Resource Planning and Bandwidth Allocation in Hybrid Fiber-Coax Residential Networks

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Abstract
The introduction of new high bandwidth services such as video-on-demand by cable operators will put a strain on existing resources. It is important for cable operators to know how many resources to commit to the network to satisfy customer demands. In this paper, we develop models of voice and video traffic to determine the effect on demand growth on hybrid fiber-coax networks. We obtain a set of guidelines that network operators can use to build out their networks in response to increased demand. We begin with one type of traffic and generalize to an arbitrary number of high-bandwidth CBR-like services to obtain service blocking probabilities. We also consider the effect of supporting variable bit rate (VBR) packet-switched traffic in addition to CBR services. These computations help us to determine how cable networks would function under various conditions (i.e., low, medium, and heavy loads). We also consider how the growth rate of the popularity of such services would change over time, and how this impacts network planning. Our findings will help cable operators estimate how much bandwidth they need to provision for a given traffic growth model and connection blocking requirement.

I. INTRODUCTION
As broadband integrated voice and data access finally becomes a reality for millions of consumers worldwide, there will inevitably be steady growth in demand for high-bandwidth services that will make careful deployment planning by cable operators a necessity. It is envisioned that video on demand (VOD), large file downloads (e.g. data, music, and video), and voice over IP (VoIP) will become common over cable networks.

In a hybrid fiber-coax (HFC) network, illustrated in Fig. 1, the residences are connected to a coax tree and branch network which terminates at a fiber node. At the fiber node, the traffic signals are converted from electrical to optical and transmitted to the Cable Modem Termination System (CMTS) located in the Cable Head End Office. The Data Over Cable System Interface Specification (DOCSIS) protocol is the current industry standard for the physical and medium access control (MAC) layers of upstream/downstream communications between a residential cable modem (CM) and the CMTS over HFC networks [1]. The spectral allocation for the upstream and downstream cable modem services and the downstream broadcast TV are illustrated.
in Fig. 2. DOCSIS allows a choice of various frequency channel widths and modulation techniques for the upstream and downstream communications. So Fig. 2 illustrates a typical implementation wherein the upstream uses 1.6 MHz frequency channels with QPSK modulation and the downstream uses 6 MHz frequency channels with 64-QAM modulation for the DOCSIS cable modem applications. With QPSK, each 1.6 MHz upstream channel provides a data rate of 2.56 Mb/s and with 64-QAM, each downstream channel provides a data rate of 30 Mb/s [1].

The new services carried over HFC are characterized by a plethora of throughput rates and delay requirements, along with varying degrees of burstiness [2]. An important goal of the system architecture and protocol design for hybrid fiber-coax (HFC) networks has been efficient service integration and real-time statistical multiplexing of various traffic types. With this objective in view, the upstream part of the MAC protocol has been carefully designed to operate with the best efficiency possible, based on contributions from many researchers [3], [4], [5], [6], [7], [8], [9], [10], [11]. The DOCSIS protocol essentially permits statistical packet multiplexing and scheduling with suitable Quality of Service (QOS) for the downstream channels. This is because the downstream channels (unlike the upstream) have no contention due to collisions. However, there are service blocking issues in the downstream due to random call arrivals/departures and inadequate downstream bandwidth. The traffic engineering and bandwidth growth planning for the downstream is becoming increasingly more complex due to the mix of various emerging services that require disparate bandwidth and latency requirements. In this paper, we focus on these downstream issues for a mix of high-bandwidth CBR-like data services of interest, e.g., VOD, large file downloads. Even though these services are transported using TCP/IP or UDP/IP over DOCSIS protocol, they are CBR-like in that they require high instantaneous bandwidth for relatively long durations, they have very high burstiness and low delay tolerance. Essentially, they need to be treated like CBR flows in order to be given adequate QOS treatment as well as for bandwidth allocation and resource planning purposes. In addition, CBR transmission has been proposed as an efficient method for transporting video by Hadar and Segal [12], who argue that the benefits of using CBR include minimizing the connection duration and allowing the use of simpler resource allocation and admission control mechanisms than those that are needed if VBR transmission is used. The mix of various CBR and VBR services in upstream channels has been analyzed and studied in the literature [2],
The work reported here can be extended in the future to consider a mix of CBR and VBR (statistically multiplexed) services in the downstream channels. The applicability of our analytical models in this paper is to produce a line card deployment schedule for a cable operator under various applications traffic growth scenarios while serving a mix of heterogeneous high-bandwidth CBR-like downstream services.

The remainder of this paper is organized as follows. In Section II we develop an analytical model of the downstream data channels when the load is provided by multiple types of CBR data services. We use this model to develop expressions for the probability that a customer will experience blocking by being unable to access a desired service or set of services. We also consider VBR packet traffic and its effect on HFC network performance. In Section III we use the model that we developed in Section II to produce a line card deployment schedule for a cable operator under various usage growth scenarios. In Section IV, we summarize our results and discuss how they may be extended to examine other types of data traffic such as compressed video and best effort downloads using TCP.

II. ANALYTICAL MODEL

For the case where the network supports a single CBR service with a bandwidth of $B$, the analysis is straightforward. The system can be in one of $M+1$ states, where $M = \lfloor B_{\text{max}}/B \rfloor$ is the maximum number of users that can be accommodated using a bandwidth of $B_{\text{max}}$. The blocking probability is given by the Erlang-B loss formula,

$$P_B = B(M, \rho) = \frac{\rho^M/M!}{\sum_{n=0}^{M} \rho^n/n!},$$

where $\rho = \lambda/\mu$ is the channel utilization, $\lambda$ is the mean arrival rate of requests for the service, assuming Poisson arrivals, and $1/\mu$ is the average time that the service is used by a customer, where the service time distribution is arbitrary.

It is certain that the bandwidth on HFC networks will be shared by multiple application types. VOD and large file downloads are examples of CBR-like services requiring sustained bandwidth in the downstream channels. Therefore, we consider the case where there are two CBR services available on the downstream channels. We refer to these two services as Service 1 and Service 2. The state of the downstream data channels is denoted by an ordered pair $(n_1, n_2)$, where $n_1$ and $n_2$ are the number of instances of Service 1 and Service 2 that are active, respectively. An example of a state diagram that is generated by two CBR services is shown in Fig. 3. In the figure, the maximum bandwidth available on the downstream digital channels is 30 Mb/s, which corresponds to a single 6 MHz channel's being active. The request rates for Service 1 and Service 2 are $\lambda_1$ and $\lambda_2$ respectively, assuming both services’ arrivals can be characterized by Poisson processes; the average time a customer uses Service 1 or Service 2 is $1/\mu_1$ and $1/\mu_2$. The state transition rates along each column and row are the same as those in a system supporting a single CBR service with bandwidth $B_1$ or $B_2$, respectively. The bandwidth limitations produce the triangular shape of the state space seen in the figure.

Using the state transitions shown in Fig. 3 and well known analytical techniques (see for example [13]), we can develop a set of balance equations that, together with the normalization condition, let us compute the state probabilities, which have the
Fig. 3. Two-dimensional state diagram that results when $B_{\text{max}} = 30$ Mb/s, $B_1 = 12$ Mb/s, and $B_2 = 9$ Mb/s.

$$p(n_1, n_2) = \rho_1^{n_1} \rho_2^{n_2} \left( \frac{1}{n_1! n_2!} \sum_{i,j} \sum_{iB_1 + jB_2 \leq B_{\text{max}}} \rho_1^i \rho_2^j \frac{i! j!}{i! j!} \right)^{-1},$$  

where $\rho_1 = \lambda_1/\mu_1$ and $\rho_2 = \lambda_2/\mu_2$ are the utilization levels for the two service types. The analysis uses the fact that a state $(n_1, n_2)$ has a non-zero steady-state occupancy probability if $n_1 B_1 + n_2 B_2 \leq B_{\text{max}}$, i.e., if the bandwidth usage associated with being in state $(n_1, n_2)$ does not exceed the maximum available downstream bandwidth.

To quantify the performance of the downstream data delivery system for a given value of $B_{\text{max}}$, we compute the blocking probabilities of the two services, which are

$$P_{B_1} = \sum_{i=0}^{M_1} p(N_1(i), i)$$  

and

$$P_{B_2} = \sum_{j=0}^{M_2} p(j, N_2(j)).$$

where $M_1 = \lfloor B_{\text{max}}/B_1 \rfloor$ and $M_2 = \lfloor B_{\text{max}}/B_2 \rfloor$ are respectively the maximum number of instances of Service 1 or Service 2 that can be supported in the absence of any instances of the other service. $N_1(i)$ is the maximum number of instances of Service 1 that can be supported given that $i$ instances of Service 2 are active, and $N_2(j)$ is the maximum number of instances of Service 2 that can be supported given that $j$ instances of Service 1 are active. Thus, $N_1(0) = M_1$ and $N_2(0) = M_2$.

Formally, we have

$$N_1(n_2) = \left\lfloor \frac{B_{\text{max}} - n_2 B_2}{B_1} \right\rfloor$$
and
\[ N_2(n_1) = \left\lfloor \frac{B_{\text{max}} - n_1 B_1}{B_2} \right\rfloor. \] (6)

In the example shown in Fig. 3, \( P_{B_i} = p(2, 0) + p(1, 1) + p(1, 2) + p(0, 3) \).

We can generalize the preceding discussion to the case where we have \( S \) CBR services whose respective bandwidths are \( B_1, B_2, \ldots, B_S \). The requests for each service are assumed to follow Poisson arrival processes. The state of the system is given by the value of the vector \( n = [n_1, n_2, \ldots, n_S] \), where \( n_k \) is the number of instances of service \( k \) that are occupying the downstream channels. The \( S \)-dimensional state diagram associated with the system is a generalization of the two-dimensional state diagram shown in Fig. 3 and leads to a set of \( S \) local balance equations for an arbitrary state \( n \). Applying the normalization condition and solving yields the following expression for \( p(n) \):
\[
p(n) = \prod_{k=1}^{S} \frac{\rho_k^{n_k}}{n_k!} \left( \sum_{\{n_k \mid \sum_{k=1}^{S} n_k B_k \leq B_{\text{max}}\}} \prod_{k=1}^{S} \frac{\rho_k^{n_k}}{n_k!} \right)\]
\[ \times \left( \sum_{\{n_k \mid \sum_{k=1}^{S} n_k B_k \leq B_{\text{max}}\}} p(n) \right) \]
\[ \times \left( \sum_{\{n_k \mid \sum_{k=1}^{S} n_k B_k \leq B_{\text{max}}\}} p(n) \right) \]
\[ - 1, \]
\[ (7) \]

where \( \rho_i = \lambda_i / \mu_i \) is the utilization level of Service \( i \), and which is a product form solution with respect to the elements of the vector \( n \).

To compute the blocking probability for the \( i^{th} \) CBR service, we define the vector \( m_i \) that contains all elements of the vector \( n \) except for \( n_i \):
\[
m_i = [n_1, n_2, \ldots, n_{i-1}, n_{i+1}, \ldots, n_S]. \]
(8)

We define the maximum number of instances of Service \( i \) that the system can support when the system is supporting the other services in the amounts given by \( m_i \) to be \( N_i(m_i) \). It is found by dividing the residual bandwidth by \( B_i \) and rounding down to the nearest integer, giving
\[
N_i(m_i) = \left\lfloor \frac{1}{B_i} \left( B_{\text{max}} - \sum_{k=1}^{S} n_k B_k \right) \right\rfloor. \]

The blocking probability for Service \( i \) is found by summing over the probabilities of states where the number of instances of Service \( i \), given \( m_i \), is the maximum:
\[
P_{B_i} = \sum_{\{m_i \mid \sum_{k=1}^{S} n_k B_k \leq B_{\text{max}}\}} \cdots \sum_{\{m_i \mid \sum_{k=1}^{S} n_k B_k \leq B_{\text{max}}\}} p(m_i, N_i(m_i)). \]
(10)

For the numerical results that follow, it is instructive to think of the blocking probability as a function of \( \rho_1, \rho_2, \) and \( c \):
\[
P_{B_i} = P_{B_i}(c, \rho_1, \rho_2). \]
(11)

where, \( c \) is the number of downstream RF channels required to provide at least \( B_{\text{max}} \) total downstream bandwidth. If we assume that each downstream RF channel can carry 30 Mb/s (e.g., using 64-QAM modulation in a 6 MHz RF channel), then
B_{max} and c are related as follows:

\[
c = \left\lceil \frac{B_{max}}{30} \right\rceil.
\] (12)

For the numerical results related to traffic engineering, we obtain the value of B_{max} first and then the value of c using Eqs. 10 and 12, respectively. This value of c would be the number of downstream channels that are required to produce a desired set of values for the service blocking probabilities \( \{P_{B_i}\}_{i=1}^S \).

In the near future, access networks will support a variety of services and, therefore, a variety of traffic types. The network will provide circuit-switched conductivity for delay- and loss-sensitive traffic like real-time voice and video while using packet-switching for best effort traffic. This model has been in existence for many years, going back to the introduction of the ISDN concept in the 1980s. In Fig. 4, which is based on Fig. 12-20 from [14], we show a schematic of a channel access system for two traffic types for the downstream channels in a HFC network. This function would be located at the head end shown in Fig. 1. As was the case in our previous analysis, there are c 30 Mb/s channels available. In this model, circuit-switched connection requests, which require 30 Mb/s and therefore occupy all the bandwidth of the channel, arrive at a rate of \( \lambda_1 \) requests/sec and follow a Poisson process. The connection duration is exponentially distributed with a mean of 1/\( \mu_1 \), so that \( \mu_1 \) is the connection completion rate. If all c channels are occupied, incoming connection requests are blocked. Packets arrive at the controller at a rate of \( \lambda_2 \); packet lengths are assumed to follow an exponential distribution with a mean value of 1/\( \mu_2 \) sec at a data rate of 30 Mb/s. If all the channels are busy, incoming packets are sent to the queue shown in the figure. For the purpose of the analysis we assume that the queue length is infinite.

In this model, variable bit rate packet traffic competes with constant bit rate circuit-switched traffic for bandwidth resources. The behavior of the system will depend on the contention resolution scheme that the channel access controller uses. The simplest case occurs when all arrivals are treated using a first-in/first-out (FIFO) queuing discipline. The first analysis of this type of system was done by Bhat and Fisher in 1976 [15]. A summary of their results appears in [14], with a detailed development for the case where the number of channels is 1. In this simplest case, the blocking probability for circuit-switched
calls is

\[ P_B = \frac{\nu_1 + \nu_2}{1 + \nu_1} \]  

(13)

where \( \nu_1 = \lambda_1/\mu_1 \) and \( \nu_2 = \lambda_2/\mu_2 \) are the utilization levels for circuit-switched and packet-switched traffic, respectively. The other metric of interest, the average amount of time that a packet spends in the system, is

\[ E\{T\} = \frac{1}{\mu_2} \left[ \frac{1}{1 - \nu_2} + \frac{\alpha \nu_1}{1 + \nu_1} \right], \]  

(14)

where \( \alpha = \mu_2/\mu_1 \) is the ratio of circuit-switched and packet-switched average message durations. The second fraction in the brackets represents the effect of the circuit-switched traffic which reduces the channel availability. If \( \alpha \ll 1 \) or \( \nu_1 \ll 1 \), then \( \mu_2E\{T\} \rightarrow 1/(1 - \nu_2) \), which is the normalized mean delay for a packet in a \( M/M/1 \) queue. The form of equations (13) and (14) indicates that, if only one channel is available, the two types of traffic can mutually interfere so that acceptable performance is possible only if both traffic types’ utilization levels are kept low.

If the number of channels is greater than 1, the blocking probability for circuit-switched calls can be approximated by

\[ P_B \approx \frac{\nu B(c - 1, v)}{c - \nu_2 + \nu B(c - 1, v)}, \]  

(15)

where \( B(\cdot, \cdot) \) is the Erlang-B formula from equation (1) and \( v = \nu_1 + \nu_2 \). If \( \nu_1 \gg \nu_2 \), equation (15) reduces to the recursion relation for the Erlang-B function, so that \( P_B \approx B(c, \nu_1) \). The mean packet delay is given in [15] and requires much more complex calculations to obtain, especially as \( c \) becomes large.

Another possible contention resolution scheme that has been explored, particularly in [16], involves the use of preemptive priority for circuit-switched traffic. In this type of scheme, circuit-switched calls are able to seize channel resources from packets if there are no channels available. The preempted packets are returned to the queue until bandwidth becomes available for them to be retransmitted. This approach eliminates the effect of the packet traffic on the circuit-switched traffic, since packets are automatically cleared from the channels that they are using if new circuit connections need the resources. Thus, the blocking probability for a single CBR service is given by the Erlang-B blocking probability in equation (1). If multiple CBR services are contending for channel resources, the blocking probability for the \( i \)th service is given by equation (10). As was the case with the FIFO discipline, obtaining the average packet delay is more difficult. For the single channel case, [15] gives the following expression for the average packet delay:

\[ E\{T\} = \frac{1}{\mu_2} \left[ \frac{(1 + \nu_1)^2 + \alpha \nu_1}{[1 - \nu_2(1 + \nu_1)][1 + \nu_1]} \right], \]  

(16)

which reduces to the mean delay of a \( M/M/1 \) queue when \( \nu_1 = 0 \).

In Fig. 5, we plot the value of \( \mu_2E\{T\} \) from (16) for several values of \( \alpha \). In each subplot, values of \( \mu_2E\{T\} \) are shown on the corresponding contour lines. For a given value of \( \nu_1 \), \( \mu_2E\{T\} \) becomes unbounded as \( \nu_2 \rightarrow 1/(1 + \nu_1) \). This limit is indicated in each of the four subplots in Fig. 5 by a dashed line. As the average circuit-switched connection duration increases beyond the average packet duration, the waiting time in the queue increases significantly. Even if the circuit-switched traffic
operates at low utilization levels, the mean packet delay can be many tens or hundreds of seconds if the time required to transmit a single packet is on the order of a fraction of a percent of the time required to support a circuit-switched connection.

For the case where circuit-switched connections occupy only a portion of the bandwidth of a single channel whose bandwidth is $B$ bits/sec, the situation becomes more complex. We examine the situation where there is one type of CBR service whose bandwidth is $B_1$ bits/sec. As the number of instances of the CBR service that are using the channel increases, the bandwidth available for packet-switched traffic falls. Let $b$ be the average size of the packet in bits, so that the average packet transmission time is $b/B$ seconds/packet. If there is only one type of CBR service using the channel, the packet transmission rate is $(B - kB_1)/b$ packets/sec when $k$ instances of the CBR service are using the channel, for $0 \leq k \leq \lfloor B/B_1 \rfloor$. We show an example of the state diagram that results in Fig. 6 for the case where $\lfloor B/B_1 \rfloor = 2$.
The balance equations for this system at the boundary states are

\[(\lambda_1 + \lambda_2)p_{00} = \frac{B}{b} p_{01} + \mu_1 p_{10}\] (17)

\[(\lambda_1 + \lambda_2 + \mu_1) p_{10} = \lambda_1 p_{00} + \frac{B - B_1}{b} p_{11} + 2\mu_1 p_{20}\] (18)

\[(\lambda_2 + 2\mu_1) p_{20} = \mu_1 p_{10} + \frac{B - 2B_1}{b} p_{21}\] (19)

The remaining equations are

\[(\lambda_1 + \lambda_2 + \frac{B}{b}) p_{0j} = \mu_1 p_{1j} + \frac{B}{b} p_{0,j+1} + \lambda_2 p_{0,j-1}\] (20)

\[(\lambda_1 + \lambda_2 + \frac{B - B_1}{b} + \mu_1) p_{1j} = 2\mu_1 p_{2j} + \lambda_1 p_{0j} + \frac{B - B_1}{b} p_{1,j+1} + \lambda_2 p_{1,j-1}\] (21)

\[(\lambda_2 + 2\mu_1 + \frac{B - 2B_1}{b}) p_{2j} = \lambda_1 p_{1j} + \lambda_2 p_{2,j-1} + \frac{B - 2B_1}{b} p_{2,j+1}\] (22)

for \(j = 1, 2, \ldots\). These can be solved using moment generating functions of the form \(G_i(z) = \sum_{j=0}^{\infty} p_{ij} z^j\) to get the mean packet delay. We note that because CBR traffic is allowed to preempt the packet traffic, the blocking probability for circuit-switched services is

\[P_B = \frac{v_1^2}{2 + 2v_1 + v_1^2}\] (23)

### III. NUMERICAL RESULTS

In this section we use the theoretical models we developed in Section II to develop a hypothetical deployment schedule for a cable operator. We consider a single HFC fiber node serving a community of 100 households. We assume low initial penetration (10% usage of digital HFC services) in the year 2004, and we assume that the usage level versus time follows a
sigmoid curve given by the expression
\[
\Pi = \left(1 + e^{-4s(t-t_0)}\right)^{-1},
\]
where \(t_0\) is the time where \(\Pi = 0.5\) and \(s\) is the slope of the curve (i.e. the penetration growth rate) at time \(t = t_0\). In Fig.7 we plot curves for three values of \(s\), where \(\Pi(2004) = 0.1\). Note that increasing \(s\) from 0.05 to 0.1 produces a greater leftward shift and contraction of the curve than does an increase in \(s\) from 0.1 to 0.15.

![Adoption curves for data over cable modem using growth rates of 5%, 10%, 15% when \(\Pi = 0.5\), assuming 10% penetration in 2004.](image)

For the two services that we are considering, we want the blocking probability for each not to exceed \(P_m = 0.001\). In order to determine the number of channels that are required to satisfy this design goal, we need to compute \(P_{B_1}\) and \(P_{B_2}\) as functions of the service utilization levels \(\rho_1\) and \(\rho_2\) for each value of \(c\) (see Eqs. 10, 11 and 12). To compute the utilization levels, we assumed that the average time that a customer spends using the VOD service (Service 1) and the high-speed download service (Service 2) are \(1/\mu_1 = 2\) hours and \(1/\mu_2 = 15\) minutes, respectively. The maximum value for the arrival rates of requests for the two services was obtained by assuming that each customer requests a service at the same rate that that service completes. In other words, at peak usage, a customer requests the VOD service every two hours and the download service every 15 minutes. Thus, if \(H\) households are using the network’s digital channels, the maximum utilization of each service is \(\rho_1 = \rho_2 = H\). For each ordered pair \((\rho_1, \rho_2)\) such that \(0 \leq \rho_1 \leq H\) and \(0 \leq \rho_2 \leq H\), we find the smallest value of \(c\) that satisfies the design criteria given by the inequalities \(P_{B_1}(c, \rho_1, \rho_2) \leq P_m\) and \(P_{B_2}(c, \rho_1, \rho_2) \leq P_m\).

In Fig.8, we show a contour plot of the surface \(c_{\text{min}}(\rho_1, \rho_2)\) that is generated when we consider two services with downstream bandwidths \(B_1 = 2\) Mb/s and \(B_2 = 10\) Mb/s. Because channels can be assigned in groups of 5, the surface in Fig.8 has the staircase shape indicated by the contour lines. The value taken by \(c_{\text{min}}(\rho_1, \rho_2)\) at a given point in the \((\rho_1, \rho_2)\) plane is found
by examining the value of the closest contour line to the left of the point. The numbers superimposed on each of the contour lines in the figure give the value of \( c_{\text{min}}(\rho_1, \rho_2) \) in the region immediately to the right of the contour line; the surface in the region that includes the origin has a value of 5. Thus, for example, \( c_{\text{min}}(80, 60) = 40 \).

![Graph](image)

**Fig. 8.** Number of channels required to achieve blocking probabilities for Service 1 and Service 2 less than \( P_m = 0.001 \) when \( B_1 = 2 \) Mb/s and \( B_2 = 10 \) Mb/s, plotted vs. service utilization levels \( \rho_1 \) and \( \rho_2 \).

The jagged appearance of the contour lines occurs because we used a \( 100 \times 100 \) array of ordered pairs to determine the behavior of \( c_{\text{min}}(\rho_1, \rho_2) \). Using more points in the grid will produce smoother contour lines. By examining the figure, we see that each of the contour lines is approximately linear with a slope of approximately -5. Interestingly, this is the negative of the ratio \( B_2/B_1 \). In order to determine whether the slope of the contour lines is determined by the ratio \( B_2/B_1 \) in general, we examined other values of \( B_1 \) and \( B_2 \) and obtained plots of \( c_{\text{min}}(\rho_1, \rho_2) \). For the case where \( B_1 = 5 \) Mb/s and \( B_2 = 10 \) Mb/s, we obtained the results are shown in Fig. 9. By examining this figure, we see that each of the contour lines is again approximately linear, taking the sampling grid in the \((\rho_1, \rho_2)\) plane into consideration. In this case, the slope is close to -2, which is again the negative of the ratio of the two CBR service bandwidths. This observation suggests that for the general case where there are \( S \) CBR services, the channel count thresholds in the \( S \)-dimensional space associated with the utilization vector \( \boldsymbol{\rho} = [\rho_1, \ldots, \rho_S] \) forms a hyperplane whose normal vector is determined by the ratios of the service bandwidths. In this case, there is a region of the \((\rho_1, \rho_2)\) plane where the two service blocking probabilities \( P_{B_1} \) and \( P_{B_2} \) are both greater than the maximum acceptable blocking probability \( P_m \). This region lies to the right of the line marked, “> 50.” Supporting a network load in this region
would require adding additional downstream digital channels.

![Graph showing the number of channels required to achieve blocking probabilities for Service 1 and Service 2 less than $P_{\text{fr}} = 0.001$ when $B_1 = 5$ Mb/s and $B_2 = 10$ Mb/s, plotted vs. service utilization levels $\rho_1$ and $\rho_2$.]

Once we know how many downstream digital channels are required to achieve a given level of service accessibility, we can use the projected growth curves in Fig. 7 to determine a set of deployment schedules for the downstream line cards. These will allow the network operator to provide an acceptable level of service to his customers while avoiding the excessive costs associated with rolling out unnecessary equipment. We expect that, in practice, the downstream line cards cannot be procured with the granularity of a single downstream RF channel. Therefore, in illustration of our results, we assume that line cards come with a granularity of 5 downstream RF channels per unit. So each time a line card is deployed by the service provider, an addition of $5 \times 30$ Mb/s = 150 Mb/s of downstream bandwidth occurs.

For each year from 2005 to 2025, inclusive, we computed the number of households in the set of 100, attached to the fiber node, that are using digital services. This number, $H$, corresponds to the maximum utilization for each of the two CBR services. Plotting this point on the graph in Fig. 8 yields the minimum number of channels needed to support peak load while maintaining a service blocking probability of at most 0.001. Plotting the required number of channels vs. time for each of the three growth rates that we considered (in Fig. 7) produces the graph shown in Fig. 10. The number of downstream channels grows in steps of five because of the above-stated assumption regarding deployment granularity (i.e., five RF channels per line card). This graph is similar to the adoption growth curves in that a peak growth rate of 5% produces a rollout schedule that
is considerably less aggressive than the ones that result from a growth rate of 10% or 15%.

![Graph showing number of 30 Mbps channels required for different years](image)

Fig. 10. Number of 30 Mb/s channels required to achieve a maximum blocking probability of 0.001 for Service 1 and Service 2 plotted versus time using the adoption curves in Fig. 7.

IV. SUMMARY

In this paper, we examined some of the issues associated with deploying resources in HFC networks to support emerging downstream data services. We developed a theoretical model that allows us to obtain the blocking probability of an arbitrary CBR-like service out of a set of such services that are simultaneously supported over the HFC. We primarily focused on VOD and large file download applications that are essentially CBR-like services requiring sustained bandwidth in the downstream channels. For these types of applications, we presented suitable connection blocking and traffic forecasting models. Using these models, we obtained a set of downstream bandwidth deployment and line-card rollout schedules for a variety of usage growth scenarios. These results can be particularly useful for HFC-based ISPs to minimize cost and connection blocking probability while planning and deploying the HFC resources to accommodate higher take rates and traffic growth over time. We extended some of these results to the case where there is a mixture of CBR and VBR (statistically multiplexed) services. If the CBR services have preemptive priority over the VBR packet traffic, then the results of our analysis that considers only CBR services can be applied.

REFERENCES