Achieving induced transparency with one- and three-photon destructive interference in a two-mode, three-level, double-Λ system

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Electromagnetically induced transparency (EIT) [1] achieved with a three-level Λ system has been demonstrated to be able to significantly reduce the absorption of a probe field tuned to a strong one-photon resonance. In two seminal studies [2,3], Harris has described how a pair of short and intense probe and pump pulses can evolve, in a three-state Λ-type EIT configuration, into a temporally matched pair that propagates losslessly in the medium after a characteristic initial propagation distance.

In this Rapid Communication we show that with a two-mode, three-level, double-Λ system it is possible to produce a pair of temporally, amplitude, and group velocity (TAG) matched ultrashort probe pulse [4,5]. The key difference, in comparison with the conventional EIT Λ scheme [1–3], is the suppression of a dark state population by an efficient multiphoton destructive interference [6], leading to a unique type of efficient induced transparency. Specifically, we show (1) under suitable conditions both temporal profiles, amplitudes, and group velocities of two probe pulses can be well matched, (2) substantial suppression of a dark state due to a robust destructive interference between a one- and a three-photon excitation channel, resulting in a unique type of induced transparency and remarkable suppression of probe pulse absorption, and (3) no requirement on having maximum atomic coherence in order to achieve 100% photon flux conversion efficiency.

Before presenting our work, we first cite several works on Raman double-Λ system. These works include lasing without inversion [7], cavity QED [8], optical phase conjugation [9], efficient parametric frequency conversion [10], and efficient Raman scattering [11,12]. We point out that all these studies (except Ref. [12]) require a four-level system and many rely on steady-state solutions to atomic responses (especially in the case of efficient frequency conversion [10]). Finally, we also point out several recent studies on double-Λ systems [13].

In the present study we consider a lifetime broadened three-state atomic medium interacting with a pulsed two-mode probe field and a two-mode continuous wave (cw) pump field (Fig. 1). The probe fields (pulse length τ at the entrance of the medium and angular frequencies ω_{pn},n=1,2) and pump fields (angular frequencies ω_{pum}) couple the transition |1⟩→|2⟩ and |2⟩→|3⟩, respectively. We assume that the probe lasers are weak so that ground-state depletion can be neglected. The equations of motion for the atomic response and the probe fields can be written as (throughout the present work n=1,2 unless specified)

\[
\frac{\partial A_n^{(n)}}{\partial t} = id_{pn}A_n^{(n)} + i\Omega_{\alpha n}A_3 + i\Omega_{\beta n}^*,
\]

\[
\frac{\partial A_3}{\partial t} = id_3A_3 + i\Omega_{\alpha 3}A_1 + i\Omega_{\beta 3}A_2^*,
\]

\[
\frac{\partial \Omega_{\beta n}}{\partial z} + \frac{1}{c}\frac{\partial \Omega_{\alpha n}^*}{\partial t} = i\kappa_{12}A_2^{(n)}.
\]

Here, A_n^{(n)} is the part of the state |2⟩ amplitude that carries the polarization at frequency ω_{pum}, d_{pn} = δ_{pn} + iγ_2/2, where δ_{pn} is the detuning of the probe laser (ω_{pum}) from the |1⟩→|2⟩ resonance and γ_2 is the decay rate of the state |2⟩. In addition, A_3 is the amplitude of state |3⟩ and d_3 = δ_3 + iγ_3/2, where δ_3

![Energy-level diagram for a three-state double-Λ system interacting with a two-mode probe and two-mode control fields.](image)

FIG. 1. Energy-level diagram for a three-state double-Λ system interacting with a two-mode probe and two-mode control fields. In reference to ultracold ⁸⁷Rb atomic vapor, we choose |1⟩=5S_{1/2}(F=1,M_F=-1), |2⟩=5P_{1/2}(F=2,M_F=0), |3⟩=5S_{1/2}(F=2,M_F=1).
\[ = \omega_{p1} = \omega_{1} - \omega_{2} - \omega_{1,2} \text{ is the two-photon detuning between states } [1] \text{ and } [3] \text{ and } \gamma_{3} \text{ is the decay rate of state } [3]. \text{ We assume that two-photon resonances are always maintained so that } \delta_{1}=0 \text{ and take } \gamma_{3}=0. \text{ Finally, } 2\Omega_{pn}(2\Omega_{om}) \text{ is the Rabi frequency of the probe (pump) field for the relevant frequency mode, } \kappa_{12}=2\pi N a_{pn}|D_{12}|^2/(hc) \text{ with } D_{12} \text{ and } N \text{ being the dipole moment for transition } [1] \rightarrow [2] \text{ and concentration, respectively. We note that the major approximations used in deriving Eqs. (1a)–(1c) are the undepleted ground state (}A_{1}=1\text{) and the neglect of far-off-resonant terms such as crossmode stimulated emission with nonvanishing two-photon detunings (these approximations should always be accurate if the fields at } a_{pn} \text{ are sufficiently weak). No other approximations have been made in our semiclassical theory, Eqs. (1a)–(1c).

To solve Eqs. (1a)–(1c) we begin by assuming that } |\delta_{pn}| \gg \gamma_{2}, |\delta_{om}| \gg 1, |\delta_{2}| \gg |\Omega_{om}|, \text{ and } \Omega_{om} \ll |\Omega_{pn}|. \text{ These conditions ensure that the ground state remains undepleted and adiabatic processes remain effective. Let } \alpha_{p}^{(n)}, \alpha_{n}, \text{ and } \Lambda_{pn} \text{ be the time Fourier transforms of } A_{2}^{(n)}, A_{3}, \text{ and } \Lambda_{pn} \text{, respectively; we obtain } (n, m=1,2; n \neq m)

\[ \alpha_{p}^{(n)} = \frac{[|\Omega_{cm}| - (d_{3} + \omega)(d_{p} + \omega)]\Lambda_{pn}^{*}}{D} \text{ (2a)} \]

\[ \alpha_{n} = \frac{\Omega_{z}(d_{p} + \omega)\Lambda_{p1}^{*} + \Omega_{z}(d_{p} + \omega)\Lambda_{p2}^{*}}{d_{p} + \omega} \text{ (2b)} \]

\[ \frac{\partial \Lambda_{pn}^{*}}{\partial z} - i \omega \Lambda_{pn}^{*} = i \kappa_{12} \alpha_{2}^{(n)}, \text{ (n=1,2)} \text{ (2c)} \]

where } \omega \text{ is Fourier transform variable,}

\[ D = (d_{3} + \omega)(d_{p} + \omega)(d_{3} + \omega) - |\Omega_{z}|^{2}(d_{p} + \omega) - |\Omega_{2}|^{2}(d_{p} + \omega) \text{ (2d)} \]

Equations (2a)–(2d) can be easily solved, yielding

\[ \Lambda_{pn}^{*} = e^{-\alpha_{p}^{(n)}(W_{p}^{(n)} e^{-L_{c}} + W_{n}^{(n)} e^{-L_{c}})} \text{ (3)} \]

where } \alpha_{p} = L_{0}^{-1/2}[(|\Omega_{cm}| - (d_{3} + \omega)(d_{3} + \omega))/2], \text{ and } \beta_{12} = \Omega_{e}^{(2)} \Omega_{om} e_{c}^{2}, \text{ etc.} \text{ Equation (3) gives the explicit Fourier transform of the probe fields } A_{2}^{(n)} \text{ at the entrance of the medium. Since the frequencies of the two modes are nearly the same, this represents a nearly } 100\% \text{ photon flux conversion from } \Omega_{om} \text{ to } \Omega_{pn}. \text{ On the other hand, } m \text{ is an even integer, we have } \Lambda_{p2}^{*}(z,t)=0 \text{ for } m \neq 1 \text{ and the state of the probe field oscillates between the two frequency modes as a function of propagation distance.}

Noting that there are no restrictions on having maximum atomic coherence between states } [1] \text{ and } [3], \text{ we note that the quadratic approximation for } \alpha_{p} \pm \Delta \text{ is accurate if it is sufficiently accurate to simply replace the coefficients in } W_{+}^{(m)} \text{ at } \omega=0. \text{ With these approximations, we obtain}

\[ W_{+}^{(m)} = \frac{|\Omega_{2}|^{2}}{|\Omega_{om}|^{2}}[\Lambda_{pn}^{*}(0, \omega) - S_{1} \Lambda_{pn}^{*}(0, \omega)], \text{ (4a)} \]

\[ W_{om}^{(m)} = \frac{|\Omega_{om}|^{2}}{|\Omega_{2}|^{2}}[\Lambda_{pn}^{*}(0, \omega) + S_{2} \Lambda_{pn}^{*}(0, \omega)], \text{ (4b)} \]

where } S_{1} = (\Omega_{z}^{(2)})^{2} \delta_{pn}^{*} + (\Omega_{1}^{(2)})^{2} \delta_{om}^{*}, S_{2} = (\Omega_{z}^{(2)})^{2} \delta_{om}^{*} + (\Omega_{1}^{(2)})^{2} \delta_{pn}^{*} \text{ (4c)} \text{ and } |\Omega_{2}|^{2} = |\Omega_{om}|^{2} + |\Omega_{pn}|^{2}. \text{ The second unique feature of the present work is a robust multiphoton destructive interference. We note that both } \Lambda_{p1}^{*} \text{ and } \Lambda_{p2}^{*} \text{ have a velocity component [the first terms in Eq. (5)] that decays in exactly the same way. In the case where}
This implies that at a sufficient depth into the medium the assuming $V_s$ changes from conventional FWM to a parametric FWM. Consequently, both two-photon ($\Omega_{p1}^{*}$ excitation balances $\Omega_{PFWM}^{*}$ excitation) and three-photon ($\Omega_{p1}^{*} + \Omega_{SR}^{*}$ excitation) destructive interferences can occur after characteristic propagation distances under suitable driving conditions. If, on the other hand, one extinguishes the control field $\Omega_2$, the generated field and the three-photon destructive interference disappear, and one loses the TAG pulse pair. These features of the dark state in the present study do not have a counterpart in the conventional EIT process. We emphasize that in contrast to the conventional EIT scheme the unique type of induced transparency is critically dependent upon the production and propagation of an internally generated field and upon two distinctive relaxation processes for each velocity component. There is no equivalent mechanism in the conventional $\Lambda$-type EIT system.

The above results are the consequence of linearization of the coefficients and exponents permitted by the assumption of good adiabatic behavior in the atomic response. Corrections to this adiabatic theory can be derived analytically to account for probe pulse spreading and additional attenuation. Due to space limitation, solutions to Eq. (3) using quadratic approximation with different pulse lengths and delays will be presented elsewhere. In the following we give numerical examples to demonstrate the validity of our analytical solutions.

For numerical simulations we consider cold $^{87}$Rb atomic vapor. In the first case we take $\Omega_{p1}(0,0)\tau_1=1$ and $\Omega_{p2}(0,0)\tau_2=0$. From Eq. (6) we find that the first destructive interference occurs at $z_1=0.00865$ cm. We thus take the medium thickness to be $z_m=z_{m1}=0.01$ cm. In Fig. 2 we plot the normalized probe field intensities as a function of $z/z_m$ for the case of $z_m=0.01$ cm (the inset) and $z_m=1.0$ cm. The numerical solutions are obtained by solving Eqs. (1a)–(1c) without any approximations. The results from the two meth-

\[ \Omega_{p2}(0,t)=0 \]
ods are so that they cannot be distinguished on the graph. Note that the peak of \( |\Omega_{p2}(z,t,\tau)|^2/|\Omega_{p1}(0,0)|^2 \) (the inset) indicates a conversion of 91% is near \( z=0.008 \) 6.5 cm, as predicted.

It is instructive to consider a case where \( \lambda^*_p = (\Omega^*_p/\Omega^*_q) \lambda^*_q \) is satisfied and \( \Omega^*_p(0,0) = \Omega^*_q(0,0) = 1 \). In such a matched injection case Eq. (8) predicts that the fields \( \Omega^*_p(0,0) \) and \( \Omega^*_q(0,0) \) form a perfectly matched pair, propagate with identical group velocity, and experience very little attenuation and pulse distortion. Extensive numerical simulations for such injection-matched pairs agree well with the analytical solution Eq. (7), and typically less than 15% attenuation of the probe fields can be achieved for an extended propagation distance of 1 cm. This is a remarkably high transparency for such a highly resonant and optically thick medium. In a separate study we will systematically investigate the advantages of such matched injection conditions and explore applications of this effect in other wave propagation problems.

We have investigated a unique type of induced transparency resulted from multiphoton destructive interference. We have shown the formation of two TAG matched ultraslow pulse pairs. In addition, we have discussed the key differences between the present scheme and the conventional EIT scheme, and we have shown that the unique effect and the underlying physics do not have equivalent counterparts in the conventional EIT system. We emphasize that the unique type of induced transparency is the result of two distinctive relaxation processes and is critically dependent on the three-photon destructive interference involving the internally generated field. The robust three-photon destructive interference and the efficient multiphoton-based induced transparency predicted here are also expected to occur in Doppler broadened media under modified driving conditions due to the broad linewidths. Of course, the requirement for near adiabatic behaviors of the system response will also be subject to appropriate modifications.

The TAG matched propagation of a pair of ultraslow probe pulses in a highly transparent medium discussed here may be applied to other multiwavelength experiments in the ultraslow propagation regime. The unique type of highly efficient induced transparency enabled by one- and three-photon destructive interference may provide yet another way to achieve lossless propagation in a highly dispersive resonant medium. This could lead to intriguing applications in the field of optoelectronics.

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[14] For instance, with \( |\delta_p| \geq |\gamma_2| \), we require \( (\delta_p - \delta_{p2})|\Omega_{p1}|^2|\Omega_{p2}|^2 = (\delta_p|\Omega_{p1}|^2 + \delta_{p2}|\Omega_{p2}|^2)^2 \). Thus, if one chooses \( \delta_p=\delta_{p2} \), then \( |\Omega_{p2}|^2 = (3+48)|\Omega_{p1}|^2 \) gives matched group velocity. We also require that \( \text{Re}[\ldots] \geq \text{Im}[\ldots] \) which can be easily satisfied because of the condition \( |\delta_p| \geq |\gamma_2| \).