OPTICAL DETECTOR NONLINEARITY: A COMPARISON OF FIVE METHODS

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Abstract

We derived a set of unified equations for five methods to evaluate nonlinearity of power meters and detectors. We performed computer simulations of these methods. The simulations assist in design of a measurement system to meet a target accuracy. Measurements verified the simulations.

Definitions and Basic Expressions

Detector nonlinearity is defined as the relative difference between the measured power $P_n$ and the actual power $P$ due to the nonlinear response. This difference is called nonlinearity error. It is a function of $P$ and depends on the calibration power $P_c$.

\[ \Delta_{NL}(P; P_c) = \frac{P_n - P}{P}. \]

The nonlinearity error can also be written in terms of output $V$, where $V$ can be electric current, voltage, or the reading of a power meter, by setting $P = g(V)$, where the conversion function $g(V)$ is the inverse function of the response function:

\[ \Delta_{NL}(V; V_c) = \frac{g(V) V_c - 1}{g'(0) V}. \]

It is often desirable to characterize the nonlinearity of an uncalibrated detector. In this case, the nonlinearity error is

\[ \Delta_{NL}(V; 0) = \frac{-g(V)}{g'(0) V} - 1. \]

Nonlinearity error calibrated at $V_c$ can be obtained by the relation

\[ \Delta_{NL}(V; V_c) = \Delta_{NL}(V; 0) - \Delta_{NL}(V_c; 0). \]

A polynomial is usually used to represent the conversion curve:

The nonlinearity is then

\[ g(V) = V + \sum_{k=2}^{n} b_k V^k. \]

\[ \Delta_{NL}(V; 0) = \sum_{k=2}^{n} b_k V^{k-1}. \]

Measurement Methods

We consider five methods. Three of them are based on superposition. The other two are the attenuation method and the differential method. A schematic diagram of the setup for all these methods is shown in Figure 1.

Integral-step superposition [1]. A set of power levels $j$ and the corresponding outputs $V_p$, where $j$ is an integer, is obtained using the superposition of the two incident beams. The conversion curve is then obtained by linear least-squares curve fitting.

Modified superposition [2]. $P$ is considered as an arbitrary unknown in the conversion polynomial for the individual powers from the two beams. Using superposition of all possible combinations of the two beams yields

\[ P_1 + P_2 = V_{12} + \sum_{k=2}^{n} b_k V_{12}^k, \]

where the subscripts 1, 2 refer to beams 1 and 2. There are more equations than unknowns when enough measurements are made. The conversion curve can then be obtained by curve fitting.

Triplet superposition [3]. For a group of triplet measurements, that is, two individual beams and their combination by superposition, we get the following equation by canceling the two unknown powers:

\[ (V_{12} - V_1 - V_2) + \sum_{k=2}^{n} b_k (V_{12}^k - V_1^k - V_2^k) = 0. \]

The conversion curve can be obtained from this equation.
**Attenuation.** The outputs \( V \) and \( V_r \), with and without a filter in the optical path, are measured. \( \tau \) is the transmittance of the filter. The conversion curve is obtained from

\[
(V_r - \tau V) + \sum_{k=2}^{n} b_k (V_r^k - \tau V^k) = 0
\]

if the exact value of \( \tau \) is known. If \( \tau \) is unknown, the conversion curve is obtained from

\[
\frac{V_r}{V} = \tau + \sum_{k=2}^{n} b_k (V_r^{k-1} - \tau V^{k-1})
\]

**Differential [4].** A small AC power \( \Delta P \) is superimposed onto a DC power \( P \). Measurements of the AC output \( h(V) = \Delta V(V) \) are made at different DC outputs \( V \). We can first fit the curve

\[
\frac{1}{h(V)} = \sum_{k=0}^{n-1} c_k V^k.
\]

When the AC input is sufficiently small, the conversion curve is

\[
g(V) = \int_0^V \sum_{k=0}^{n-1} c_k V^k dV'
\]

\[
= V + \sum_{k=2}^{n} b_k V^k,
\]

where \( b_k = \frac{c_{k-1}}{c_0} \).

**Computer Simulations and Measurements**

A functional form is assumed for the response curve, and simulated data are created for different methods. Then noise with known standard deviation is added to the data. The conversion curve is obtained by linear least-squares fitting. Errors are calculated by comparing the resultant and original curves. Thus, the methods are compared for different situations.

Conclusions from the simulations are:

1. Systematic error due to the truncation of polynomials is the same for all the methods.
2. Uncertainty is improved if low-order polynomials are used.
3. The combined uncertainty shows that a third-order polynomial is preferred in most cases.
4. The differential method has the least uncertainty; the attenuation method with unknown \( \tau \) has the largest uncertainty. All the other methods give almost the same uncertainty.

Measurements were made for all the methods under the same conditions. The results verified the computer simulations.

![Figure 1. Measurement setup.](image)

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**References**


