many proposals for gates in the literature, including two-qubit gates based on exchange interaction, strong coupling of spin and photon, superexchange coupling, and capacitive coupling.^{15–17,19–22} In recent experiments in silicon, the two-qubit gates, including controlled-phase (CPHASE) gate and controlled-not (CNOT) gate, are mediated by the exchange interaction between electrons in the nearby QDs.^{15–17} In this study, we focus on a CPHASE gate mediated by the exchange interaction, which was studied experimentally.¹⁵ Most of the results will also apply to other two-qubit gates based on the exchange interaction.

Environmental noise can destroy the quantum coherence necessary for quantum computation. For a spin qubit in silicon, nuclear noise can be suppressed with isotopic enrichment.²³ However, low-frequency $1/f$ charge noise remains ubiquitous in solid state devices. Consequently, spin dephasing is typically dominated by charge noise.^{24,25} For QD devices, charge noise can be measured by monitoring the charge offset drift in a single

electron transistor, where noise causes fluctuation of the electron chemical potential $\delta\mu$ in a single QD and shifts the Coulomb blockade spectrum.²⁶ The power spectral density $S_{1/f}(\omega) = \int_{-\infty}^{\infty} \langle \delta\mu(0)\delta\mu(\tau) \rangle \cos(\omega\tau) d\tau$ of energy fluctuation due to charge noise is typically

$$S_{1/f}(\omega) = A/\omega, \quad (1)$$

where A is the amplitude of charge noise.^{27,28} Although $S_{1/f}(\omega)$ usually exhibits $1/\omega$ dependence, the exponent of ω can be different for different devices. The energy fluctuations \sqrt{A} ranges from 0.1 to 10 μeV depending on material and experimental details.^{27–30}

For two electrons in a double quantum dot (DQD), where the spins can be used to define a single singlet-triplet (S-T0) qubit or two single-spin qubits coupled by exchange interaction, the $1/f$ charge noise can significantly affect the spin coherence and the number of gate operations. Usually, the detuning fluctuation from charge noise is assumed to be the dominant source of decoherence.^{31–35} When that is the case, the symmetric operation can increase the number of qubit operations as demonstrated in recent experiments.^{36–38} Furthermore, many theoretical papers on the optimal operation of spin qubit are based on reducing the sensitivity to detuning fluctuation from charge noise.^{39–43} However, the assumption that spin decoherence is dominantly due to detuning fluctuation may not always be satisfied, especially for spin qubits in small silicon QDs formed in accumulation mode. In this work, we show that tunneling fluctuation can play an important role for two electrons in a DQD used for a two-qubit CPHASE gate. The dominance of tunneling noise can significantly modify the optimal operation of a spin qubit.

We consider two electrons in a gate-defined DQD for a two-qubit CPHASE gate mediated by the exchange interaction. First,

¹Joint Quantum Institute, National Institute of Standards and Technology and University of Maryland, Gaithersburg, MD 20899, USA; ²Quantum Measurement Division, National Institute of Standards and Technology, Gaithersburg, MD 20899, USA and ³Shenzhen Institute for Quantum Science and Engineering, and Department of Physics, Southern University of Science and Technology, Shenzhen 518055, China
Correspondence: Peihao Huang (p Huang.cmp@gmail.com)

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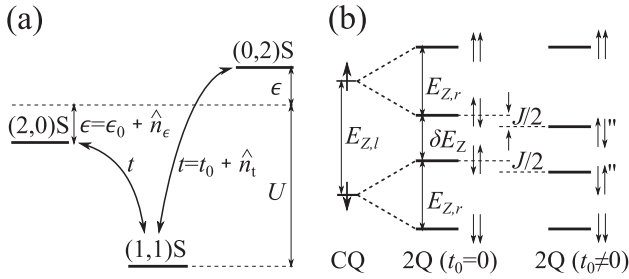


Fig. 1 Schematic energy diagrams. **a** Energy diagram of singlet states for two electrons in a DQD. Detuning $\epsilon = \epsilon_0 + \hat{n}_\epsilon$ and tunneling $t = t_0 + \hat{n}_t$ are indicated, in which \hat{n}_ϵ (\hat{n}_t) is detuning (tunneling) fluctuation from charge noise. **b** The energy diagram of control-qubit (CQ) states, two-qubit (2Q) states without exchange interaction (i.e. $t_0 = 0$), and two-qubit eigenstates with exchange interaction J

we introduce the Hamiltonian that describes two electrons in a DQD. The system Hamiltonian is $H = H_Z + H_C$, where H_Z is the Zeeman term, and H_C is the Hamiltonian for the charge degree of freedom.¹⁵ In the case of a silicon DQD, we assume that the electron Zeeman splitting and thermal energy are well below the valley splitting, so that we consider only the lowest valley state.^{44–47} The basis states can be denoted as $|N_l, N_r, X\rangle$, where N_l (N_r) is the number of electrons in the left (right) QD, X is the two-electron spin state (S for the spin singlet and T_+ , T_0 , or T_- for the spin triplet). In the $(1, 1)$ charge regime, the lowest four states $|1, 1\rangle X$ are degenerate in the absence of magnetic field, and the energies of the double occupation states $|2, 0\rangle S$ and $|0, 2\rangle S$ are higher because of the strong electron-electron repulsion. ϵ is the detuning defined relative to the symmetric operation point. The energy of the double occupation states can be raised or lowered by the detuning. The tunneling t between the two dots can couple $|1, 1\rangle S$ to the double occupation states. Figure 1a shows the energy diagram of the relevant singlet states, and the coupling between the states. For a two-qubit logic gate in ref.¹⁵, which is the experiment most relevant to our theory, the DQD is operated in the asymmetric regime (large ϵ) of the $(1, 1)$ charge regime. We need to consider only the lower energy double occupation state, shown as $|2, 0\rangle S$ in Fig. 1a with the left dot at lower energy. Then, the system Hamiltonian is

$$H_Z = \bar{E}_Z \sum_m |1, 1\rangle T_m \langle 1, 1| T_m| + \frac{\delta E_Z}{2} (|1, 1\rangle T_0 \langle 1, 1| S| + h.c.), \quad (2)$$

$$H_C = (U - \epsilon) (|2, 0\rangle S \langle 2, 0| S| + \sqrt{2} t (|1, 1\rangle S \langle 2, 0| S| + h.c.), \quad (3)$$

where $\delta E_Z = (E_{Z,l} - E_{Z,r})$ (or $\bar{E}_Z = (E_{Z,l} + E_{Z,r})/2$) is the difference (or average) in the Zeeman splitting of the left (l) and right (r) dots, the index $m = -1, 0$, or 1 , and U is the energy due to the electron-electron repulsion.

Here, we include noise due to charge fluctuation. The electrical field due to charge noise causes a fluctuation of the electrical potential of each QD, and thus leads to the detuning noise \hat{n}_ϵ of a DQD, $\epsilon = \epsilon_0 + \hat{n}_\epsilon$. \hat{n}_ϵ is usually considered as the decoherence source of a spin qubit in a DQD.^{31–35,40,41,43} Charge noise also causes fluctuation of the tunneling barrier and results in tunneling noise \hat{n}_t , $t = t_0 + \hat{n}_t$. Here, \hat{n}_ϵ and \hat{n}_t vary randomly in time. The fluctuation of both ϵ and t provides a mechanism for spin dephasing as shown below.

The Hamiltonian without noise describes the physics of a two-qubit controlled logic gate using the two electrons in the DQD based on the exchange interaction as we now describe. The electron spin in the left (right) QD act as a control-qubit (target-qubit). The qualitative energy diagram of the two-qubit states is

shown in Fig. 1b. If there is no tunneling, $t_0 = 0$, then, H_Z alone governs the spin degree of freedom. For a finite difference in the Zeeman splitting δE_Z at two dots, the four spin eigenstates are $|\uparrow\uparrow\rangle = |(1, 1) T_+\rangle$, $|\uparrow\downarrow\rangle = |(1, 1) T_0\rangle + |(1, 1) S\rangle$, $|\downarrow\uparrow\rangle = |(1, 1) T_0\rangle - |(1, 1) S\rangle$, and $|\downarrow\downarrow\rangle = |(1, 1) T_-\rangle$. The energy splitting of the target-qubit does not depend on the control-qubit's state and there is no two-qubit gate. However, if $t_0 \neq 0$, the tunnel coupling results in an effective exchange interaction J , which lowers the energies of states $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$.^{15,48} As a consequence, the energy splitting of the target-qubit depends on the state of the control-qubit, allowing the two spins to operate as a controlled logic gate. Then, a Ramsey-like pulse sequence will result in a two-qubit CPHASE gate between the two electron spin qubits.^{15,48}

The tunneling fluctuation and the detuning fluctuation from the $1/f$ charge noise will cause the spin decoherence during the CPHASE gate operation. In the following, we focus on the decoherence and the number of operations, and show the importance of tunneling fluctuation in a recent two-qubit gate experiment.

RESULTS

In this section, we first summarize the derivation of an effective Hamiltonian allows us to identify how the noises appear during gate operation in the $(1, 1)$ charge regime. More details are provided in the Supplementary Information S1. Then, we present the decoherence formula, the spectral density of the effective noise, and the spin decoherence results.

Effective Hamiltonian

When $t_0 \ll U - \epsilon_0$, which is generally satisfied in experiments, an effective two-qubit Hamiltonian can be obtained for the system. The way to obtain the effective Hamiltonian is through an Schrieffer–Wolff transformation that decouples the higher energy states. Since we are considering only one of double occupation states, the procedure can be simplified. Consider the Hamiltonian without noise. The Hamiltonian H_C without noise can be diagonalized and the corresponding eigenstates can be denoted as $|1, 1\rangle S'$ and $|2, 0\rangle S'$. Here the prime denotes that the new state is close to the original state. The higher eigenstate $|2, 0\rangle S'$ is decoupled from the rest of the Hamiltonian up to the first order of $\frac{t_0}{U - \epsilon_0 U - \epsilon_0}$ (Supplementary Information S1), which is in general small. The lower eigenstate $|1, 1\rangle S'$ is approximately $|1, 1\rangle S$ with certain admixture from $|2, 0\rangle S$,

$$|1, 1\rangle S' \approx |1, 1\rangle S + \frac{\theta}{2} |2, 0\rangle S, \quad (4)$$

where the admixture factor $\theta = -2\sqrt{2}t_0/(U - \epsilon_0)$. The charge admixture lowers the energy of $|1, 1\rangle S'$, which results in an effective exchange interaction J . Due to this charge admixture, the charge noise also couples to the qubit subspace. $|1, 1\rangle T_+\rangle$ and $|1, 1\rangle T_-\rangle$ are decoupled from $|1, 1\rangle T_0\rangle$ and $|1, 1\rangle S'$, so, the effective Hamiltonian for the states ($|1, 1\rangle T_0\rangle$, $|1, 1\rangle S'$) that are affected by noise is

$$H' = \begin{bmatrix} 0 & \frac{\delta E_Z}{2} \\ \frac{\delta E_Z}{2} & J + \hat{n}' \end{bmatrix}, \quad (5)$$

$$\hat{n}' = \sqrt{2}\theta\hat{n}_t - (\theta^2/4)\hat{n}_\epsilon, \quad (6)$$

where $J = \langle S|H_C|S'\rangle \approx -\frac{2t_0^2}{U - \epsilon_0}$ is the exchange interaction, and the noise \hat{n}' is formally $\hat{n}' = (\partial J/\partial t_0)\hat{n}_t + (\partial J/\partial \epsilon_0)\hat{n}_\epsilon$. Both tunneling noise \hat{n}_t and detuning noise \hat{n}_ϵ act on the spin qubit subspace but in different orders of the admixture factor θ . The effective noise due to the tunneling fluctuation is first order in the small charge admixture, $\partial J/\partial t_0 \propto \theta$; while the noise due to the detuning

fluctuation is a second order effect, $\partial J/\partial \epsilon_0 \propto \theta^2$. Tunneling noise is a first order effect because tunneling noise $\sqrt{2}\hat{n}_t|(1,1)S\rangle\langle(2,0)S|$ is coupled to $|(1,1)S\rangle$ in the qubit subspace even without charge admixture, while $-\hat{n}_t|(2,0)S\rangle\langle(2,0)S|$ is decoupled from the qubit subspace and requires admixture to couple to $|(1,1)S\rangle$. Even if the tunneling noise is smaller than the detuning noise, which is generally true as shown below, the effect of tunneling noise may still be dominant.

The eigenstates of H'' without noise can be obtained and denoted as $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$ (see Fig. 1b). In the basis of $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$, the effective Hamiltonian including noise is

$$H'' = \begin{bmatrix} \frac{J}{2} + \frac{\Omega_J}{2} + \hat{n}_{\uparrow\downarrow,\uparrow\downarrow}'' & \hat{n}_{\uparrow\downarrow,\downarrow\uparrow}'' \\ \hat{n}_{\downarrow\uparrow,\uparrow\downarrow}'' & \frac{J}{2} - \frac{\Omega_J}{2} + \hat{n}_{\downarrow\uparrow,\downarrow\uparrow}'' \end{bmatrix}, \quad (7)$$

where $\Omega_J = \sqrt{J^2 + \delta E_Z^2}$ is the energy splitting of $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$, $\hat{n}_{\uparrow\downarrow,\uparrow\downarrow}'' = (1 + J/\Omega_J)\hat{n}'/2$, $\hat{n}_{\uparrow\downarrow,\downarrow\uparrow}'' = \hat{n}_{\downarrow\uparrow,\uparrow\downarrow}'' = (\delta E_Z/\Omega_J)\hat{n}'/2$, $\hat{n}_{\downarrow\uparrow,\downarrow\uparrow}'' = (1 - J/\Omega_J)\hat{n}'/2$.

When $J \ll \delta E_Z$, which is satisfied in a two-qubit gate experiment,¹⁵ states $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$ are approximately spin product states, where $|\uparrow\downarrow\rangle \approx (|(1,1)T_0\rangle + |(1,1)S'\rangle)/\sqrt{2}$, $|\downarrow\uparrow\rangle \approx (|(1,1)T_0\rangle - |(1,1)S'\rangle)/\sqrt{2}$, and the effective noise on $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$ are $\hat{n}_{\uparrow\downarrow,\uparrow\downarrow}'' \approx \hat{n}_{\downarrow\uparrow,\downarrow\uparrow}'' \approx \hat{n}'/2$. In this limit, the control-qubit and target-qubit are well defined. The control-qubit state effectively selects the subspace of the system (See Fig. 1b). If control-qubit is spin-down (spin-up), then, the system is in the subspace of $|\downarrow\uparrow\rangle$ and $|\uparrow\downarrow\rangle$ ($|\uparrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$). Thus, for a given state of the control-qubit, we can reduce the decoherence of a two-qubit system to the decoherence of a two-level system. We focus on pure spin dephasing, which is generally much faster than spin relaxation.

Decoherence formula

In this subsection, we develop the expression that determines the effect of noise. For two states $|\alpha\rangle$ and $|\beta\rangle$ of interest, the system dephases as $\exp[-\phi(\tau)]$,^{49,50}

$$\phi(\tau) = \int_{\omega_0}^{\infty} d\omega J_{\alpha\beta}^{(zz)}(\omega) [2\sin(\omega\tau/2)/\omega]^2, \quad (8)$$

$$J_{\alpha\beta}^{(zz)}(\omega) = \frac{2}{\hbar^2} \int_{-\infty}^{\infty} \langle \hat{h}_{\alpha\beta}^{(z)}(0) \hat{h}_{\alpha\beta}^{(z)}(\tau) \rangle \cos(\omega\tau) d\tau, \quad (9)$$

where $\hat{h}_{\alpha\beta}^{(z)} = (\hat{n}_{\alpha\alpha} - \hat{n}_{\beta\beta})/2$ is the relative noise of the two states $|\alpha\rangle$ and $|\beta\rangle$ of interest, $J_{\alpha\beta}^{(zz)}(\omega)$ is the spectral density for the noise, and the cutoff frequency ω_0 represents the inverse of the measurement time of coherence dynamics.

Here, we emphasize the difference between a two-qubit gate system and a $S-T_0$ qubit before our detailed discussion of decoherence in the system. A two-qubit gate system with two electrons in a DQD shares many similarities with a $S-T_0$ qubit in a DQD; however, there is an important difference. The difference is due to the fact that spin dephasing depends on the relative noise of two states rather than the noise of each individual states. For a $S-T_0$ qubit, the qubit is encoded in states $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$, the effective noise is $\hat{h}_{\uparrow\downarrow,\downarrow\uparrow}^{(z)} = (\hat{n}_{\uparrow\downarrow,\uparrow\downarrow}'' - \hat{n}_{\downarrow\uparrow,\downarrow\uparrow}'')/2 = (J/\Omega_J)\hat{n}'/2$. Increasing δE_Z , which reduces the ratio J/Ω_J , can reduce the effective noise $\hat{h}_{\uparrow\downarrow,\downarrow\uparrow}^{(z)}$ and suppresses the decoherence of $S-T_0$ qubit, as shown in a recent experiment.²² However, in a two-qubit gate system, for a given state of control-qubit, only one of $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$ is involved. The relative noise will be either $\hat{n}_{\uparrow\downarrow,\uparrow\downarrow}''$ or $\hat{n}_{\downarrow\uparrow,\downarrow\uparrow}''$, which is not suppressed with increasing δE_Z . Therefore, in contrast to a $S-T_0$ qubit, spin decoherence in a two-qubit logic gate is not suppressed by increasing δE_Z .

Spectral densities

In order to study spin dephasing in the system, we need the corresponding spectral density. In a DQD, charge noise can induce detuning noise that arises from the non-identical noise on the two QDs, and tunneling noise that arise from fluctuations in barrier height. When the control-qubit is initialized to be spin-down, the relevant two states are $|\downarrow\uparrow\rangle$ and $|\uparrow\downarrow\rangle$. The effective noise is $\hat{h}_{\downarrow\uparrow,\uparrow\downarrow}^{(z)} = \hat{n}'/4$. (Dephasing for the spin-up control-qubit is the same, since $\hat{n}_{\uparrow\downarrow,\uparrow\downarrow}'' \approx \hat{n}_{\downarrow\uparrow,\downarrow\uparrow}''$ when $J \ll \delta E_Z$). Since the charge noise is believed to be from noise-producing defects, homogeneously distributed in the plane of the device, fluctuation of the tunnel barrier height due to charge noise is of the same order as detuning fluctuations. For non-correlated noises, the effective noise spectral density is given by (Supplementary Information S2)

$$J_{\downarrow\uparrow,\uparrow\downarrow}^{(zz)}(\omega) = A_{\text{eff}}/\omega, \quad (10)$$

where $A_{\text{eff}} = \frac{A}{8\hbar^2} [2\theta^2(\partial t_0/\partial E_b)^2 + \frac{\theta^4}{16}]$. The first term accounts for tunneling noise; the second for detuning noise. $\partial t_0/\partial E_b$ converts the barrier fluctuation to fluctuations of the tunneling rate. In the WKB approximation,

$$\partial t_0/\partial E_b \approx t_0/(2\Delta_b), \quad (11)$$

where $\Delta_b \equiv \sqrt{(E_b - E_0)\hbar^2/(2m^*l_b^2)}$, E_0 is the orbital energy of a single QD, E_b and l_b are the effective barrier height and width. With knowledge of $J_{\downarrow\uparrow,\uparrow\downarrow}^{(zz)}(\omega)$, the spin dephasing dynamics in $\exp[-\phi(\tau)]$ can be calculated from Eq. (8).

Equations (10) and (11) indicate that the relative strength of $\sqrt{2}t_0/\partial E_b \approx \sqrt{2}t_0/(2\Delta_b)$ and $\theta/4 = \sqrt{2}t_0/(2(U - \epsilon_0))$ determines whether tunneling noise or detuning noise dominates. If $\Delta_b > U - \epsilon_0$, detuning noise dominates; if $\Delta_b < U - \epsilon_0$ tunneling noise dominates.

Spin decoherence results

The spin dephasing has $\exp[-A_{\text{eff}}\tau^2 \ln(1/(\omega_0\tau))]$ dependence (Supplementary Information S3), approximately $\exp[-(\tau/T_\phi)^2]$ dependence, where T_ϕ is the spin dephasing time. We have evaluated Eq. (8) and fitted $\exp[-\phi(\tau)]$ to $\exp[-(\tau/T_\phi)^2]$, and found that $\beta \lesssim 2$ (Supplementary Information S4). Therefore, the spin dephasing shows $1/T_\phi \propto A_{\text{eff}}^{1/2} \approx \sqrt{A_{\text{eff}}}$ scaling, i.e. the dephasing rate $1/T_\phi$ has approximately $1/(U - \epsilon_0)^2$ dependence for detuning noise and $1/(U - \epsilon_0)$ dependence for tunneling noise.

Figure 2a shows the detuning dependence of the dephasing rate of the target-qubit including only detuning noise and only

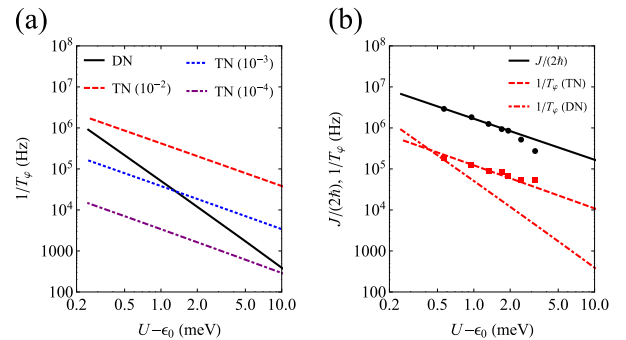


Fig. 2 Spin dephasing $1/T_\phi$ as a function of detuning. **a** $1/T_\phi$ as a function of detuning ϵ_0 due to tunneling noise (TN) only or detuning noise (DN) only. For tunneling noise, we choose representative values $\partial t_0/\partial E_b = 10^{-2}$ (Dashed), 10^{-3} (Dotted), and 10^{-4} (Dash-dotted). **b** $J/(2\hbar)$ and $1/T_\phi$ as a function of detuning ϵ_0 , where $A = (2\mu\text{eV})^2$ and $\partial t_0/\partial E_b = 3.2 \times 10^{-3}$. The dots are the experimental data for $J/(2\hbar)$ (circle) and $1/T_\phi$ (square).¹⁵

tunneling noise (the control-qubit is spin-down). The dephasing due to detuning and tunneling noise show different detuning dependence, which enables the identification of different noise sources. For tunneling noise, results have been shown for different values of $\partial t_0/\partial E_b$, which modifies the crossover between tunneling noise and detuning noise. The dephasing due to tunneling noise can dominate over detuning noise in a wide range of detuning (note that $U = 25$ meV).

Figure 2b shows a log-log plot of $J/(2\hbar)$ and dephasing rate $1/T_\phi$ for only tunneling noise and only detuning noise. The experimental data shown as dots is extracted from ref. ¹⁵. The calculated $1/T_\phi$ due to detuning noise shows approximately $1/(U - \epsilon_0)^2$ dependence, which is different from the experimental data, while $1/T_\phi$ due to tunneling noise shows approximately $1/(U - \epsilon_0)$ dependence. To match the experimental spin dephasing, we find $\partial t_0/\partial E_b = (3.2 \pm 0.2) \times 10^{-3}$, which for WKB estimate $\partial t_0/\partial E_b \approx t_0/(2\Delta_b) \approx 4 \times 10^{-4}$. We attribute the discrepancy to the simplicity of WKB method, the simplicity of the model barrier used, and the exponential dependence of tunneling on the parameters. (Note that the value of $\partial t_0/\partial E_b$ also depends on the choice of the amplitude A of charge noise.) $J/(2\hbar)$ and $1/T_\phi$ are almost parallel, indicating that they show the same $1/(U - \epsilon_0)$ dependence, and that the dephasing is dominated by the tunneling noise. This parallel dependence doesn't change with variation of a_{cz} or V_{c20} .

The dominance of tunneling noise is counter to what is usually assumed. To understand the qualitative behavior, we consider the WKB approximation Eq. (11). Tunneling noise is dominant because Δ_b is small (lower tunnel barrier E_b and bigger distance between QDs) compared to $U - \epsilon_0$ (U is big in small dots). This tends to be satisfied in small silicon QDs using accumulation mode.

The detuning dependence of the number of two-qubit operations can be different for detuning and tunneling noise. When detuning noise is dominant, the dephasing rate $1/T_2^* \approx 1/T_\phi \propto 1/(U - \epsilon_0)^2$, which increases faster than the exchange interaction ($J \propto 1/(U - \epsilon_0)$) as ϵ_0 moves towards U (more asymmetry). Thus, when detuning noise is dominant, the number of CZ operations $N_{CZ} = JT_2^*/(2\hbar)$ reduces as the DQD becomes more asymmetric, as suggested in the recent experiments of a $S - T_0$ qubit in a GaAs DQD.^{36–38} However, when tunneling noise is dominant, J and $1/T_\phi$ have the same $1/(U - \epsilon_0)$ dependence, and the number of two-qubit operation N_{CZ} is not reduced as DQD becomes more asymmetric. Furthermore, if there is finite spin dephasing $1/T_\phi^{(1)}$ due to the single qubit mechanisms, then, the total dephasing rate $1/T_2^* \approx 1/T_\phi + 1/T_\phi^{(1)}$ has a slower scaling compared to $1/(U - \epsilon_0)$ (Supplementary Information S5). Consequently, when tunneling noise is dominant, N_{CZ} can increase as DQD becomes more asymmetric (i.e. ϵ_0 approaches U), which is consistent with the experiment in ref. ¹⁵. This defines the regime of optimal operation when tunneling noise is dominant.

DISCUSSION

Note that the value of $\partial t_0/\partial E_b$ we obtained for experiment depends on the choice of the amplitude A of charge noise. If A is chosen to be $(1 \mu\text{eV})^2$ rather than $(2 \mu\text{eV})^2$, then, we have $\partial t_0/\partial E_b = (6.4 \pm 0.4) \times 10^{-3}$ to reproduce the results. Suppose that the amplitude of charge noise has been measured, then, the value of $\partial t_0/\partial E_b$ can be determined. Therefore, our theory also provides a metrology tool to further investigate the tunneling noises in those systems and characterize the system parameters, which is essential for the improvement of qubit behavior.

To reduce the effect of the tunneling noise, one needs to reduce the parameter $\partial t_0/\partial E_b$ as suggested by our theory. From a simple WKB approximation, one may reduce the parameter $\partial t_0/\partial E_b$ by using higher barrier and longer distance between two dots. Due to the valley physics, more detailed investigation will be needed in order to reduce $\partial t_0/\partial E_b$.

We should emphasize that the theory we developed here is general and can be applied to silicon, to spin qubits in GaAs QDs, to spin qubits on donor atoms, and even to hole spin qubit as long as other degrees of freedom are well separated from the problem of interest. Thus, tunneling noise from charge noise can have significant effect in those systems. The tunneling noise also has a similar effect on logical spin qubits, such as the $S - T_0$ qubit, which will affect the optimal operation of logical spin qubits.

In the discussion so far, we have assumed that the tunneling and detuning noises are uncorrelated. We do not think that it is likely that the noise will be correlated because the two types of noise are from different events at different locations. In the Supplementary Information. We also derived expressions for spectral density when the noise is partially and fully correlated, and discussed the consequences if the noise is correlated (Supplementary Information S2 and S6). We find that, for the regime of the experiment, the tunneling contribution dominates and it does not matter whether or not the tunneling and detuning noises are correlated.

In conclusion, spin decoherence due to detuning and tunneling noises from $1/f$ charge noise is studied in a two-qubit gate system. The amplitude of tunnel noise is smaller than the detuning noise in general. However, the contribution of the detuning noise to spin decoherence is second order in the charge admixture, while the contribution of tunneling noise is first order. As a consequence, decoherence due to tunneling noise can dominate over detuning noise for spin qubits in a DQD. The different orders of contribution lead to different detuning dependence of spin dephasing, which enables the identification of the noise source. Decoherence is dominated by the tunneling noise from charge noise rather than detuning noise in a recent experiment of a two-qubit logic gate. We identified the condition when tunneling noise dominates. Furthermore, we find that when detuning noise dominates, symmetric operation indeed helps improve the number of two-qubit operation as suggested in recent experiments; however, this is not the case when tunneling noise dominates. The results highlight the importance of tunneling noise and its consequences on the optimization of spin qubit operation. The theory developed also provides a metrology method to investigate the tunneling noises and characterize the system parameters, which is essential for the improvement of qubit behavior.

METHODS

To study spin decoherence in the system, we first obtain an effective Hamiltonian. For a weak tunneling with $t_0 \ll U - \epsilon_0$, the four lowest eigenstates are in the $(1, 1)$ charge configuration. In this limit, we first diagonalize H_C without noise and eliminate the double occupation state $|(2, 0)S\rangle$. Then, an effective two-qubit Hamiltonian including the effect from charge noise is obtained (Supplementary Information S1). From the effective Hamiltonian and spectral density of $1/f$ charge noise, the spin dephasing dynamics is evaluated.

Parameters

We use similar parameters as in the experiment on a two-qubit gate.¹⁵ The applied magnetic field $B_0 = 1.4$ T so the average Zeeman energy $\bar{E}_Z = 0.162$ meV. The Zeeman energy difference $\delta E_Z = 0.17 \mu\text{eV}$ (40 MHz) due to a different g-factor modulation at each dot. The tunneling amplitude $t_0 = 2.63 \mu\text{eV}$ (or $900/\sqrt{2}$ MHz) [an extra factor of $1/\sqrt{2}$ due to a difference in the expression of J]. The onsite Coulomb energy (estimated from the charge stability diagram) $U = 25$ meV. The condition $t_0 \ll U - \epsilon_0$ is always satisfied when $U - \epsilon_0 > 50 \mu\text{eV}$. We choose a cutoff frequency $\omega_0 = 1 \text{ s}^{-1}$, and an amplitude $A = (2 \mu\text{eV})^2$ for $1/f$ charge noise. To extract experimental data from Fig. 3 of ref. ¹⁵, we convert the voltage V_{c2} to detuning, $\epsilon_0 = a_{cz}(V_{c2} + V_{c20})$. We choose $a_{cz} = 0.19$ and $V_{c20} = 110.4$ mV, so that the calculated J matches the experimental points in the detuning regime when ϵ_0 is close to U .

DATA AVAILABILITY

The main data supporting the finding of this study are available within the article and its Supplementary Information files. Additional data can be provided upon request.

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AUTHOR CONTRIBUTIONS

P.H. developed the theory and performed the calculations. G.B. supervised the project. All authors researched, collated, and wrote this paper.

ADDITIONAL INFORMATION

Supplementary information accompanies the paper on the *npj Quantum Information* website (<https://doi.org/10.1038/s41534-018-0112-0>).

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