LATEST RESULTS FROM THE PROTON GYROMAGNETIC RATIO IN WATER AND RELATED EXPERIMENTS

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ABSTRACT

The results of the latest measurement made at the National Institute of Standards and Technology (NIST) of the proton gyromagnetic ratio, $\gamma_p^\text{low}$, are presented, and the resultant value of the quantized Hall resistance, $R_H$, and the fine structure constant, $\alpha$, are compared. A discussion of possible sources for the $(-0.102 \pm 0.043)$ ppm discrepancy between the absolute ohm and this measurement is included along with a new method to measure $\hbar/e^2$ by counting electrons in a storage ring.

INTRODUCTION

The low magnetic field method of measuring the proton gyromagnetic ratio, $\gamma_p^\text{low}$, involves two experiments. (The prime indicates that a spherical sample of pure H$_2$O at a temperature of 25 °C is used, and "low" implies that a procedure like that described here is employed and this generally results in low fields.) First, we measure the dimensions of a precision single-layer solenoid by an inductive technique in which the position of the current in the wire is located. In the second part, we measure the proton precession frequency, $\omega_p$, by standard NMR techniques. $\gamma_p^\text{low}$ is then obtained from: $\gamma_p^\text{low} = \omega_p/\xi$, where $\xi$ is the coil constant calculated from the measured dimensions which is equal to the magnetic flux density for unit current, and $I$ is the current in the solenoid.

We are continuing to measure $\gamma_p^\text{low}$, testing for systematic errors. Our present value of $\gamma_p$ can be expressed in terms of the volt and ohm based on the following adopted values of the Josephson frequency-to-voltage quotient and the quantized Hall resistance:

$$[2\pi/e]_{\text{Lab}} = 483597.9 \text{ GHz/V, and } [R_H]_{\text{Lab}} = 25812.807 \Omega.$$ 

These values, which we call laboratory (Lab) values, have been adopted by the Comité International des Poids et Mesures, CIPM. The result is

$$\gamma_p^\text{low} = 2.67515427 \times 10^8 \text{ s}^{-1} \Omega^{-1} \text{ (±0.11 ppm).}$$

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contribution to our knowledge of the values of the fundamental constants, particularly the fine structure constant, $\alpha$, since\(^3,2\)

$$\alpha^{-1} = \left[ \frac{(\mu_p/\mu_B)[R_{H}\text{Lab}][2e/h]\text{Lab}]}{2\mu_0 R_{\infty}[\gamma_p]\text{Lab}} \right]^{1/3} , \quad (1)$$

and also the quantized Hall resistance, $R_H$, since

$$R_H = \frac{\mu_0 c \alpha^{-1}}{2} \left[ \frac{\mu_0^2 c^3 (\mu_p/\mu_B)[R_{H}\text{Lab}][2e/h]\text{Lab}}{16 R_{\infty}[\gamma_p]\text{Lab}} \right]^{1/3} . \quad (2)$$

In these equations $\mu_0$ is the permeability of free space, $c$ is the speed of light in vacuum, $\mu_p/\mu_B$ is the magnetic moment of the proton in units of the Bohr magneton, and $R_{\infty}$ is the Rydberg constant for infinite mass. These quantities are known to one part in $10^8$ or better. Note that the three electrical constants, $R_H$, $2e/h$, and $\gamma_p$, must be measured in the same laboratory (Lab) units, and that there is a cube root dependence on the measured quantities. From Eq. (1): $\alpha^{-1} = 137.0359840(51) \pm 0.037$ ppm, and from Eq. (2): $R_H = 25812.8046(95) \pm 0.037$ ppm.

This value of $\alpha^{-1}$ agrees fairly well with the QED value, the difference being $(-0.054 \pm 0.038)$ ppm, but differs by somewhat more than two combined standard deviations from the NIST absolute ohm realization, the difference being $(-0.102 \pm 0.043)$ ppm. The agreement with the precise QED value is satisfying, but the difference between our value and the NIST ohm value, which also has a relatively small uncertainty, is disconcerting. We plan to continue our measurements to test further for any errors.

**Difference Data from Mar '89 to Feb '88**

![Graph showing pitch and radius difference data](image)

**Figure 1.** The difference between the pitch and radius data measured from March 9 to 23, 1989 and similar pitch and radius data measured during February 22-24, 1988. The error bars are the standard deviation of eight measured values for each solenoid turn number collected during March 9-23, 1989.
LATEST EXPERIMENTAL RESULTS

Since reporting the above value of \( \gamma^*_p \) (low), we have remeasured the critical dimensions of the solenoid and find only a small difference from our previous measurements. Figure 1 shows the measured difference between the recent data taken on March 9-23, 1989 and one set of earlier data taken on Feb. 22-24, 1988.

Using these results the difference in the calculated field is \((0.078 \pm 0.15) \text{ ppm}\). The statistical scatter in this difference is smaller \((\pm 0.05 \text{ ppm})\), but the actual uncertainty is thought to be \(\pm 0.15 \text{ ppm}\) because some important corrections are not included in our latest measurement. From this latest result we conclude that there is no evidence that our value of \( \gamma^*_p \) will change, but further measurements are required.

PROPOSED MEASUREMENT OF \(\hbar/e^2\)

A recently proposed experiment will also be discussed briefly in this paper. The concept is based on the premise that storage rings offer the possibility of realizing a current by counting the electrons per second passing through a toroidal SQUID current comparator. In effect, this technique has the potential for accurately measuring the quantity \(\hbar/e^2\) and providing a stringent test of the quantum Hall effect.

If one measures \( K \), the number of electrons per second that constitute a current \( J \), then \( J = eK \), where \( e \) is the electron charge. In a storage ring, \( K \) will equal \( NF_c \), where \( N \) is the number of particles stored in the ring and \( f_c \) is the frequency of revolution of these electrons. If a

![Figure 2. Electrons circulate in a ring and pass through a SQUID current comparator. To keep the high field regions, \( H_c \), from affecting the SQUID, both mumetal and superconducting shields are required. The superconducting shield can extend into the SQUID to help attenuate the fields leaking through the holes. (The SQUID design is best seen in Fig. 3.) A LINAC is used to inject the electrons or ions.](image-url)
current equal to this ring current is passed through a resistor, R, producing a voltage drop, V, which is compared to the voltage on a Josephson junction, then \( R = \frac{2 \hbar}{\pi e^2} \) where \( f_j \) is the Josephson frequency, \( n \) is the Josephson step number, and \( h \) is the Planck constant. This equation is very similar to the quantum Hall relation, and this is the experiment would help test that effect by providing an independent measure of \( h/e^2 \).

Figure 2 is a schematic drawing of an electron storage ring that has \( N \) electrons orbiting at a frequency of revolution, \( f_r \), determined by the geometry but phase-locked to the rf drive frequency applied to keep the electrons from slowing as they give off synchrotron radiation when bending in the magnetic field. The key to the experiment is a toroidal SQUID current comparator that measures the electron current precisely.

Figure 3. Current comparator cross section. The key to the current comparator is the toroid. An electron beam current, \( J_b \), and a servo current, \( J_s \), pass through the toroid and a supercurrent \( J_b J_s \) is induced in the toroid. Because of the gap in the toroid, this current must go around the inside, some of it passing through the SQUID. When the impedances are matched, half the flux is in the SQUID and half inside the toroid. The signal increases linearly with \( z \) and is not very sensitive to \( h/d \), except when \( d \) is very small. The magnetic shields are not shown.

This SQUID current comparator measures the ratio of the beam current to a current known in terms of the laboratory units for voltage and resistance. For this purpose, a servo current is also passed through the current comparator to maintain a null at the SQUID. Figure 3 shows some details of the proposed comparator including the construction of a superconducting toroid. The key to the concept is that the toroid is not closed, so that the supercurrent, \( J_b J_s \), caused by any current difference between the beam, \( J_b \), and the servo current, \( J_s \), will have to flow on the inner surface of the toroidal structure unless it passes through the dc SQUID detector connected across the gap. The current in the SQUID, \( J_{in} \).
depends on the ratio of the SQUID inductance, $L_{16}$, to the toroid inductance, $L_T$:

$$J_{16} = \frac{L_T}{(L_T + L_{16})} (J_x J_y).$$  \hspace{1cm} (3)

The toroid serves the very important role of making the SQUID response the same for the beam current, $J_x$, and the servo current, $J_y$. It is possible to use more than one such toroid, if necessary, to achieve the desired performance. Also, the overlap of this toroid should be very long and tight, so that the only flux that gets inside the toroid is from the super current, $J_x J_y$. In effect we have an ideal current transformer, but we can have only one turn, or one pass, through the toroid for the beam current.

Using a small storage ring such as NJII II at the Electrotechnical Laboratory in Japan we might expect to measure this current with 1 to 10 ppm accuracy in a preliminary experiment. By making a smaller all cryogenic storage ring one might obtain accuracies better than 0.1 ppm, thus impacting on our knowledge of the fundamental constants and related physical phenomena.

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REFERENCES