# Frame-Based Randomized Scheduling of Packets with Random-Deadlines for Multi-Flow Wireless Networks

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**Abstract**

The use of wireless communications in industrial applications has motivated various advances in manufacturing automation by allowing more flexibility in installing wireless sensors and actuators than their wired counterparts. The main challenge in industrial wireless deployment is the strict timing and reliability requirements in these systems. Industrial wireless networks are commonly characterized by strict packet deadlines. As a result, Time Division Multiple Access (TDMA) protocols have been widely exploited in various technologies due to their ease of implementation and packet collision avoidance. Moreover, the use of frame-based protocols is motivated by the need for short processing times at the edge nodes of the network. In this work, we consider the problem of scheduling multiple data flows over a wireless network operating in an industrial environment. These flows are characterized by random strict deadlines for each packet following a given probability distribution. Each of these flows may represent the data coming from a sensor to the controller or the control commands from the controller to an actuator. A randomized frame-based scheduling scheme is analyzed where each time slot in the frame is assigned to a data flow randomly.

## 1. Introduction

Wireless communications technology is a key enabler of advances in various applications due to its better coverage, more flexibility, and massive connectivity. Better coverage is achieved because wireless signals can cover locations where wires cannot reach either due to the long distances or a harsh environment. Different applications have different requirements and different performance indicators. Industrial wireless is motivated to allow better process and factory automation where more communications devices can be installed and larger amounts of data can be transferred.

Due to the criticality of the data transfered in many industrial environments, industrial wireless has strict requirements on the delay and reliability of the transferred data. Consequently, industrial wireless protocols are developed to meet these requirements. Examples of wireless protocols for process

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automation include wireless Highway Addressable Remote Transducer Protocol (WirelessHART) and the International Society of Automation (ISA) protocol ISA100.11a [1, 2] and for factory automation include wireless communications with ultra-high performance protocol (WirelessHP) [3].

Specifically, strict packet deadlines are required in industrial wireless networks. The stochastic and broadcast nature of the wireless channels can cause data errors. As a result, packets may be lost or may have to be retransmitted, which cause delays. Hence, improving transmission control schemes in industrial wireless has been extensively studied for transmission of packets with strict deadlines [4]-[7].

In order to overcome the challenges of the broadcast nature of the wireless channel, time division multiple access (TDMA)-based medium access control (MAC) protocols are used to avoid packet collisions. Generally, TDMA-based MAC protocols allow the control of the transmitted packets in any time slot, and hence, the network can achieve bounded transmission delay [8]. TDMA-based MAC is used in industrial wireless to eliminate the possibility of packet collision and hence it increases the likelihood of packets getting delivered by their deadlines.

In TDMA-based networks, scheduling plays a crucial rule in the network performance by determining the packets to be transmitted at any given time slot. Moreover, scheduling can improve reliability by allowing multiple copies of data to be transmitted over the network. As a result, scheduling has been widely discussed to achieve the requirements of wireless sensor networks (WSN). In [9], scheduling in TDMA-based networks is addressed where various performance metrics are discussed including latency and energy consumption. Additionally, the network parameters that impact the scheduling algorithms are studied. Heuristic scheduling algorithms have been surveyed in [10].

In this work, we consider a randomized frame-based scheduling policy for multiple data flows with strict deadlines. In the proposed scheduling policy, each time slot is assigned to a flow following some probability distribution. In existing industrial wireless communications protocols, the schedule is commonly evaluated once every transmission frame composed of many time slots. The distribution of the transmission probabilities and the schedule may be re-evaluated at the beginning of each new frame. The route of each data flow is assumed to be known before the schedule is evaluated. The data flows are not assumed to be periodic. Instead, the packet generation process and packet deadlines are assumed to be probabilistic with some defined probability mass functions. A similar model is previously considered in [11] for the case of a deterministic scheduling policy obtained by solving a Markov decision problem.

The consideration of data flows with random deadlines is motivated by event-based signals that may be affected by random events or random processing delays. In [12]-[15], the concept of data flows with random deadlines is discussed. In this paper, we use random deadlines in a different setting compared to the existing literature and we also consider the effects of the wireless channel.

In this work, a randomized scheduling policy is presented and the average number of packets missing their deadlines per frame is derived . The ability of the scheduler to achieve a required performance metric for a set of flows is discussed. Numerical methods are used to assess the performance of the scheduling policy for various parameter settings.

The rest of the paper is organized as follows. We introduce the network model in Section 2. The performance analysis is presented in Section 3. The performance optimization through formulating the minimization problem of the packets missing their deadlines is discussed in Section 4. Numerical results are presented in Section 5. Finally, Section 6 presents the concluding remarks.

## 2. System Model

We denote the $M$ flows in a wireless network by $\mathcal{F} = \{F_1, F_2, ....F_M\}$. These flows are to be scheduled over a single frequency bands. The $m$th flow has a predefined route $\phi_m$ with a number of hops $h^*_m$. In $F_m$, each packet should be delivered over all the hops successfully before it gets to its destination. The flow $F_m$ has a new packet when the deadline of the previous packet expires.

The scheduling frame has a fixed length of $T$ time slots. The scheduling frame length is commonly defined in the literature as the least common multiple of the packet generation periods of the field devices [16]. This definition is valid only in the case of periodic data flows. The value of $T$ is commonly referred to as a hyper-period in industrial wireless networks protocols. In each time slot, at most one transmission occurs, as we assume a single wireless frequency band is used.

The wireless link between any two nodes in the network modeled as a binary erasure channel. It is represented by the success transmission probability $\rho_{i,j}$ between the nodes $i$ and $j$. We assume that $\rho_{i,j} = \rho$, for all $(i,j)$, for the sake of simplicity even though the proposed algorithms can still be analyzed in the more general case with different values for $\rho_{i,j}$. The value of $\rho$ is determined by the wireless channel and the wireless nodes parameters such as required error rate, the transmission power, and the modulation and coding scheme. The state of the wireless channel is independent of the packet generation process.

Each packet in $F_m$ is characterized by a required deadline for delivery to the destination which is denoted by $D_m$. The parameter $D_m$ is modeled as positive integer random variable. The values of $D_m$ are denoted by $d_m \in \mathcal{B}_m$ and drawn from the set $\mathcal{B}_m = \{h^*_m, h^*_m + 1, \ldots, D^*_m\}$, where $D^*_m$ is finite. The probability mass function of $D_m$ is denoted by $f_m(.)$ with the mean $\mu_m$ and the variance $\sigma^2_m$. The deadlines are strict such that a packet is discarded if not successfully received at the destination prior to its deadline. Each packet is generated and released as the deadline of the previous packet in the same flow expires.

We assume that a network manager takes the role of the schedule generation at the beginning of the hyper-period. The hyper-period takes a fixed value which long enough compared to the average packet deadlines to have a negligible scheduling processing overhead.

In this paper, we exploit the ratio of the average number of packets missing their deadlines to the average number of packets generated in a hyper-period as the performance metric to evaluate the performance of a randomized scheduling policy. The obtained performance can be compared to a preset value for schedulability testing or admission control.

## 3. Performance Analysis

In this section, we analyze the network performance of a randomized scheduling policy. The performance criterion is the average number of packets missing their deadlines in a hyper-period. The first step is evaluating the probability of a packet to miss its deadline. Then, the stochastic packets arrival process is defined and studied to determine the average number of packets missing their deadlines over the wireless network.

### *3.1. Probability of a Packet Missing Its Deadline*

We consider a randomized scheduling policy which is characterized by the transmission probabilities for various flows. At a time slot, the flow $F_m$ is to be scheduled for transmission by the probability $p_m$. The values of $p_m$ are set based on various system characteristics. These values are constrained by $\sum_{m=1}^{M} p_m = 1$ because only a single flow is scheduled at each time slot and hence the transmission decision of a flow has to follow a probability mass function.

We define $q_m(t_m, h_m, p_m)$ as the probability of a packet in $F_m$ to miss its deadline if it has $h_m$ hops remaining in its route and $t_m$ time slots remaining before its deadline expires given that the probability for a packet to be scheduled for transmission is $p_m$. In order to evaluate $q_m(t_m, h_m, p_m)$, we list the three events that may occur to a packet in $F_m$ at any time slot. These events are the packet is transmitted and successfully received, the packet is transmitted but fails to reach the following node in its route, and the packet is not scheduled. As a result, the value of $q_m(t_m, h_m, p_m)$ is expressed through evaluating the probability not to have $h_m$ successful transmissions in the following $t_m$ time slots as follows

$$q_m(t_m, h_m, p_m) = \sum_{i=0}^{h_m-1} \binom{t_m}{i} (p_m\rho)^i (1 - p_m\rho)^{t_m - i}, \text{ for } h_m \leq t_m, \tag{1}$$

Equation (1) is calculated only in cases where $h_m \leq t_m$ where the corresponding packets have not missed their deadlines yet. The initial conditions of the flow states, including the number of remaining hops and the remaining time slots of the flow packets while the schedule is being built, are not considered in this analysis because of their negligible effects on performance. The proposed policy is randomized and the hyper-period is long enough compared to the average packet deadlines such that the packets at the start and the end of the observation interval has negligible effect compared to the total number of missed packets.

Finally, we obtain the average probability $\bar{q}_m(p_m)$ of a packet in the flow $F_m$ to miss its deadline given that the scheduling probability is $p_m$. During the schedule evaluation, the exact states of the flows are not known to the network manager because we use frame-based scheduling where the schedule is determined before the frame transmission. As a result, the average probability is calculated at the arrival instant of the packet where the value of the deadline at the arrival instant is not known and follows the random distribution $f_m(.)$. The average probability is expressed as follows

$$\bar{q}_m(p_m) = \sum_{d_m=h_m^*}^{D_m^*} f_m(d_m) q_m(d_m, h_m^*, p_m). \tag{2}$$

*3.2. Average Number of Packets Missing their Deadlines*

In the following, we evaluate the average number of packets missing their deadlines of each flow during $T$. We start by introducing the random variable $X_m$ which depicts the number of packets of $F_m$ that are generated within $T$. Also, we set the random sequence $\mathcal{T}_{m,x} = (\mathcal{T}_m(1), \mathcal{T}_m(2), ..., \mathcal{T}_m(x))$ to represent the sequence of deadlines of $x$ packets of the flow $F_m$ within $T$. We denote the sum of the elements of this random sequence by $\Sigma_{m,x}$ and we express it as follows

$$\Sigma_{m,x} = \sum_{i=1}^{x} \mathcal{T}_m(i). \tag{3}$$

The limiting values of $X_m$ are then evaluated using the limiting values of the random variable $D_m$. When all the packet of $F_m$ have the maximum deadline, the number of packet generated within $T$ is minimum such that

$$X_m^{(\min)} = \left\lceil \frac{T}{D_m^*} \right\rceil, \quad h_m^* \leq D_m^*, \tag{4}$$

where $\lceil . \rceil$ is the ceiling function. On the other hand when all packets have the minimum deadline, which equals $h_m^*$, the maximum value of the random variable $X_m$ occurs. As a result, this maximum value is calculated as

$$X_m^{(\max)} = \left\lceil \frac{T}{h_m^*} \right\rceil. \tag{5}$$

The probability distribution of $X_m$ is then calculated. The event $\{X_m = x_m\}$ happens when the deadlines of the first $x_m - 1$ packets of $\mathcal{T}_{m,x_m}$ are summed to a value below $T$ while the deadlines of the first $x_m$ packets are summed to be greater than or equal to $T$. The probability of this event is denoted by $\Pr(X_m = x_m)$ and evaluated as follows

$$\Pr(X_m = x_m) = \Pr(\Sigma_{m,x_m-1} < T, \Sigma_{m,x_m} \geq T). \tag{6}$$

This same event can also be represented by having all the events in which $\Sigma_{m,x_m-1}$ takes values between 0 to $T-1$ and the deadline of the $x_m$th packet is greater than or equal to $T - \Sigma_{m,x_m-1}$. The sum of the probabilities of the corresponding events defines the case in which the $x_m$th packet is the last packet of the flow $F_m$. The expression of $\Pr(X_m = x_m)$ can be stated also as follows

$$\Pr(X_m = x_m) = \begin{cases} \sum_{l=(x_m-1)h_m^*}^{T-1} \Pr(\Sigma_{m,x_m-1} = l, \mathcal{T}_m(x_m) \geq T - l), \\ \qquad\qquad\qquad\qquad \text{for } X_m^{(\min)} \leq x_m \leq X_m^{(\max)}, \\ 0, \qquad\qquad\qquad\qquad \text{otherwise.} \end{cases} \tag{7}$$

By independence of the deadlines of the packets of the same flow, we are able multiply the probabilities of the two independent events in the above expression to obtain their joint probability expression. In order to calculate the probability of the last packet deadline to be greater than or equal to $T-l$, we use the deadline probability distribution as follows

$$\Pr(\mathcal{T}_m(x_m) \geq T - l) = \sum_{\mathcal{T}_m(x_m)=T-l}^{D_m^*} f_m(\mathcal{T}_m(x_m)). \tag{8}$$

Then using the independence of the packets' deadlines, , the expression in (7) can be evaluated as

$$\Pr(X_m = x_m) = \sum_{l=0}^{T-1} \left( \Pr(\Sigma_{m,x_m-1} = l) \sum_{\mathcal{T}_m(x_m)=T-l}^{D_m^*} f_m(\mathcal{T}_m(x_m)) \right). \tag{9}$$

Furthermore, the $\Pr(\Sigma_{m,x_m-1} = l)$ is calculated using the deadline probability distribution as follows

$$\Pr(\Sigma_{m,x_m-1} = l) = \sum_{\mathcal{T}_{m,x_m-1}|\Sigma_{m,x_m-1}=l} \left( \prod_{x=1}^{x_m-1} f_n(\mathcal{T}_m(x)) \right), \tag{10}$$

where $\mathcal{T}_{x_m-1}|_{\Sigma_{m,x_m-1}=l}$ is a random sequence of length $x_m - 1$ where the sum of all its packets' deadlines equals $l$. Hence, the sum in the above expression includes all the combinations of the packets' deadlines of $F_m$ that lead to this value.

As a result, the average number of packets missing their deadlines in the flow $F_m$ is evaluated by obtaining the sum of probabilities of all the packets in $F_m$ to miss their deadlines over the distribution of $X_m$. These events of packets missing their deadlines in $F_m$ are independent of each other with average probability of $\bar{q}_m(p_m)$. Thus, the average number of packets missing their deadlines is expressed as follows

$$\bar{N}_m = \sum_{x=X_m^{(\min)}}^{X_m^{(\max)}} \Pr(X_m = x) \sum_{i=1}^{x} \bar{q}_m(p_m). \tag{11}$$

By rearranging the terms in the sums, the expression is evaluated as follows

$$\bar{N}_m = \bar{q}_m(p_m) \sum_{x=X_m^{(\min)}}^{X_m^{(\max)}} x\Pr(X_m = x). \tag{12}$$

Moreover, the average number of packets missing their deadlines in all the $M$ flows over $T$ is expressed as follows

$$\bar{N} = \sum_{m=1}^{M} \bar{N}_m. \tag{13}$$

On the other hand, the average number of all packets generated by all the flows over $T$ is calculated as follows

$$\bar{N}_{\mathrm{T}} = \sum_{m=1}^{M} \sum_{x=X_m^{(\min)}}^{X_m^{(\max)}} x\Pr(X_m = x). \tag{14}$$

Finally, the ratio of the average number of packets missing their deadlines to the average number of generated packets is evaluated as follows

$$R_{\mathrm{Missed}} = \frac{\bar{N}}{\bar{N}_{\mathrm{T}}} = \frac{\sum_{m=1}^{M} \bar{q}_m(p_m) E[X_m]}{\sum_{m=1}^{M} E[X_m]}, \tag{15}$$

where $E[X_m]$ is the expected value of $X_m$.

This obtained ratio can generally be used for admission control when a randomized scheduling policy is employed for flow scheduling. If $M-1$ flows have been admitted into the network and the value of $R_{\mathrm{Missed}}$ is calculated to be less than or equal to a prescribed threshold, such as 10%. Upon arrival of the $M$th flow, the ratio $R_{\mathrm{Missed}}$ is computed again to decide about admitting this flow to the network. If it does not exceed 10%, the new flow is admitted.

In addition, the value of $R_{\mathrm{Missed}}$ for any given choice of $\{p_m : m = 1, ..., M\}$ serves as an upper bound to the value of $R_{\mathrm{Missed}}$ for the optimal randomized schedule. The optimal value typically can be found by minimizing $R_{\mathrm{Missed}}$ over all possible distributions $\{p_m : m = 1, ..., M\}$. However, this obtained upper bound may be loose.

## 4. Minimization of $R_{\mathrm{Missed}}$

In this section, we consider the problem of minimizing $R_{\mathrm{Missed}}$ over the values of decision probabilities $p_m$. The minimization problem is stated as follows

$$\begin{aligned} &\min_{p_m} R_{\text{Missed}} \\ \text{s.t.} \quad & p_m \geq 0, \qquad \forall 1 \leq m \leq M, \\ & \sum_{m=1}^{M} p_m = 1. \end{aligned} \tag{16}$$

The denominator of $R_{\text{Missed}}$ is independent of $p_m$ and hence it can be removed from the optimization problem to restate it as

$$\begin{aligned} &\min_{p_m} \sum_{m=1}^{M} \bar{q}_m(p_m) \sum_{x=X_m^{(\min)}}^{X_m^{(\max)}} x.\Pr(X_m = x) \\ \text{s.t.} \quad & p_m \geq 0, \qquad \forall 1 \leq m \leq M, \\ & \sum_{m=1}^{M} p_m = 1. \end{aligned} \tag{17}$$

We then substitute for $\bar{q}_m(p_m)$ from (2) and denote $E[X_m]$ by $W_m$ which is independent of $p_m$.

$$\begin{aligned} &\min_{p_m} \sum_{m=1}^{M} W_m \sum_{d_m=h_m^*}^{D_m^*} f_m(d_m) q_m(d_m, h_m^*, p_m) \\ \text{s.t.} \quad & p_m \geq 0, \qquad \forall 1 \leq m \leq M, \\ & \sum_{m=1}^{M} p_m = 1, \end{aligned} \tag{18}$$

where $W_m$ represents the average number of packets of $F_M$ generated in a hyperperiod which depends only on the deadline distribution and not on the transmission probabilities.

The optimization problem is finally rewritten substituting $q_m(d_m, h_m^*, p_m)$ from (1). The objective function is a positive non-convex polynomial function of degree $D_m^*$.

$$\begin{aligned} &\min_{p_m} \sum_{m=1}^{M} W_m \sum_{d_m=h_m^*}^{D_m^*} f_m(d_m) \sum_{i=0}^{h_m^*-1} \binom{d_m}{i} (p_m\rho)^i (1 - p_m\rho)^{d_m - i} \\ \text{s.t.} \quad & p_m \geq 0, \qquad \forall 1 \leq m \leq M, \\ & \sum_{m=1}^{M} p_m = 1. \end{aligned} \tag{19}$$

In the following, we apply a generalization of Lagrange duality theory to relax the constrained minimization problems with non-convex objective functions. In this generalization, the Lagrange multiplier terms are nonlinear combinations of the constraints. Later, the upper bound of the objective function is obtained through solving a semidefinite programming (SDP) problem [17, 18] through a sum-of-squares (SOS) optimization algorithm [19].

We start by defining the function $G_m(p_m) = \sum_{d_m=h_m^*}^{D_m^*} f_m(d_m) q_m(d_m, h_m^*, p_m)$ to include all the terms depending on $p_m$ in the objective function.

$$\begin{aligned} &\min_{p_m} \sum_{m=1}^{M} W_m G_m(p_m) \\ &\text{s.t.} \;\; p_m \geq 0, \quad \forall 1 \leq m \leq M, \\ &\sum_{m=1}^{M} p_m = 1. \end{aligned} \tag{20}$$

The Lagrange dual problem is obtained by using nonlinear Lagrange multipliers, namely, $\lambda(p_m)$ and $\delta_m(p_m)$.

$$\begin{aligned} &\max_{\lambda(p_m),\delta_m(p_m)} \min_{p_m} \sum_{m=1}^{M} W_m G_m(p_m) + \lambda_m(p_m)(\sum_{m=1}^{M} p_m - 1) - \sum_{m=1}^{M} p_m \delta_m(p_m) \\ &\text{s.t.} \;\; \lambda(p_m) \geq 0, \delta_m(p_m) \geq 0, \quad \forall 1 \leq m \leq M. \end{aligned} \tag{21}$$

In order to deploy the SOS method, we bound the minimum objective function by a variable $\gamma$ in polynomial time and try to find a tight bound for the objective function [20]. In [21], it was shown that a nested family of SDP relaxations can produce the exact minimum while the degree of the polynomial can be exponential with the number of variables. It was observed that a low order relaxation usually produces the optimal solution. Note that in (20) the functions $G_m(p_m)$ are polynomials in the transmission probabilities $p_m$. We set the following problem which converges to the optimal value of (20). The objective function of (22) is the bound for the objective function of (20). This problem is then relaxed using the SOS optimization in order to get a close to optimal solution.

$$\begin{aligned} &\max_{\lambda(p_m),\delta_m(p_m),\gamma} \gamma \\ &\text{s.t.} \; \gamma \leq \sum_{m=1}^{M} W_m G_m(p_m) + \lambda_m(p_m)(\sum_{m=1}^{M} p_m - 1) - \sum_{m=1}^{M} p_m \delta_m(p_m), \forall p_m, \\ &\lambda(p_m) \geq 0, \\ &\delta_m(p_m) \geq 0, \quad \forall 1 \leq m \leq M. \end{aligned} \tag{22}$$

The relaxed problem can be written as follows where the polynomial constraint of (22) is constrained in (23) to be SOS.

$$\max_{\lambda(p_m),\delta_m(p_m),\gamma} \gamma$$
$$\text{s.t. } \gamma - \sum_{m=1}^{M} W_m G_m(p_m) - \lambda_m(p_m)(\sum_{m=1}^{M} p_m - 1) + \sum_{m=1}^{M} p_m \delta_m(p_m) \text{ is SOS,}$$
$$\lambda(p_m) \text{ is SOS,}$$
$$\delta_m(p_m) \text{ is SOS,} \qquad \forall 1 \leq m \leq M. \tag{23}$$

The optimization variables are $\gamma$ and the polynomial coefficients of $\lambda(p_m)$ and $\delta_m(p_m)$. Let $d$ be the degree of the polynomial of the first constraint in (23) where for a fixed value of $d$, the problem is solved using SDP. By increasing $d$, the SDP size increases and the obtained value of $\gamma$ is tighter to the optimal value. The initial value of $d$ is selected to be the nearest even number greater than or equal to the degree $\sum_{m=1}^{M} W_m G_m(p_m)$ which is $D^*_m$. The degree is then increased by 2 for each following level.

To check that the objective bound has converged to the optimal value, the following test is performed using the GloptiPoly tool [22]. The test checks the non-negativity of a polynomial over a semi-algebraic set through finding a sequence of moments to represent a probability measure with support in this semi-algebraic set. A sufficient rank evaluation is performed over the moment matrix which is a positive semidefinite matrix formed by the sequence of moments [19].

In summary, the following algorithm is used to obtain the optimal solution of the minimization problem.

**Algorithm 1** Sum-Of-Squares Algorithm

1. Formulate the relaxed problem (23) for a given $d$.
2. Use SDP to solve the relaxation of order $d$ [19].
3. If the result satisfies the sufficiency condition, the value of $\gamma^*(d)$ is the optimal objective and the $p^*_m$ are the optimal probabilities.
4. Otherwise, increase $d$ by 2, and repeat steps 2-3.

## 5. Numerical Results

In this section, the performance of the proposed randomized scheduling algorithm is assessed in the case of multiple flows with packets having random deadlines. The performance criterion is $R_{\text{Missed}}$. In the following, we demonstrate the performance of the optimal strategy using various system parameters. Moreover, the performance of the optimal randomized policy is compared to the basic round robin benchmark in which flows are scheduled over time in equal proportions and in circular order without prioritizing any of the flows [23]. We will refer to these strategies, respectively, as 'Optimal' and 'RR'.

In the case of symmetric flows, all the $M$ flows have the same value for $h^*_m$ and the same value for $D^*_m$, and the deadlines $D_m$ follow the same discrete uniform distribution over the range $h_m^*, ...., D_m^*$. The performance results are obtained by simulating the system using the optimal scheduling transmission probabilities, which are $p_m = 1/M$, $\forall m$ due to the use of symmetric flows. Although the simulations are done over multiple hyper-periods, occasional dips in curves are observed that are due to the finite time duration of simulations.

### *5.1. Effects of the Number of Hops*

In this subsection, we study the effect of the number of hops on $R_{\text{Missed}}$. In Fig. 1, we set $M = 2$ with asymmetric flows such that $D_1^* = 30$ and $D_2^* = 10$. We show the improvement of the performance due to the use of the optimal policy for the more constrained networks needing a larger number of hops between sources and destinations. Moreover, we show how the performance of both the optimal and round robin policies improve with channel quality.

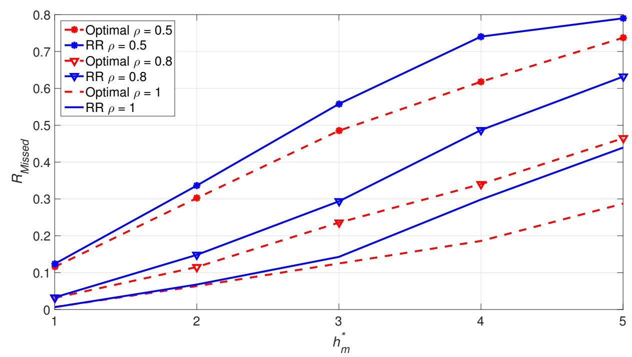


Figure 1: The ratio $R_{\text{Missed}}$ vs. $h_m^*$ for different values of $\rho$ with $M = 2$, $D_1^* = 30$ and $D_2^* = 10$

In Fig. 2, we show the ratio $R_{\text{Missed}}$ as a function of the number of hops per flow. We vary the values of $\rho$ and $D_m^*$. In this figure, the relation between

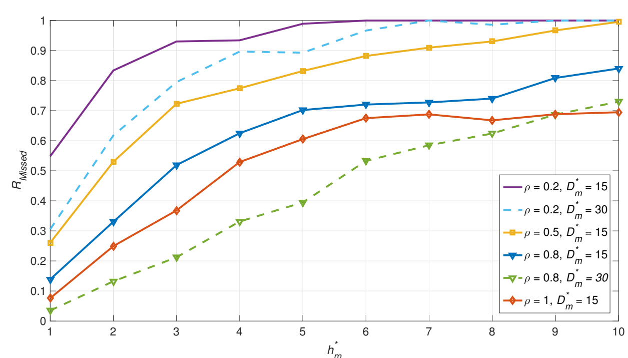


Figure 2: The ratio $R_{\text{Missed}}$ vs. $h_m^*$ for different values of $\rho$ and $D_m^*$ with $M = 3$

$R_{\text{Missed}}$ and $h_m^*$ is monotonically non-decreasing over the whole range of $h_m^*$. Generally, the slope of the curves is higher at lower values of $h_m^*$ and decreases as $h_m^*$ increases. Moreover, the performance is enhanced by having higher values of $D_m^*$ and $\rho$.

### *5.2. Effects of the Random Deadline Range*

In this subsection, we study the effects of the random deadline range on performance, specifically, the effects of $D_m^*$ on $R_{\text{Missed}}$. In Fig. 3, we set $M = 2$ with asymmetric flows such that $h_1^* = 1$ and $h_2^* = 5$. We show the improvement of the performance due to the use of the optimal policy especially for the more constrained networks with having tighter deadlines. Moreover, we show the improvement of the performance for both the optimal and round robin policies with the improvement of the channel quality. The optimal policy has an advantage over the simple round robin policy in the case of asymmetric flows.

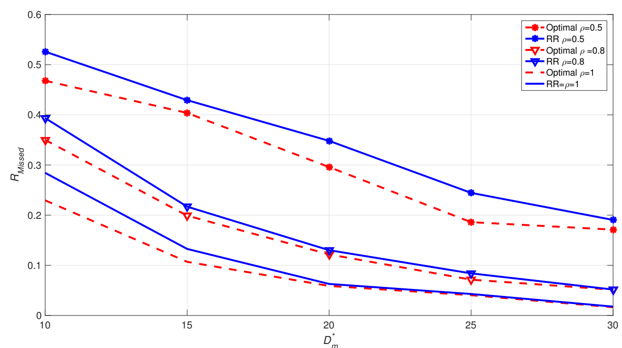


Figure 3: The ratio $R_{\text{Missed}}$ vs. $h_m^*$ for different values of $\rho$ with $M = 2$, $h_1^* = 1$ and $h_2^* = 5$

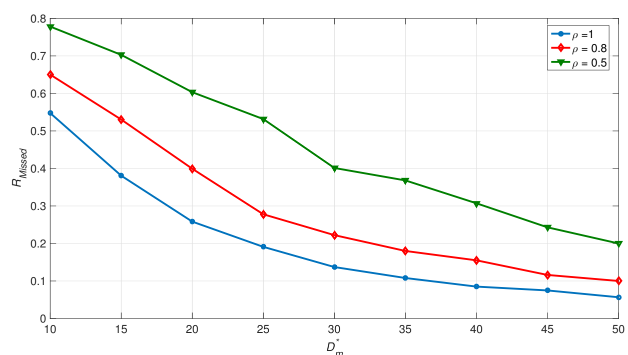


Figure 4: The ratio $R_{\text{Missed}}$ vs. $D_m^*$ for different values of $\rho$ with $h_m^* = 3$ and $M = 3$

In Fig. 4, we demonstrate the variation of $R_{\text{Missed}}$ against $D_m^*$. The range over which the random deadlines is determined through the value of $D_m^*$ where the range is wider for a higher $D_m^*$. It is observed that the value of $R_{\text{Missed}}$ decreases with both $D_m^*$ and $\rho$. As a result, the channel quality has a greater importance in the case of a tight deadline range.

### 5.3. Effects of the Channel Quality

In this subsection, we study the performance against transmission success probability. The optimal strategy has higher improvement compared to the round robin policy when the difference in the two flows parameters is higher as shown in Fig. 5. This improvement increases as the transmission success probability increases.

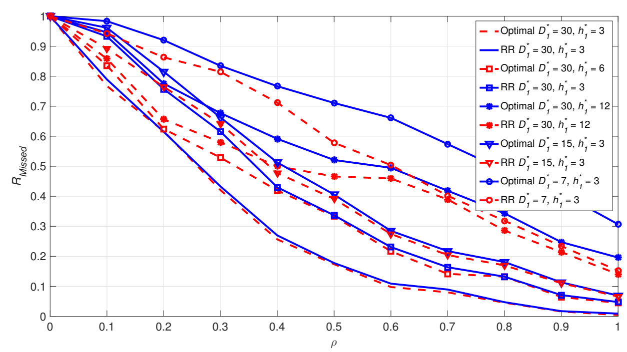


Figure 5: The ratio $R_{\text{Missed}}$ vs. $\rho$ for different values of $D_1^*$, and $h_1^*$ with $M = 1$, $D_2^* = 30$, and $h_2^* = 3$

In Fig. 6, the value of $R_{\text{Missed}}$ is demonstrated against transmission success probability where a monotonically non-increasing relationship is observed. In

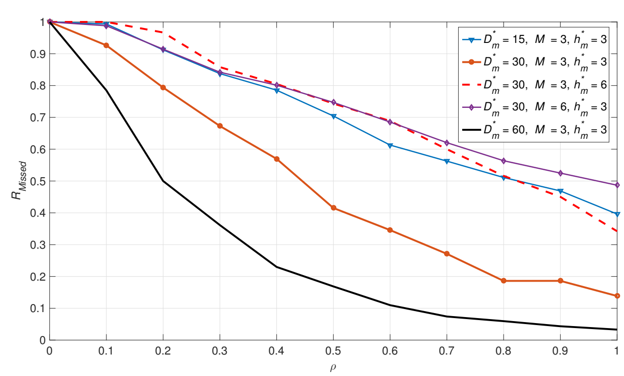


Figure 6: The ratio $R_{\text{Missed}}$ vs. $\rho$ for different values of $D_m^*$, $M$, and $h_m^*$

the case of $D_M^* = 60$, the largest variation in the curve slope is found. Hence, the importance of the channel quality is more pronounced for networks with a larger number of hops on the routes of the flows, a larger number of data flows, or tighter deadlines ranges. In the case of $D_m^* = 15$, the curve is almost linear such that any change in the value of $\rho$ leads to a corresponding change in the performance. On the other hand, in the case of $D_m^* = 60$, the performance improves significantly for small values of $\rho$ and the improvement rate decreases for the higher values of $\rho$.

### 5.4. Effects of the Number of Flows

In Fig. 7, the value of $R_{\text{Missed}}$ is demonstrated against $M$ for various settings of $\rho$. For all values of $\rho$, we observe a monotonically non-decreasing relation with higher slopes at lower $M$ and the slope decreases as $M$ increases. The use of deadline missing probability analysis in admission control can be explained using Fig. 7. If a ratio threshold is predefined, we can use the curves to determine the maximum number of admitted flows.

In the more general case of asymmetrical flows, similar analysis can be used to check the schedulability of a set of flows or admitting a new flow to the network in addition to the existing ones while keeping $R_{\text{Missed}}$ below a preset value. Furthermore, in the case of asymmetrical flows, the performance metric $R_{\text{Missed}}$ can be computed for individual flows and different benchmarks enforced for different flows. Hence, flow admission control or optimization of transmission probabilities for various flows can be carried out to meet these requirements.

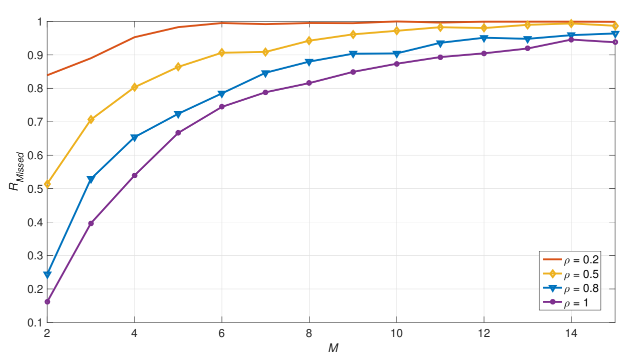


Figure 7: The ratio $R_{\text{Missed}}$ vs. $M$ for different values of $\rho$ with $h_m^* = 3$ and $D_m^* = 15$

## 6. Conclusions

In this paper, we have analyzed the performance of randomized frame-based scheduling for industrial wireless networking. The network has multiple data flows with random packet deadlines. Each flow is assigned a transmission probability and the frame schedule is composed at the beginning of each frame. We have derived the expression for the probability of a packet to miss its random deadline. Also, we derived the expression for the ratio of the average number of packets missing their deadlines to the average number of packets generated by all the flows per frame. Then, we studied the performance of the system using the optimal transmission probabilities to minimize that ratio. We have shown that the optimal policy is robust to the changes of the number of route hops when the random deadline range is relatively large. Moreover, a good wireless channel is needed for more constrained networks, i.e., networks that have a larger number of data flows, a larger number of hops on the routes of the flows, or tighter deadlines ranges. We have also shown how to use the derived expressions for flow admission control and schedulability. Lastly, the improvement in the performance by using the optimal policy is quantified against a simple round robin benchmark policy. In future work, we plan to study more efficient algorithms and heuristic alternatives.

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