Transmittance measurements for filters of optical density between one and ten


We have developed a facility for measuring the transmittance of optical filters at a wavelength of 1064 nm, using a Nd:YAG laser, a power stabilizer, and linear photodiode detectors. A direct measurement method was used for filters with optical densities (OD's) less than or equal to 4, and a reference substitution technique was used for filters with OD's as great as 10. The apparatus and data-acquisition system are described. Measurement results for a set of filters are presented. The expanded uncertainties for the measured OD and deduced absorption coefficient are determined through a detailed analysis of all the uncertainty components. © 1997 Optical Society of America

Key words: Infrared filter, optical density, radiometry, transmittance, uncertainty.

1. Introduction
Accurate spectral transmittance measurements are important for calibration of spectrometers and for determination of the optical properties of materials.1,2 High-accuracy spectrophotometers are commonly used for these measurements.2–4 Recent advances in stable laser sources and highly sensitive detectors have allowed accurate determination of the spectral transmittance of filters with extremely low transmittance.5–8 As reviewed by Gentile et al.,9 both direct and heterodyne detection methods have been employed for measuring transmittance with a dynamic range as large as 10 decades at wavelengths of 633 nm (He–Ne line), 1064 nm (Nd:YAG line), and 10.2 and 10.6 μm (CO₂ lines).

A facility has been developed at the National Institute of Standards and Technology (NIST) for transmittance measurements at a wavelength of 1064 nm. Its primary components are a Nd:YAG laser, a laser power stabilizer, and linear photodiode detectors. A direct measurement method is used for filters of optical densities (OD's) from 1 to 4, and a reference substitution technique is used to measure higher OD filters. Ionically colored glass filters with OD's from 1 to 6 have been characterized and certified as Standard Reference Materials. These filters can be used to calibrate transmittance measurements made with lasers or spectrophotometers to accurately attenuate optical power or to characterize detector nonlinearity.10 In the present paper we discuss the measurements of filters with OD's from 1 to 10, especially high OD measurements and absorption coefficient determination.

2. Theory and Instrumentation
A. Transmittance and Optical Density
For a filter plate made of a homogeneous and isotropic material with smooth and parallel surfaces, the transmittance depends on the thickness, optical constants of the material (which are wavelength and temperature dependent), the angle of incidence and polarization state of the incident electromagnetic radiation, and the degree of coherence between multiple reflected waves.2,11 At normal incidence the reflectance at the first interface is \( r = \left| \frac{n - 1}{n + 1} \right|^2 \), where \( n \) and \( k \) are the real and the imaginary parts of the refractive index (both depend on wavelength). If \( k \ll (n - 1) \), the reflectance may be calculated from \( r = (n - 1)^2 / (n + 1)^2 \). The internal transmittance \( \tau \) for radiation propagating in the direction perpendicular to the surface is \( \exp(-ad) \), where \( d \) is the thickness of the plate and \( a = 4\pi k/\lambda \) (where \( \lambda \) is the wavelength in vacuum) is the absorption coefficient. The fraction of radiation transmitted through the plate is reduced by reflections at the two interfaces and the internal absorption. Therefore the external transmittance \( T \) is

\[
T = (1 - r)^2 \tau. \tag{1}
\]
The effects of multiple reflections are negligible for filters of OD ≥ 3 because of the low reflectance (ρ ≈ 0.04) and strong absorption (τ ≤ 1) of the filter samples used in the present study. For filters of OD ≥ 2, however, expressions that include multiple reflections or interference effects should be used. The relative error in transmittance caused by neglecting interference effects is discussed in Subsection 3.D.

The optical density is defined as

$$OD = \log_{10} T.$$

For example, OD = 1 corresponds to a transmittance value of 0.1, and OD = 8 corresponds to a transmittance value of 10⁻⁸. Based on Eqs. (1) and (2), the absorption coefficient is related to the OD, the reflectance, and the thickness as

$$\alpha = \frac{OD + 2 \log_{10}(1 - \rho)}{d \log_{10} e}.$$

**B. Measurement Techniques**

The transmittance measurement setup is shown in Fig. 1. The optical source is a continuous-wave arc-lamp-pumped Nd:YAG laser with an output power of 3 W at 1064 nm. The laser beam is directed through a collimator, a polarizer and a power stabilizer, a weakly focusing lens, and then to a light-tight enclosure. A wedged quartz beam splitter inside the enclosure reflects a small portion (~4%) of the laser power to a feedback detector (FD), which controls the stabilizer. The laser power is stabilized to better than 0.3% root-mean-square fluctuation over several hours of operation. The reflection off the second surface of the wedge is sent to a monitor detector (MD) to normalize the laser power, which reduces the effect of residual power fluctuation. The stability of the optical power and the significance of the monitor detector on the measurement are discussed in detail by Zhang et al. The transmitted beam passes through a shutter and a sample (or a reference) and is then reflected by a mirror to the signal detector (SD). Attenuators are used to vary the laser power and to ensure that the detectors are operated in their linear ranges. A baffle at the middle of the enclosure prevents stray light from reaching the signal detector. A detector enclosure further reduces the stray light, which is necessary to measure filters of OD > 9.

The signal detector is a Hamamatsu S1337 series silicon photodiode, with an active area of 10 mm × 10 mm. The detector responsivity is ~0.15 A/W at a wavelength of 1064 nm. The signal detector is coupled to a built-in transimpedance amplifier that has nine feedback-resistor settings from 2 kΩ to 100 GΩ. Therefore the voltage responsivity after the amplifier is between 300 V/W and 1.5 × 10¹⁰ V/W. A 6½ digit digital voltmeter (DVM) measures the dc voltage from the amplifier. The electronic design and test results of the amplifiers used with photodiode detectors to achieve a linear dynamic range of 14 decades are discussed in Refs. 5 and 6. Another DVM simultaneously measures the output voltage from the monitor detector–amplifier.

The filter is mounted on a copper holder. A thermometer on the holder monitors the filter temperature during the measurement. Computer-controlled motors move the filter holder horizontally and vertically to position the sample or reference (the reference is either blank or another filter). An automatic data-acquisition program controls the motion of the shutter and the motors, controls the DVM’s (one of which also measures the resistance of the filter thermistor), and calculates the transmittance and optical density for each measurement.

Six measured values determine the relative transmittance of the sample filter at a single position, namely,

$$T_{relative} = \frac{[(V_{s1} - V_{o1})/V_{m}]}{[(V_{s1} - V_{o1})/V_{m}]}_{reference},$$

where V is the output voltage, subscripts s and m indicate the signal detector and the monitor detector, respectively, and subscripts 0 and 1 indicate shutter closed and open, respectively. The signal when the shutter is closed (Vs1) is subtracted from the output signal (Vs1) to eliminate background. The DVM integration time is approximately 5 s (bandwidth 0.2 Hz).

The reference is left blank for transmittance measurements for filters with OD’s 1 to 4. Hence the sample transmittance is the same as the relative transmittance calculated from Eq. (4). The gain setting cannot be changed during each measurement. The signal-to-noise ratio is lower with higher-OD filters, since the minimum measurable voltage of the DVM is 1 μV. For filters of OD ≥ 5, a reference substitution method was used, similar to the step-down method employed by Eckeler et al. to measure the spectral transmittance down to OD = 4 with a spectrophotometer. The transmittance of the reference filter was measured at a fixed position by use of a lower gain setting. A higher gain setting was used for filters of OD ≥ 4, which are indicated by a small increase in the relative error due to noise.
to measure the transmittance of a high OD filter relative to that of the reference filter. The transmittance of the sample filter is calculated by

$$T_{\text{sample}} = T_{\text{relative}} \times T_{\text{reference}},$$

where $T_{\text{reference}}$ is the transmittance of the reference filter at the fixed position. An OD 3 filter was used as the reference for filters of OD 5, 6, and 7; an OD 4 filter was used as the reference for filters of OD 8; and an OD 8 filter was used as the reference for filters of OD 9 and 10.

3. Results and Discussion

The filters used in this study were made of an ioni-
cally colored glass material (NG-9) manufactured by Schott of Mainz, Germany. The filters are un-
coated and optically polished. Each surface is flat to
within one-tenth of the He–Ne wavelength (633 nm), and the wedge angle is less than 5 μrad (1 arc sec). The filters are 51 mm × 51 mm, with thicknesses between 1 and 10.5 mm. The OD is determined by
the thickness of the filter.

The laser beam incident on the filter was perpen-
dicular to the surface (angle of incidence, $<2^\circ$). This was achieved by observation of the reflected beam with an infrared sensing card. The slight translation of the beam through the filter (maximum 0.12 mm) has little effect on the measurement because of the high degree of spatial uniformity of the detector. The beam diameter at the filter was
≈3-mm full width at half-maximum, and the beam
divergence is less than 2 mrad. The detector was
tilted $\approx 1^\circ$ to eliminate interreflections between the
detector and the filter. The temperature of the lab-
boratory was between 22 and 24 °C, except during
the study of the temperature effect as discussed in
Subsection 3.B.

A. Detector Linearity and Noise Equivalent Power

The linearity of the detector was tested by measuring
the OD at different power levels. Attenuation filters
were used to change the incident power. The ampli-
fier gain was optimized to yield the best signal-to-
noise ratio without saturating the DVM. The OD of
two filters measured with different laser powers is
shown in Fig. 2. It can be seen that the detector is
extremely linear at optical powers of less than 1 mW.
With an input optical power of 1 nW, the power reach-
ing the detector after the OD 4 filter is only $10^{-13}$ W,
resulting in a low signal-to-noise ratio and hence a
large standard uncertainty. Measurements of zero
signal (i.e., with the detector blocked) indicated that
the noise power is of the order of $10^{-14}$ W for a 5-s
measurement time (i.e., bandwidth of 0.2 Hz), which
agrees with the manufacturer-specified noise equiva-
lent power. The laser power at the filter position
was limited to 20 mW to avoid excessive heating of
the filter. Therefore the dynamic range of the mea-
surement is $\approx 12$ decades by the reference substitu-
tion method.

B. Temperature Dependence of the Absorption
Coefficient

The temperature of the filter was varied to allow us to
investigate the effect on the optical density. The central air conditioner in the building was used to change the temperature in the laboratory from 23 to
27 °C. The filter temperature was assumed to be
the same as that of the copper holder. The filter
temperature was $\approx 1^\circ$ higher than room tempera-
ture owing to the heating of the positioning motors.
To reduce the filter temperature below 24 °C, cold N$_2$
gas from a liquid-nitrogen tank was passed through
the enclosure. The enclosure was closed after the
cooling. The OD and the filter temperature were
measured as the filter temperature increased. The
change of the filter temperature ($1$ to $2^\circ$ per hour)
was slow enough to obtain a correlation between the
OD and temperature. The measurements were
made at the same position of the filter to eliminate
the effect of spatial nonuniformity. A linear fit
shows that a $1^\circ$ temperature rise increases the OD
by $0.00052$ for an OD 2 filter and by $0.0008$ for an OD
3 filter. The change in OD is almost proportional to
the filter thickness, indicating that it is not caused by
a change in the reflectance (which depends on the
refractive index $n$) but is caused by a change in the
internal absorption. The thickness change is negli-
The spatial nonuniformity and interference effects

The spatial variation in OD depends on the individual filter. Transmittance measurements were performed either at 9 positions in a 3 × 3 matrix with 10-mm spacing or at 25 positions in a 5 × 5 matrix with a 5-mm spacing around the center of the filter. These measurements were repeated at least once for all positions. Because the surfaces of the filters are extremely flat and parallel, the spatial variation is attributed to the inhomogeneity of the material except for OD 1 filters. For OD 1 filters, interference effects of multiple reflections must be taken into consideration.

For normal incidence the transmittance of a plate with two parallel, optically smooth surfaces for completely coherent radiation is

\[
T = \frac{(1 - \rho)^2 \tau}{1 + \rho^2 \tau^2 - 2\rho \tau \cos(4\pi nd/\lambda)}
\]  

The denominator oscillates as \( \xi = 2nd/\lambda \) varies. The amplitude of oscillation is estimated to be 0.8% for OD 1 filters and 0.08% for OD 2 filters. For OD 1 filters (\( d = 1.04 \) mm), a variation of either 0.18 \( \mu m \) in \( d \), of 0.17 \( \mu m \) in \( \lambda \), or of 0.016% in \( n \) would change the transmittance from a maximum to a minimum. The linewidth \( \Delta \nu \) of the laser is between 1 and 5 \( cm^{-1} \). Therefore the laser radiation is not completely coherent.11 From the formula for partially coherent radiation given by Zhang,11 the amplitude of oscillation can be calculated from \( \Delta T/T \approx 2\rho \).
where \( \sin(c) = \sin(c)/c \). Using \( \Delta \nu = 1 \text{ cm}^{-1} \), the standard uncertainty in OD caused by interference effects \((\sigma_{\text{interf}})\) is estimated to be 0.0016 for OD 1 filters, 0.0001 for OD 2 filters, and negligible for filters of OD \( \geq 3 \). The large spatial variation for OD 1 filters is likely caused by interference effects, since the thickness variation is of the order of 0.2 \( \mu \text{m} \). Variations in the range of \( \pm 0.0025 \) OD were observed for an OD 1 filter as the filter temperature was varied from 21 to 27 °C, which could be explained by a slight temperature dependence of \( n \). The OD variation is less than \( 2\sigma_{\text{interf}} \), indicating that the calculated \( \sigma_{\text{interf}} \) on the basis of \( \Delta \nu = 1 \text{ cm}^{-1} \) is still a conservative estimate.

**E. Measurement Uncertainty**

The uncertainty for a measurement at a single position is calculated according to the guidelines given by Taylor and Kuyatt.\(^{20}\) The standard uncertainty associated with the measurement repeatability \((\sigma_{\text{repeate}})\) at the same position is shown in Table 1. The repeatability is better for the OD 5 and OD 6 filters than for the OD 4 filter, because the substitution technique has improved the signal-to-noise ratio. The reproducibility \((\sigma_{\text{reprod}})\) of the experimental setup was calculated based on various sets of measurements with filters of OD 1, 2, and 3, performed during a couple of weeks when the samples had been unmounted and remounted. The reproducibility of approximately 0.00035 was obtained and used to calculate the measurement uncertainty for all filters. The standard uncertainty associated with the detector nonlinearity \((\sigma_{\text{nonlin}})\) was estimated to be 0.0002. The standard uncertainty caused by the temperature variation \((\sigma_{\text{temp}})\) of \( \pm 0.5 \) °C is also given in Table 1. Since the absorption coefficient changes with temperature, \( \sigma_{\text{temp}} \) is proportional to the filter thickness. For filters with OD \( \geq 5 \), the uncertainty due to the reference measurement \((\sigma_{\text{ref}})\) needs to be included. Filters of OD 5, 6, and 7 were measured with an OD 3 filter used as the reference; filters of OD 8 were measured with an OD 4 filter used as the reference; and filters of OD 9 and 10 were measured with an OD 8 filter used as the reference. The combined uncertainty for a measurement at a single location is\(^{20}\)

\[
\sigma_{\text{comb}} = (\sigma_{\text{repeate}}^2 + \sigma_{\text{reprod}}^2 + \sigma_{\text{nonlin}}^2 + \sigma_{\text{temp}}^2 + \sigma_{\text{ref}}^2)^{1/2},
\]

where \( \sigma_{\text{reprod}} = 0.00035 \) and \( \sigma_{\text{nonlin}} = 0.0002 \) for all filters. The expanded uncertainty (95% confidence) is twice the combined uncertainty (see Table 1). An absolute difference of 0.001 in OD corresponds to a relative difference \( \Delta T/T \) of 0.23% in transmittance. Therefore the relative expanded uncertainty in transmittance is calculated using the \( \Delta T/T \) values in Table 1 and \( \sigma_{\text{comb}} \) values in Table 2.

### Table 1. Uncertainty in OD for a Single Measurement at a Fixed Position

<table>
<thead>
<tr>
<th>Nominal OD</th>
<th>Repeatability ( \sigma_{\text{repeate}} )</th>
<th>Temperature Variation ( \sigma_{\text{temp}} )</th>
<th>Reference ( \sigma_{\text{ref}} )</th>
<th>Combined Standard Uncertainty ( \sigma_{\text{comb}} )</th>
<th>Expanded Uncertainty ( 2\sigma_{\text{comb}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>OD 1</td>
<td>0.00006</td>
<td>0.00007</td>
<td>NA</td>
<td>0.00041</td>
<td>0.0008</td>
</tr>
<tr>
<td>OD 2</td>
<td>0.00006</td>
<td>0.00015</td>
<td>NA</td>
<td>0.00043</td>
<td>0.0009</td>
</tr>
<tr>
<td>OD 3</td>
<td>0.00020</td>
<td>0.00022</td>
<td>NA</td>
<td>0.00050</td>
<td>0.0010</td>
</tr>
<tr>
<td>OD 4</td>
<td>0.00007</td>
<td>0.00030</td>
<td>NA</td>
<td>0.00084</td>
<td>0.0017</td>
</tr>
<tr>
<td>OD 5</td>
<td>0.00002</td>
<td>0.00038</td>
<td>0.00050</td>
<td>0.00077</td>
<td>0.0015</td>
</tr>
<tr>
<td>OD 6</td>
<td>0.00002</td>
<td>0.00045</td>
<td>0.00050</td>
<td>0.00081</td>
<td>0.0016</td>
</tr>
<tr>
<td>OD 7</td>
<td>0.00010</td>
<td>0.00053</td>
<td>0.00050</td>
<td>0.00130</td>
<td>0.0026</td>
</tr>
<tr>
<td>OD 8</td>
<td>0.00010</td>
<td>0.00060</td>
<td>0.00084</td>
<td>0.00149</td>
<td>0.0030</td>
</tr>
<tr>
<td>OD 9</td>
<td>0.00010</td>
<td>0.00067</td>
<td>0.00149</td>
<td>0.00197</td>
<td>0.0039</td>
</tr>
<tr>
<td>OD 10</td>
<td>0.00060</td>
<td>0.00075</td>
<td>0.00149</td>
<td>0.00624</td>
<td>0.0125</td>
</tr>
</tbody>
</table>

\(^{a}\)The reproducibility and nonlinearity components are the same for all the filters.

### Table 2. Measurement Results for a Set of Filters with OD’s from 1 to 10

<table>
<thead>
<tr>
<th>Filter No.</th>
<th>( d ) (mm)</th>
<th>Filter Temp. (°C)</th>
<th>( \sigma_{\text{spatial}} )</th>
<th>( \sigma_{\text{interf}} )</th>
<th>Optical Density OD ± ΔOD</th>
<th>Absorption Coefficient ( a ) ± Δα (mm⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-01</td>
<td>1.044</td>
<td>24.7</td>
<td>0.00129</td>
<td>0.0016</td>
<td>1.0111 ± 0.0042</td>
<td>2.1518 ± 0.0113</td>
</tr>
<tr>
<td>22-01</td>
<td>2.177</td>
<td>25.4</td>
<td>0.00065</td>
<td>0.0001</td>
<td>2.0785 ± 0.0016</td>
<td>2.1609 ± 0.0035</td>
</tr>
<tr>
<td>32-01</td>
<td>3.157</td>
<td>23.8</td>
<td>0.00021</td>
<td>NA</td>
<td>2.9931 ± 0.0011</td>
<td>2.1572 ± 0.0023</td>
</tr>
<tr>
<td>42-01</td>
<td>4.194</td>
<td>24.1</td>
<td>0.00072</td>
<td>NA</td>
<td>3.9679 ± 0.0022</td>
<td>2.1580 ± 0.0020</td>
</tr>
<tr>
<td>54-01</td>
<td>5.410</td>
<td>25.1</td>
<td>0.00026</td>
<td>NA</td>
<td>5.1140 ± 0.0016</td>
<td>2.1615 ± 0.0014</td>
</tr>
<tr>
<td>64-01</td>
<td>6.391</td>
<td>25.3</td>
<td>0.00023</td>
<td>NA</td>
<td>6.0325 ± 0.0017</td>
<td>2.1606 ± 0.0012</td>
</tr>
<tr>
<td>75-01</td>
<td>7.526</td>
<td>24.8</td>
<td>0.00160</td>
<td>NA</td>
<td>7.1015 ± 0.0041</td>
<td>2.1619 ± 0.0015</td>
</tr>
<tr>
<td>84-01</td>
<td>8.418</td>
<td>25.2</td>
<td>0.00960</td>
<td>NA</td>
<td>8.1084 ± 0.019</td>
<td>2.2082 ± 0.0054</td>
</tr>
<tr>
<td>94-01</td>
<td>9.413</td>
<td>24.9</td>
<td>0.00200</td>
<td>NA</td>
<td>9.1038 ± 0.0056</td>
<td>2.2183 ± 0.0016</td>
</tr>
<tr>
<td>105-01</td>
<td>10.549</td>
<td>24.7</td>
<td>0.00900</td>
<td>NA</td>
<td>10.1885 ± 0.022</td>
<td>2.2162 ± 0.0048</td>
</tr>
</tbody>
</table>

\(^{a}\)Expanded uncertainties are given as ± values.
mittance is less than 0.25% for filters of OD 1 to OD 3, less than 0.4% for filters of OD 4 to OD 6, less than 1% for filters up to OD 9, and less than 3% for filters of OD 10.

4. Conclusions

A Nd:YAG laser, a power stabilizer, and several linear photodiode detectors were employed to characterize the optical densities of transmission filters (51 mm × 51 mm) at a wavelength of 1064 nm for a laser beam normally incident on the filter. The optical density of these filters ranged from OD 1 to OD 10 depending on filter plate thickness. The expanded uncertainty for the OD measurements at a single position was estimated to be <0.001 for filters of OD 1 to OD 3, <0.002 for filters of OD 4 to OD 6, <0.004 for filters of OD 7 to OD 9, and <0.013 for filters of OD 10. The spatial inhomogeneity of the material is a major source of uncertainty. The standard uncertainty in OD associated with the spatial nonuniformity ranges from 0.0002 to 0.01 for the measured filters. Interference effects introduced a large uncertainty in the measurement of OD 1 filters. The absorption coefficient for each filter was obtained with a relative expanded uncertainty between 0.06% and 0.52%. The effect of temperature on the absorption coefficient was investigated at temperatures from 21 to 27 °C, which showed that the absorption coefficient at 1064 nm increases with temperature by ≈0.026% °C⁻¹.

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References and Notes

13. Catalog. Photodiodes (Hamamatsu Photonics K.K., Hamamatsu City, Japan, 1994), pp. 14–15. The use of trade and company names is for identification only and does not imply endorsement by the National Institute of Standards and Technology, nor does it imply that the material or product identified is necessarily the best available for the purpose.
17. S. G. Kaplan, Optical Technology Division, National Institute of Standards and Technology, Gaithersburg, Md. 20899 (personal communication).