Electronic limitations in phase meters for heterodyne interferometry

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Limitations imposed by the phase meters used in heterodyne interferometers are evaluated. These instruments measure the phase relationship between electrical signals generated by the heterodyning process, allowing the interferometers to resolve fractions of an optical fringe. Measurements indicate that the phase meters used in currently available heterodyne interferometers probably limit achievable accuracy to a greater extent than barriers imposed by the optics. We show that a new class of time interval counters offers a means of greatly improving accuracy in these instruments.

Keywords: Phase meters; heterodyne interferometry

Introduction

Commercial heterodyne interferometers claim displacement measurement accuracies of approximately 10 nm, and because the interferometers are based on well-known optical wavelengths, such accuracies have been accepted with little questioning. However, more demanding requirements, which push displacement measurement uncertainties down to 1 nm and below, have led to the investigation of remaining sources of error. Reasonable attention has been given to the fidelity of the heterodyning process, which converts the optical path difference between beams that have traversed the test and reference legs to a phase difference between electrical signals from the test and reference photodetectors. This article deals with the errors introduced in the next step: the measurement of the electrical phase angle.

Heterodyne interferometers

A heterodyne interferometer, shown schematically in Figure 1, uses a dual-frequency laser to produce two electrical signals: a reference signal, $V_R$, which is at the laser difference frequency $\Delta f = f_1 - f_2$, and a measurement signal, $V_M$, which is at the difference frequency but Doppler-shifted by the motion of the test reflector $(\Delta f \approx \Delta f(t))$. This Doppler shift can equivalently be thought of as a time-varying phase shift, $\phi(t) = 2\pi \int \Delta f(t) \, dt$. If $v(t)$ is the test mirror or reflector speed, then

$$\Delta f(t) = \frac{2\pi v(t)}{\lambda}$$

where $\lambda$ is the index of refraction of the medium along the light path and $\lambda$ is the wavelength of the light.

For commercial heterodyne interferometers, the difference frequency is typically between 250 kHz and 20 MHz. At rest, the reference and measurement signals have the same frequency and their phase relationship is proportional to the position of the test reflector. The ability to measure this phase angle determines how accurately one knows the change in position of the test reflector. In a simple Michelson interferometer, the phase is related to displacement by

$$\phi(t) = 4\pi n L(t)/\lambda,$$

where $L(t)$ is the displacement of the test reflector relative to the reference. From the above relationship, 1° of phase represents a test reflector displacement of $\lambda/720$, or about 0.9 nm for He-Ne laser wavelengths.

Angular resolution

Most phase meters measure phase angle by measuring time ratios. In interferometry, absolute accuracy is not necessary, but stability, linearity, and resolution are. The angular resolution obtainable depends on the signal frequency and the timing resolution:
Time interval analyzers (TIAs) are the instruments that tend to be able to travel this capability out to 1 MHz at the time we were beginning this study, at the time we were beginning this study, a new class of counter/timer known generically as time interval analyzers (TIAs). These instruments can be configured as fast, high-frequency phase meters.

**Phase meters**

Time interval analyzers measure phase by measuring the ratio of the time between zero crossings of the two signals to the period of one of the signals. As shown in *Figure 3*, the phase relationship between $V_R$ and $V_M$ (both the same frequency), is given by

$$\Phi = \left(\frac{T_I}{T_R}\right)360^\circ$$

where $T_I$ is the time interval between the positive going zero-crossings of $V_R$ and $V_M$, and $T_R$ is the period of $V_R$.

Unlike conventional digital timers, TIAs can resolve times in the picosecond range by counting periods of a reference clock and then interpolating on the ends. A common time interpolation scheme uses an analog ramp generator to charge a circuit at a known rate, $I$, during the interpolation period. The height of the ramp, $V$, at the end of the period is proportional to the interpolation interval, $T_I \propto V/I$. Several methods are then used to determine the ramp height, $V$. One way is to measure the time it takes to discharge the circuit back to its starting point using a lower discharge rate, $I/k$. This technique converts a short time interval to a longer one that can be measured using the digital timer. However, to enhance time interval resolution by a factor

$$\Delta \alpha = 360^\circ f \Delta t$$

where $\Delta \alpha$ is the angular resolution in degrees, $f$ is the signal frequency in hertz, and $\Delta t$ is the timing resolution in seconds.

This relationship is displayed graphically in *Figure 2*, where it can be seen that to resolve 0.1° at 200 kHz requires a time resolution of about 1 nsec. At 2 MHz the same 0.1° resolution requires a time resolution of 100 psec. For a 20 MHz signal, 10 psec resolution is required. In the application that led to this study,10 the objective was to resolve 0.1 nm using a laser with a difference frequency of about 500 kHz and using an optical configuration that gives a fringe spacing of $\lambda/8$. This translates to a phase measurement resolution of 0.4° or a time resolution of 2 nsec. The chosen heterodyne frequency limits the update rate to 2 μsec, which limits the motional slew rate to about 10 mm/s. (The slew rate is calculated by assuming a conservative maximum of one fourth of a fringe per measurement update cycle.) The phase meter must meet or exceed this update rate to maintain the slew rate.

Before this investigation, standards for phase angle (the solid curve crossing *Figure 2*) were only available up to 50 kHz,11 although the measurement uncertainties were ±0.01° at that frequency. Extending this capability out to 1 MHz at 0.1° required an improvement in timing resolution by a factor of two. More challenging was the need to make new phase measurements every microsecond (existing standards required seconds of averaging). Fortunately, at the time we were beginning this study, several manufacturers had recently introduced a new class of counter/timer known generically as time interval analyzers (TIAs). These instruments
4). For static heterodyne frequency. It works by generating a triangle wave from an interferometer with a 20-MHz heterodyne frequency. Its phase interpolation is done by generating a triangle wave from the reference signal. The triangle wave voltage level at the time of the measurement signal zero crossing is digitized using a fast analog-to-digital converter, similar to one of the methods used in TIAs. The principal limitation of this method of phase interpolation is the perfection of the triangle wave, particularly at the peak of the triangle. Small glitches affect the measured voltage level and hence the inferred phase. The errors that were measured for this phase meter (see below) are believed to originate from this source.

For comparison, we also tested a typical digital phase meter (B). This meter works by generating pulses whose widths are proportional to the phase angle difference between the two input waveforms. The pulses are averaged and scaled, and the resultant DC waveform is converted into a 16-bit digital signal. The obvious disadvantage of this method from the interferometry standpoint is the need to average over several hundred periods of the input signals.

Tests

The performance of these phase meters was measured using a dual-channel, phase-adjustable source as a phase standard (Figure 4). For static mode tests, where both signals are at the same frequency, the measured phase nonlinearity of the source ranges from 0.03° at 250 kHz up to 0.2° at 20 MHz. The static mode would roughly correspond to interferometric measurements made with the test reflector at rest. For dynamic tests, the same source was used; however, the two signals were programmed to different frequencies to produce a linear rate-of-change in phase. This corresponds to

![Diagram Illustrating How a Time Interval Analyzer Can Be Used as a Phase Meter](image.png)

**Figure 3** Diagram illustrating how a time interval analyzer can be used as a phase meter. \( \Phi \) is the measured phase and \( V_R \) and \( V_M \) are the reference and measurement signals, respectively. A typical interpolation scheme is also illustrated.

of 1,000 using this means requires an accurately known \( k \) and 1,000 clock periods. For a 100-MHz clock, for example, the interpolation would take 10 \( \mu \)sec. An alternative method of measuring \( V \) that overcomes these long interpolation times is to use a fast analog-to-digital converter. With these converters, \( V \) can be resolved to a part in a 1,000 in about 100 nsec with resultant time resolutions of 10 to 100 psec. This, for example, is the principle of operation for phase meter C used in this study.

On the other hand, one of the other TIAs examined (A) uses a completely different approach. With its 500-MHz clock it can resolve 2 nsec digitally. To measure the fractional time difference between the event and the next clock edge, it launches a pulse corresponding to that event down a series of 10 delay lines. Latches located at the junction of each delay line are triggered by the next clock. By interrogating these latches, it can be determined how far down the delay line the event traveled before the next clock pulse arrived. This technique provides a single shot time resolution of 200 psec.

The phase meters used in commercial interferometers operate in a similar manner, by measuring the time between the zero-crossings of the two signals. However, the interpolation techniques vary significantly from one manufacturer to the next.

Phase meter D is from an interferometer that has a 250 kHz heterodyne frequency. It interpolates fringes by a factor of 1,000 by timing the average of 10 cycles against an internal 25-MHz clock. This yields an update rate of only 25 kHz. Fringe tracking, however, is done at the full 250-kHz heterodyne frequency, which limits the test reflector slew rate in a simple Michelson interferometer configuration to about 20 mm/s.

Phase meter E is from an interferometer with a ~2 MHz heterodyne frequency. It works by generating an internal phase-locked clock at 32 times the reference frequency. Zero crossing times of the measurement signal are then counted against this internal clock. Although a simple and robust method of fringe interpolation, in this incarnation it suffers from a least quantization count of ~11°. Clearly, the phase resolution could be increased by using a faster clock.

Phase meter F is from an interferometer with a 20-MHz heterodyne frequency. Its phase interpolation is done by generating a triangle wave from the reference signal. The triangle wave voltage level at the time of the measurement signal zero crossing is digitized using a fast analog-to-digital converter, similar to one of the methods used in TIAs. The principal limitation of this method of phase interpolation is the perfection of the triangle wave, particularly at the peak of the triangle. Small glitches affect the measured voltage level and hence the inferred phase. The errors that were measured for this phase meter (see below) are believed to originate from this source.

For comparison, we also tested a typical digital phase meter (B). This meter works by generating pulses whose widths are proportional to the phase angle difference between the two input waveforms. The pulses are averaged and scaled, and the resultant DC waveform is converted into a 16-bit digital signal. The obvious disadvantage of this method from the interferometry standpoint is the need to average over several hundred periods of the input signals.
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DUAL CHANNEL SOURCE USED TO TEST PHASE METERS

Figure 4  Block diagram of the dual channel source used to test phase meters. Sinusoidal signals are synthesized using two phase-adjustable function generators connected to a common frequency reference. Phase resolution is ±0.1°, whereas uncertainties as low as ±0.03° may be achieved by characterizing the source.

the more typical case of phase measurements made “on the fly” while the test reflector is moving. In this mode, new phase measurements should ideally be made at every zero crossing of the measurement signal. This phase standard and the techniques for characterizing it are described elsewhere in greater detail.12,13

Static results

Results of measurements performed at 250 kHz under static conditions for four different meters are plotted in Figure 5. Each point on each of the plots represents an average of 1,000 readings; however, the amplitude of the errors was about the same even for a single reading, indicating that the errors are not primarily due to random noise. Results shown in this plot are from a digital phase meter (B), two TIAs (A and C), and a phase meter used with a commercial interferometer (D). Phase meter D has a resolution of a part in 1,000 and is linear to ±1 count (±0.4°). The other three meters have higher resolution and show nonlinearities on the order of ±0.1°. The scale on the right shows the equivalent reflector displacement error (in nm) for a simple Michelson interferometer. The results indicate that, ignoring optical sources of error, the interferometer’s phase meter will limit its resolution and linearity to about ±0.4 nm. However, the electronic limitation could be improved by a factor of 4, to 0.1 nm, by using one of the other meters.

At 2 MHz, static measurements were performed on two TIAs (A and C) and a phase meter from a different commercial interferometer (E). The results of these tests are plotted in Figure 5A. Here the interferometer phase meter (E) has an angular resolution of only about 1.1° but is still linear to ±1 count (about 10 nm), which is its specification. Note that the phase error is well behaved in that it builds up linearly between digitization steps. The TIA errors are so small that they cannot be resolved on this plot. Expanding the scale in Figure 6A shows that the TIA nonlinearities are ±0.2° (which corresponds to about 0.2 nm, 50 times better).

At 20 MHz, two TIAs (A and C) and the phase meter from a third commercial interferometer (F) were tested. The results are plotted in Figure 6. Here the interferometer phase meter (F) has a resolution of 2.8°; however, a peak nonlinearity in excess of 10° was observed. (There were no convenient input connectors on this instrument, so signals were applied to test points on the circuit board. It is not clear what influence this may have had on the measurements.) Once again, the TIAs had better resolution and were more linear (±1°), indicating that it should be possible to improve the accuracy of this system by as much as a factor of eight by substituting a TIA for the built-in phase meter.

The tests described up to this point were performed using relatively pure synthesized sine waves simulating the signals that would be produced in an interferometer. To test the performance with optically generated signals, the setup illustrated in Figure 8 was used. Here a 360° phase angle change between the reference and test signals was generated by moving the retroreflector along a track.
Figure 6  (A) A comparison of the errors in phase measurement of several phase meters at 2 MHz using a static phase difference as in Figure 5. Meter E is the interferometer phase meter. (B) An expanded y-axis plot of (A) showing the much smaller nonlinearities in meters A and C.

Figure 7  A comparison of the errors in phase measurements of several phase meters at 20 MHz using a static phase difference. Meter F is the interferometer phase meter.

Figure 8  A block diagram of the setup used to optically generate precisely controllable phase differences. As the retroreflector is moved through half a wavelength (7.5 m for a 20 MHz heterodyne frequency), the phase of the test signal shifts 360° relative to the reference signal.

Dynamic results
The interferometer electronics are not only able to make static measurements, but also dynamic...
Figure 9 Comparison of phase measurement errors of a time interval analyzer (A) and a commercial interferometer phase meter (F). The phase shift in the abscissa is the optical phase shift generated as shown in Figure 8. Also shown for comparison are the errors measured for meter F relative to the electronically generated phase standard (offset by $-10^\circ$).

Figure 10 Comparison of the errors in dynamic phase measurement of two high-speed phase meters. The reference signal was 2 MHz and the test signal was 2.2 MHz so that the relative phase shifted through $360^\circ$ in 5 $\mu$sec. No averaging was used on these data.
measurements could be compared. By then rotating one of the detector polarizers by 90°, or by introducing a delay line, the relative phase offset could be changed by 180° or by any arbitrary amount, respectively, and the differences in phase measurements could again be compared.

Conclusions
One of the major sources of uncertainty in commercial heterodyne interferometers appears to be the phase measurement. At higher heterodyne frequencies it may be the primary source of uncertainty. Data from these instruments are quantized between 0.4° (in the low frequency instruments) to as much as 11°.

On the other hand, limitations imposed by the new time interval analyzers (under ideal signal conditions) are on the order of ±0.1° below 2 MHz. That corresponds to λ/7,000 or about 0.1 nm for a simple Michelson interferometer.

Based on these measurements, the new time interval analyzers should be capable of enhancing the phase-measuring performance of these interferometers. Improvement by a factor of four to 50 may be possible if optical limitations do not begin to dominate the improved resolution. The major obstacle to taking advantage of this increased performance (aside from the cost of the TIA) is the difficult, but not insurmountable, problem of accessing the TIA data in a real-time mode.

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References
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