Magnetic Units

Electromagnetic Units, the Giorgi System, and the Revised International System of Units

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Abstract—The centimeter-gram-second system of electromagnetic units (EMU) has been used in magnetism since the latter part of the 19th century. The International System of Units (SI), a successor to Giorgi's 1901 rationalized four-dimensional meter-kilogram-second system, was adopted by the General Conference on Weights and Measures in 1960 with the ampere as the fourth base unit. However, EMU remains in common use for the expression of magnetic data. The forthcoming revision of the SI will accentuate its differences with EMU. The permeability of vacuum will no longer be a fixed constant, which recalls the Giorgi system and prompts a review of historical arguments on the concepts of magnetic flux density and magnetic field strength in vacuum. The redefinition of the ampere in terms of the fixed numerical values of two defining constants could allow for independent experimental measurements of the permeability of vacuum, i.e., determination of the magnetic constant.

Index Terms—Magnetic units, electromagnetics, International System of Units, Giorgi system, electromagnetic units, permeability of vacuum, permeability of free space, magnetic constant.

I. INTRODUCTION

The International System of Units (**SI**), established in 1960 by the General Conference on Weights and Measures (**CGPM**), has been generally accepted by researchers in most scientific disciplines. Magnetics is a notable exception, where the centimeter-gram-second (**CGS**) system of electromagnetic units (**EMU**), as formulated by William Thomson, James Clerk Maxwell, and others, through the British Association for the Advancement of Science [Thomson 1874], remains in common use. The coexistence of the SI and EMU in magnetics, and the conversion of numerical values from one to the other, has been a source of confusion and error for students and practitioners.

Although the use of the SI in magnetics has some inconveniences, they are minor compared to the ambiguities in the EMU system. The case for the SI remains compelling for the reasons first articulated by Giovanni Giorgi at the beginning of the 20th century [Frezza 2015]. Importantly, the expected revision of the SI, to take effect in 2019, will make it incompatible with EMU.

II. ADVANTAGES AND DISADVANTAGES OF THE SI AND EMU

One of the advantages of the SI is that it unifies magnetic and electrical units, whereas CGS bifurcates into EMU and electrostatic units (**ESU**). A possible disadvantage in the SI is that two constitutive relations are recognized: $B = \mu_0(H + M)$, the Sommerfeld convention, where B is magnetic flux density, μ_0 is the permeability of vacuum, H is magnetic field strength, and M is the magnetization; and $B = \mu_0 H + J$, the Kennelly convention, where J is the magnetic polarization. Because M and J have different units, confusion is averted.

A disadvantage of the SI is that, in practice, many researchers have the urge to use tesla as the unit for H (which they could use if they instead wrote $\mu_0 H$), or they mistakenly refer to B instead of H.

Turning to EMU, the disadvantages are more serious. The unit for magnetic moment m is often expressed as "emu"; however, "emu" is not a unit, but is simply an indicator of electromagnetic units. The actual unit for m is erg per gauss or erg per oersted.

As in the SI, volume susceptibility is dimensionless, but its value in EMU is smaller than in the SI. It may be appreciated that values of volume susceptibility, reported in the literature as dimensionless in both the SI and EMU, might be difficult to compare. In EMU, volume susceptibility is often expressed in "emu," "emu per cubic centimeter," or "emu per cubic centimeter per oersted," a state of confusion originating from the misuse of "emu" as the unit for magnetic moment.

Magnetization (magnetic moment per unit volume) is commonly expressed either as M in "emu per cubic centimeter" or as $4\pi M$ in units of gauss. They are dimensionally equivalent, but they differ numerically by 4π. This double definition often leads to misunderstandings and mistakes.

Care is required when electrical and magnetic quantities are combined: the EMU of current is the abampere. Some researchers are reluctant to abandon the ampere and use mixed units in equations that do not balance dimensionally in a so-called "practical CGS system" [Knoepfel 2000].

It is much easier to verify the dimensional consistency of equations in the SI than in EMU. For example, one may try to verify dimensional balance in the equation for the magnetic moment m of a circular loop of radius a carrying a current I, $m = \pi I a^2$, in the two systems of units: $[\mathrm{A \cdot m^2}] = [\mathrm{A \cdot m^2}]$ versus $[\mathrm{erg \cdot G^{-1}}] = [\mathrm{abamp \cdot cm^2}]$.

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III. PERMEABILITY OF VACUUM IN THE REVISED SI

The forthcoming revision of the SI [CIPM 2017], in which fixed values will be assigned to the Planck constant h and the elementary charge e [Newell 2018], will accentuate the philosophical differences between EMU and the SI: μ_0 is fixed at unity in the former but will be measurable in the latter [Goldfarb 2017], and quantities will no longer be strictly convertible by factors of 4π and powers of 10 [Davis 2017].

In the revised SI, μ_0 will be derived from fixed constants $h,\ e$, and the speed of light c, and the experimentally determined fine-structure constant α

$$(\mu_0)_{\text{experimental}} = (2h/ce^2)_{\text{fixed}} \times (\alpha)_{\text{experimental}}. \quad (1)$$

The recommended value of μ_0, initially equal to $4\pi \times 10^{-7}$ H/m to 9 significant figures, will change slightly over time as better measurements of α are made. Magnetics researchers will have to choose to work and publish in one of two incompatible systems: EMU, which has a long tradition, or the SI, which unifies all metrology and which has been adopted by international convention. Of course, the adoption of the SI by magneticians accustomed to working in EMU will require not only relatively straightforward conversions of units, but the conversion of equations (e.g., insertions and deletions of μ_0 and 4π).

IV. GIORGI'S RATIONALIZED MKS SYSTEM

The demotion of μ_0 from its immutable value of $4\pi \times 10^{-7}$ H/m within the SI might seem to violate the sanctity of a fixed constant. However, it is actually a return to the origins of the SI.

Giorgi [1901] introduced the rationalized, four-dimensional, meter-kilogram-second (**MKS**) system in which the fourth, electromagnetic unit was initially not specified. His system was "rationalized" in the sense used by Heaviside [1893]: to remove the irrational number 4π in the expression of Maxwell's equations. (In advocating for their respective systems of units, both Heaviside and Giorgi often used "rational" to additionally mean "logical.") Giorgi noted, "In my system, [μ_0] is not a numeric, nor do I assume any special value for it; it is a physical quantity, having dimensions, and to be measured by experiment" [Giorgi 1902].

A few years later, he recommended the international (artifact) ohm as the fourth unit in an "MKSΩ" system. He regarded both μ_0 and the permittivity of vacuum ε_0 as subject to experimental refinement, with $\mu_0 \approx 1.256 \times 10^{-6}$ H/m and $\varepsilon_0 \approx 8.842 \times 10^{-12}$ F/m, and both subject to the condition $(\mu_0\varepsilon_0)^{-1/2} = c \approx 3 \times 10^8$ m/s. He noted that his four-dimensional system "is neither electrostatic nor electromagnetic, because neither the electric nor the magnetic constant of free ether is assumed as a fundamental unit" [Giorgi 1905].

The path from Giorgi's proposal toward the SI did not proceed quite as he originally envisioned.

In 1935, the International Electrotechnical Commission (**IEC**) recommended adoption of the "Giorgi system," with the fourth unit to be determined. In 1938, the IEC Technical Committee on Electric and Magnetic Magnitudes and Units (**EMMU**), while recognizing that the fourth unit could be any one of the ohm, ampere, volt, henry, farad, coulomb, or weber, recommended "as the connecting link between the electrical and mechanical units, the permeability of free space with the value of $\mu_0 = 10^{-7}$ in the unrationalized system or $\mu_0 = 4\pi \times 10^{-7}$ in the rationalized system" [IEC 1964]. No unit for μ_0 was specified, so the EMMU recommendation was effectively the three-dimensional MKS system with μ_0 as a scaling factor with respect to CGS. A suggestion at the IEC meeting to name the new system "MKSμ_0" was not approved [Carr 1950], probably because μ_0 is not a unit.

In its deliberations, the EMMU was informed by recommendations from the Consultative Committee for Electricity (**CCE**) of the International Committee for Weights and Measures (**CIPM**), under the authority of the CGPM [CCE 1935], and of the Symbols, Units, and Nomenclature (**SUN**) Commission of the International Union of Pure and Applied Physics [SUN 1936]. Both recommended the unrationalized and rationalized values of μ_0 above. Although the CCE recommendation did not specify the unit for μ_0, the SUN Commission recommended henry per meter as the fourth unit for the MKS system.

The CCE and the SUN Commission also recommended "absolute" MKS units, that is, a system based on fundamental mechanical units of length, mass, and time, and convertible from CGS by powers of 10 and, in the case of rationalized MKS, 4π. Several years prior to the 1938 IEC meeting, Giorgi [1934] had warned, "the evil of the 4π and irrationality of units would be perpetuated, because the permeability of free space would receive the exact value of $4\pi \times 10^{-7}$ instead of being the result of a measurement." Giorgi extended the classical definition of an absolute system of units by noting the equivalence of mechanical and electrical energy, and thus applied the term to his four-dimensional MKS system.[1]

The IEC later adopted both the ampere as the fourth unit and rationalization of the Giorgi system in 1950 and affirmed the unit and rationalized value of μ_0 in 1956 [IEC 1964]. But by then, international leadership in systems of units had already transferred to the CGPM.

In 1939, the CCE formally recommended an absolute four-dimensional MKS system, with the fourth unit being the ampere. In 1946, following the Second World War, the CIPM accepted the CCE recommendation for the ampere as the fourth unit with its present definition, which gave unit and rationalization to μ_0. In 1954, the CGPM approved the ampere as a base unit, thereby formalizing the "MKSA" practical system of units—without reference to Giorgi. In 1960, the CGPM adopted the name *Système International d'Unités,* with the abbreviation "SI," for the practical system of units [BIPM 2006].

V. IS B THE SAME AS H IN VACUUM?

Whether or not the fields B and H in vacuum are physically the same has been controversial [Roche 2000]. The question is more nuanced in the revised SI, but as discussed below, an affirmative answer is philosophically appealing.

In EMU, B and H in vacuum are equal in value and dimensions despite their different unit names, gauss and oersted. In the present SI, B and H in vacuum are mutually convertible through the fixed constant μ_0 although they have different values and dimensions. As emphasized by Birge [1935], "the intrinsic character of a physical quantity has no necessary connection with the *dimensions* assigned to the *unit* of that

[1] Toward the end of his memorandum, by way of comparison, Giorgi [1934] displayed values of ε_0 and μ_0 in *rationalized* CGS units (different from the rationalized Heaviside system): $\varepsilon_0 = 1/4\pi$ and $\mu_0 = 4\pi/c^2$ (ESU) and $\varepsilon_0 = 1/4\pi c^2$ and $\mu_0 = 4\pi$ (EMU) [Fleming 1900].

quantity." Thus, the assertion that dimensional difference between B and H implies physical difference is a misguided criticism of the present SI expressed, for example, by Chambers [1999].

However, the revised SI, with μ_0 measurable, may be susceptible to criticism by magneticians who believe that B and H in vacuum must be physically the same for a system of units to make sense; that unlike the case of the presumed ether in Giorgi's time, there should exist no measurable permeability of a vacuum [Roche 2000]. Counterarguments that B and H actually will be coupled by $(2h/ce^2)\alpha$ and not a measurable μ_0 would be unpersuasive because, in fact, α is experimental, as indicated in (1). It seems implicit in the revised SI that B and H in vacuum are physically different.

Or does it? To again quote Birge [1935], "the character of a physical quantity is a matter of philosophy."

Before the value of c was fixed in the present SI in 1983, the same considerations applied to the electric flux density (electric displacement) D and the electric field strength E in vacuum: they were mutually convertible through ε_0, a constant whose value depended on the measured value of c. Concerns about measurability in vacuum would have applied also to D and E in EMU and B and H in ESU: their ratios are equal to c^{-2}. Only those who used the CGS Gaussian system could avoid cognitive dissonance.

Page [1974, 1976] summarized his philosophy and that of the IEC at the time, which is applicable to all unit systems: "$\boldsymbol{E}$ and $\boldsymbol{B}$ can be defined *in vacuo* in terms of forces on moving charges. $\boldsymbol{D}$ and $\boldsymbol{H}$ are defined *in vacuo* in terms of $\boldsymbol{E}$ and $\boldsymbol{B}$ and the magnetic constant (μ_0) of the unit system used." Out of vacuum, $\boldsymbol{B}$ and $\boldsymbol{H}$ generally have different properties and boundary conditions: $\boldsymbol{B}$ is solenoidal, $\boldsymbol{H}$ is not, and they are clearly different in magnetic materials.

Thus, in essence, B is the same as H in vacuum in all systems of units. In the revised SI, it is helpful to recognize $\boldsymbol{B}$ as the primary magnetic field vector, μ_0 as an experimental constant, and $\boldsymbol{H}$ as an arithmetically derived auxiliary vector. This does not affect the traditional Maxwellian definition of magnetic susceptibility, equal to M/H (not M/B), nor detract from the utility of $M-H$ and $B-H$ curves to characterize magnetic materials.

VI. MEASUREMENT OF μ_0

Equation (1) is definitional. Are there ways to measure μ_0 in the revised SI even if the measurements might have large uncertainties? Is there a way to "realize," in the sense used by standards laboratories [Zimmerman 1998], the unit henry?

One need only refer to the original definition of the ampere in the present SI in terms of the force between two parallel conductors, Ampère's force law,

$$F/l = \mu_0 I_1 I_2 / 2\pi d \tag{2}$$

where force per unit length $F/l = 2 \times 10^{-7}$ N/m, currents I_1 and $I_2 = 1$ A, and separation $d = 1$ m. This definition fixes $\mu_0 = 4\pi \times 10^{-7}$ N/A^2 $\equiv 4\pi \times 10^{-7}$ H/m in the present SI. The definition of the ampere in the revised SI will be based on the fixed value of the elementary charge e and the fixed value of the cesium hyperfine transition frequency, which defines the second. The ampere might be realized by counting electrons or, as it is represented now, in terms of the Josephson effect for voltage, the quantum Hall effect for resistance, and Ohm's law. With the unit for current I thus determined, (2) may be used to measure μ_0 instead of I.

Alternatively, the equation for torque $\boldsymbol{\tau}$ between two current loops may be used, $\boldsymbol{\tau} = \boldsymbol{m} \times \boldsymbol{B}$. One of the coils would be the source of $\boldsymbol{B}$, which may be taken as being uniform at the center of the coil. The other would be a small, exploratory magnetic dipole of magnetic moment $\boldsymbol{m}$ in units of ampere meter squared. When their axes are mutually perpendicular

$$\boldsymbol{\tau} = \mu_0 I_1 I_2 (\pi {a_1}^2 / 2a_2) \tag{3}$$

where I_1 is the current in the dipole loop of radius a_1 and I_2 is the current in the $\boldsymbol{B}$ coil of radius a_2.

If, instead of the Sommerfeld convention, one used the Kennelly convention, as in Chikazumi [1997], the torque equation would be $\boldsymbol{\tau} = \boldsymbol{j} \times \boldsymbol{H}$, where $\boldsymbol{j}$ is the magnetic dipole moment, equal to $\mu_0 \boldsymbol{m}$, in units of weber meter. In either case, the torque would be a measure of μ_0.

A measurement of ε_0 with a Thompson–Lampard calculable capacitor for capacitive impedance [Thompson 1959] and the quantum Hall effect (**QHE**) for resistance [Jeckelmann 2001], and the use of $(\mu_0 \varepsilon_0)^{-1/2} = c$, would be a way to determine μ_0.

Any measurement of self- or mutual inductance, such as with a calculable inductor for inductive impedance [Harrison 1967] and the QHE for resistance, could be used to measure μ_0, but with greater uncertainty than with a calculable capacitor.

Actually, none of the above is likely to be used to measure μ_0. Although measurable, the recommended value of μ_0 will be adjusted periodically based on (1).

In the revised SI, the henry could be realized by using a Maxwell-Wien bridge to measure an unknown inductance in terms of a known capacitance and a known resistance, or with a calculable inductor and the value of μ_0 from (1) [Overney 2010, CCEM 2017].

VII. CONCLUSION

In the revised SI, μ_0 will be a measurable constant (proportional to the experimental fine-structure constant α) that serves to couple electric current and mechanical force in vacuum. B in vacuum is defined in terms of current. H in vacuum will be a quantity defined in terms of B and μ_0. Days before he died in 1935, Glazebrook [1936], the chair of the SUN Commission, wrote, "It is of course true, and is perhaps a little unfortunate, that $[\mu_0]$ measures also the permeability, i.e., the ratio of B/H in the space—'free space'—in which our measurements are being conducted, and this has rather masked the fact that it comes into the expressions of the engineer as the coefficient or link which enables him to express electrical and magnetic forces absolutely." To people concerned with absolute systems of units, μ_0 was (and still is) just a constant, the magnetic constant. But while μ_0 and ε_0 are not fundamental constants in the revised SI, their product does represent a fundamental constant: $(\mu_0 \varepsilon_0)^{-1/2} = c$.

As emphasized by Jackson [1999] in his canonical textbook, systems of units are chosen for convenience, and the number of base units and the dimensions of physical quantities in those units are arbitrary. As a case in point, he notes that fundamental constants are assigned dimensionless values of unity in some areas of theoretical physics. That is not going to change with the revision of the SI.

Table 1. Conversion of units for magnetic quantities. In the right column, $\{\mu_0\}$ refers to the numerical value of μ_0, the recommended value of which may change slightly over time. Factors of 4π originate from the conversion of unrationalized EMU to rationalized SI units. In the absence of units, a dimensionless quantity is labeled with its associated system of units (EMU or SI). The arrows (→) indicate correspondence, not equality.

SI Symbol	SI Quantity	Conversion from EMU and Gaussian Units to SI Units [a]
Φ	Magnetic flux	1 Mx = 1 G·cm^2 → 10^{-8} Wb = 10^{-8} V·s
B	Magnetic flux density, magnetic induction	1 G → 10^{-4} T = 10^{-4} Wb/m^2
μ	Permeability [b]	1 (EMU) → $\{\mu_0\}$ H/m = $\{\mu_0\}$ N/A^2 = $\{\mu_0\}$ Wb/(A·m)
H	Magnetic field strength, magnetizing force	1 Oe → $10^{-4}/\{\mu_0\}$ A/m
m	Magnetic moment	1 erg/G = 1 emu → 10^{-3} A·m^2 = 10^{-3} J/T
j	Magnetic dipole moment	1 erg/G = 1 emu → $10^{-3}\{\mu_0\}$ Wb·m
M	Magnetization, volume magnetization	1 erg/(G·cm^3) = 1 emu/cm^3 → 10^3 A/m
		1 G → $10^{-4}/\{\mu_0\}$ A/m
J, I	Magnetic polarization, intensity of magnetization	1 G → 10^{-4} T = 10^{-4} Wb/m^2
σ	Specific magnetization, mass magnetization	1 erg/(G·g) = 1 emu/g → 1 A·m^2/kg
χ	Susceptibility, volume susceptibility	1 (EMU) → 4π (SI)
χ_ρ, $\chi_{\rm m}$	Specific susceptibility, mass susceptibility	1 cm^3/g → $4\pi \times 10^{-3}$ m^3/kg
w, W	Energy product, volume energy density [c]	1 erg/cm^3 → 10^{-1} J/m^3
N, D	Demagnetizing factor	1 (EMU) → $(4\pi)^{-1}$ (SI)

[a] EMU are the same as Gaussian units for magnetostatics: Mx = maxwell, G = gauss, Oe = oersted. SI: Wb = weber, T = tesla, H = henry, N = newton, J = joule.
[b] In the SI, relative permeability $\mu_{\rm r} = \mu/\mu_0 = 1+\chi$. In EMU, permeability $\mu = 1+4\pi\chi$. Relative permeability $\mu_{\rm r}$ in the SI corresponds to permeability μ in EMU.
[c] In the SI, w [J/m^3] = B [T] · H [A/m] = μ_0 [Wb/(A·m)] · M [A/m] · H [A/m]. In EMU, w [erg/cm^3] = $(4\pi)^{-1}$ B [G] · H [Oe] = M [erg/(G·cm^3)] · H [Oe].

Jackson wrote, in the preface to the third edition of his book, "My tardy adoption of the universally accepted SI system is a recognition that almost all undergraduate physics texts, as well as engineering books at all levels, employ SI units throughout." It is not too late for magneticians to start using the SI. Table 1 is a listing of conversion factors from EMU to the revised SI.

In introducing his rationalized, four-dimensional system, Giorgi [1901] wrote, *Il sistema CGS, con questo, perde ogni ragione di esistere; ma non credo che il suo abbandono sarà lamentato da alcuno.* ("With this, the CGS system loses every reason to exist; but I do not think that its abandonment will be lamented by anyone.")

He may be correct, eventually.

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