Leakage Current Detection in Cryogenic Current Comparator Bridges

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Abstract—Several tests have been developed to locate leakage currents in cryogenic current comparator (CCC) resistance ratio bridges used at NIST to measure ratios of 1000 Ω/100 Ω, 6453.2 Ω/100 Ω, and 10 kΩ/100 Ω. The major advantage of the tests is that they can be performed in situ using the sensitivity of the CCC bridge. These test procedures have been used to reduce the leakage error uncertainty of CCC ratio measurements linking working standards to the quantized Hall resistance (QHR) and to the NIST calculable capacitor experiment. CCC bridges require that the current which passes through a standard resistor must equal the current through the appropriate CCC winding to very high precision. This can be difficult to verify at or below 1 pA because a large number of possible leakage paths exist. Errors due to six important leakage current paths are given, and the calculated changes in the resistance ratio are compared with measurements made with a controlled leakage resistance in a 100 Ω/1 Ω CCC bridge.

I. INTRODUCTION

This paper presents an analysis of dc electrical leakage in simple CCC [1] bridges, and describes an in situ measurement technique that can detect certain leakage currents with high sensitivity. In CCC bridges, the windings can be superconducting and can be kept close to ground potential; cryogenic windings thus reduce interwinding leakage and leakage from the windings to ground which exist in room temperature current comparator bridges [2]. To ensure that the nanovolt detector's input leakage is small, the common-mode detector input voltage must also be kept close to ground (see Fig. 1). These improvements are effective, especially if the bridge design places leakage paths in parallel with low-impedance connections [3] so that the important leakages will have a small effect on the voltages measured by the bridge. In addition, guarding and other methods of leakage suppression are sometimes used in CCC measurement systems [4]. However, at high-resistance levels, CCC bridge leakage currents are an important systematic source of error, possibly limiting the accuracy of determinations of QHR values.

II. ANALYSIS OF LEAKAGE ERRORS

A CCC bridge comparing resistors $R_1$ and $R_2$ is shown in Fig. 1(a). The primary four-terminal resistor $R_1$ is connected to a nanovolt detector input, the primary CCC winding $N_1$, and the primary current source, all of which are assumed to be well isolated from ground. The current through the secondary CCC winding $N_2$ must be the same with or without leakage due to feedback from the SQUID circuit with a constant current $I_1$. Leakage through resistance $R_L$, shown in Fig. 1(b), is assumed to be present in the bridge and flows to a shield conductor which is grounded. Then, a leakage current given by

$$I_L = \frac{V(R_1) + V(D)}{R_L} \tag{1}$$

flows through leakage path $A'$. The voltage across the inputs of the detector is $V(D)$, and $V(R_1)$ is the voltage across $R_1$ relative to the positive input of the detector. Here and elsewhere, a subscript distinguishes terms related to the leakage current. Because the bridge primary circuit is isolated from ground, a current $I_L$ must pass through the voltage link resistance $R_{VL}$ from the secondary side of the bridge. This current flows through the leakage resistance to ground and back to the secondary circuit, balancing the current returning to the secondary current source. As noted in [4], the resistance in the voltage link connecting the standard resistors will contribute to the leakage current error.
The working CCC bridges used at NIST are designed to be self-balancing. In addition to the SQUID feedback to the secondary winding, a small feedback current is derived from the detector output and goes to a third CCC winding \( N_3 \) (see Fig. 2) to balance the signal at the detector inputs. The resistance ratio and any leakage current contribution are observed in the feedback current \( I_3 \).

For the self-balancing design,

\[
\frac{N_1}{N_2} = \frac{- (N_2 I_2 + N_3 I_3)}{I_1} \\
\frac{N_1}{N_2} = \frac{V(R_1)}{R_2} \text{ if no leakage exists.}
\]

If leakage exists through path \( A' \), \( I_3 \) is equal to the sum of the current in resistor \( R_2 \) and leakage current \( h \). The leakage current is

\[
I_L = \frac{V(R_1)}{R_L}. \tag{2}
\]

The ratio error with a leakage current \( \Delta (R_1 / R_2) \) is found from the difference in the two expressions for \( I_3 \), as follows:

\[
V(R_2) = V(R_1) + I_3 R_{VL} \tag{3}
\]

\[
I(R_2) + I_L = V(R_1) \left[ \frac{1}{R_2} + \frac{R_{VL}}{R_2 R_L} + \frac{1}{R_L} \right] \tag{4}
\]

\[
\Delta (I_3) = -V(R_1) \left[ \frac{R_{VL}}{R_2 R_L} + \frac{1}{R_L} \right] \tag{5}
\]

The leakage causes \( I_3 \) to increase and \( I_3 \) to decrease. With the approximation that \( R_1 / N_1 = R_2 / N_2 \), \( \Delta (R_1 / R_2) = - \Delta I_3 R_2 / V(R_1) \), and we obtain the ratio error for leakage through path \( A' \):

\[
\Delta \left( \frac{R_1}{R_2} \right) = \frac{(R_{VL} + R_2)}{R_L}. \tag{6}
\]

The ratio errors due to six of the possible leakage paths (see Fig. 3) in the self-balancing CCC bridge are given in Table I in terms of the appropriate leakage current.

In most CCC bridges, resistive loads are added to the current loops of the reversing current sources to reduce current source noise or to measure the current in the primary arm of the bridge. The leakage for each current polarity is determined by the current source design (bipolar or reversing unipolar) and by the placement and value of the added load resistance and standard resistor. In the bridges presently used at NIST (see Fig. 2), a resistive load larger than the standard resistor is placed between the winding and the current source. The current reversal is achieved using reversing relays. For leakage paths \( C \) and \( C' \) (see Table I), the average leakage current is given by \( \text{avg} (I_L) = (I_L(+)) - (I_L(-)) / 2 \), where \( I_L(+) \) and \( I_L(-) \) are the leakage currents for the two polarities. For leakage path \( C' \) in this bridge, \( I_L(+) \) is smaller than \( I_L(-) \), and the ratio error is negative.
III. Measurements and Experimental Results

A known leakage resistance has been used to measure the magnitude and sign of the ratio error caused by the six paths shown in Fig. 3. Measurements were made using a 100 Ω/1 Ω CCC bridge. A controlled leakage resistance $R_L$ was produced using a 1 GΩ resistor. The analysis for leakage path $A'$ shows that resistance added to the voltage link ($R_{VL}$) can increase the sensitivity to leakage currents which pass between the two sides of the bridge. Therefore, in place of the low-resistance link between the two standard resistors, for some of the measurements, the connection was made with a 100 Ω resistor. The resulting leakage current errors were then compared with the calculated error (see Fig. 4). The calculated and measured errors were in agreement within 0.008 ppm for errors as large as 6.7 ppm.

A resistor ($R_{VL}$) can be substituted for the link connecting $R_1$ and $R_2$ to detect a real leakage current. Tests were done with ratios of 1000 Ω/100 Ω, 6453.2 Ω/100 Ω, and 10 kΩ/100 Ω. Using a 6453.2 Ω/100 Ω bridge and a measurement voltage $V_{R_1}$ of 0.25 V, the standard deviation for a 136 s measurement increases from 0.010 ppm with $R_{VL} = 0$ to 0.020 ppm with $R_{VL} = 20$ kΩ due to the increase in Johnson noise. If there is a 1 pA leakage current through path $A'$, the corresponding leakage error with $R_{VL} = 20$ kΩ is 0.08 ppm. However, with $R_{VL} = 0$, the ratio error for 1 pA leakage through path $A'$ is less than 0.001 ppm.

Tests for leakage path $A'$ were made for each voltage pair of two QHR samples and for each of four 1000 Ω standards, four 6453.2 Ω standards, and two 10 kΩ standards in the orientation that was used for step-down calibrations. Tests were run with $R_{VL} = 0$, 10 kΩ, and 20 kΩ. The leakage error is proportional to the voltage across the leakage path, if the leakage originates in the resistor terminals, it is driven by the measurement voltage across $R_1$ or $R_2$. The test only detects leakage from the pair of terminals (current and potential) of the standard resistor or at the QHR measurement leads which are at an elevated potential. Leakage from the unused potential leads of the QHR sample can introduce a significant additional error [3] not considered in the above derivation, but detectable using this test. No leakage current to ground was detected for the QHR samples. Leakage resistance to the shields (10$^{11}$ Ω or greater) was detected at the terminals of one of the standards, but this was not a significant source of error.

The primary and secondary current sources [5] used at NIST are normally run with an output voltage of about 2 V. Systematic leakage measurements have been made using the bridge with output voltages from 3.8 to 1.4 V for each source. Added load resistance at the point between the CCC winding and the current source was varied to change the output voltage. The measured slope of ratio error versus source output voltage with $R_{VL} = 20$ kΩ was $-0.02$ ppm/V for the primary source and less than $-0.01$ ppm/V for the secondary source. From Table I, either of the leakage paths $B'$ or $C'$, or some combination, could produce the measured error of $-0.02$ ppm/V for the primary voltage. With $R_{VL} = 0$ Ω, the estimated error from paths $B$, $C$, $B'$, and $C'$ is less than 0.005 ppm for all ratios.

A 6453.2 Ω/100 Ω bridge has been used with a 10 kΩ resistor added to the circuit in the current lead between the detector input and the $N_1$ winding to test the isolation to ground of the CCC wiring and other leakage through path $D'$. This greatly increases the voltage of $N_1$ relative to $N_2$ and to ground. No change in the resistance ratio was detected. The resulting total leakage error uncertainty estimate for NIST CCC measurements is approximately 0.005 ppm at 6453.2 Ω and 10 kΩ and 0.002 ppm at 100 Ω for step-down calibrations to 100 Ω.

IV. Conclusions

The technique described here is capable of detecting leakage currents below 1 pA and of distinguishing several different types of leakage in CCC bridge measurement systems. To ensure that all sources of detectable leakage errors can be located in the circuit, a series of measurements with added resistance in the circuit is needed. This allows limits to be placed on the leakage error; however, a direct path between the terminals of a standard resistor cannot be detected using the techniques described above. CCC bridge calibration of experimental standards therefore should be performed with the same conditions of grounding and applied measurement voltage which are used in experimental measurements.

References
