Impedance Mismatch Effects On Propagation Constant Measurements
Donald C. DeGroot, David K. Walker, and Roger B. Marks
National Institute of Technology, Microwave Metrology Group
Mail Code 813.06, 325 Broadway, Boulder, CO 80303-3328
(303) 497-7212, FAX: (303) 497-3970, degroot@boulder.nist.gov

Abstract
By measuring propagation constants of coplanar waveguide transmission lines, we show the significant systematic errors of common measurement techniques when the characteristic impedance of the lines does not match the reference impedance of the instrument.

I. Introduction
In this conference report we examine various methods for measuring the propagation constant ($\gamma$) in uniform transmission lines. We particularly focus on the port-match assumptions made in three commonly used procedures and, through a comparison to the fully-corrected solution of the Multiline Method [1], demonstrate significant errors in $\gamma$ when the characteristic impedance ($Z_0$) of the transmission line under test differs from the reference impedance of the measurement system.

The following sections describe the four measurement methods and present coplanar waveguide (CPW) transmission line measurements from each of the techniques. In the final section, we draw general conclusions regarding accurate propagation constant measurements of electronic interconnects.

II. Four Measurement Methods
a) Probe-Tip
The first method relies on transmission coefficient data from a single transmission line. It is often used when physical constraints allow only one line length, but it requires an instrument calibration at a well-defined and well-matched reference plane (see [2] as an example implementation). We designate it the “Probe-Tip Method” since this work utilizes on-wafer measurements of planar transmission lines, but the method is often applied to lines with fixed connectors as well.

To implement this technique, we perform an OSLT (open-short-load-thru) calibration of a vector network analyzer (VNA) to the tips of wafer probes. This calibration uses the constants and procedures supplied by the manufacturer of the probe and calibration substrate. The measured transmission parameters ($S_{21}$ or $S_{22}$) are then presumed to be the actual transmission coefficients of the line.

The Signal-Flow and Probe-Tip diagrams of Fig. 1 illustrate the potential source of errors in this method. Even though a calibration has been performed at the connection plane, the properties of the electrical transition between that plane and the physical device must be taken into account. We represent the transition between the CPW line and the probe-tip calibration by the error boxes labeled “Port A” and “Port B” in the Signal-Flow Diagram. The signal propagation on the length of uniform transmission line between the probes is represented as $e^{j\lambda L}$, where $L$ is the distance between probe tips. If the probes are perfectly matched to the transmission line and the contact is ideal (as might be the case if the calibration substrate is identical in every respect to the CPW under test), then the reflection terms ($A_{11}, A_{22}, B_{11}, B_{22}$) go to zero and the transmission terms ($A_{21}, A_{12}, B_{21}, B_{12}$) go to unity, allowing the error boxes to be ignored. This is the port-match assumption. If, however, $Z_0$ of the CPW is different from the calibration reference impedance $Z_{ref}$, or if the contact is nonideal, then the transition error boxes must be considered.

Solving the signal-flow diagram for the measured transmission coefficient ($S_{21}=b_2/a_1$) gives the following expression:

$$e^{-\gamma L} = S_{21} \cdot \frac{1 - r_{22} e^{-2\gamma L}}{r_{21}},$$

where $r_{22} = A_{22}B_{22}$, and $r_{21} = A_{21}B_{12}$. This equation shows that the measured $S_{21}$ will describe the true signal propagation on the line ($e^{-\gamma L}$) only when the port-match assumption is valid.

b) Ratio
The second method does not require calibration. Rather, it describes the propagation constant using the ratio of uncalibrated transmission parameters from two lines that
differ only in length. It is frequently used when lines of varying lengths are readily accessible, or when a measurement calibration is impractical (see [3] for an application of this technique).

In this case, the Port A and Port B error boxes represent all connections and transitions between the uncalibrated instrument and an arbitrary reference plane on the CPW (see the Ratio & Multiline Diagram in Fig. 1). As shown by Hayden in a slightly different formalism [4], the ratio of the measured $S_{21}$ parameters of (1) eliminates the transmission error $t_{21}$, but not the mismatch errors, leaving

\[ e^{-\gamma L} = \frac{S_{21}^L}{S_{21}^T} = \frac{1 - r_{22}e^{-2\gamma L_2}}{1 - r_{22}e^{-2\gamma L_1}}, \]  

where $L_1$ and $L_2$ are the lengths of the two lines, and $\Delta L = L_2 - L_1$.

This method has an advantage over the single-line Probe-Tip method in that the reference plane can be located a distance away from the transition where the fields will be uniform and where $\Delta L$ truly represents the additional length of uniform transmission line. However, as above, any reflections at the ends of the lines will introduce error into $\gamma$ when the ratio of measured $S_{21}$ parameters is presumed to represent the actual transmission coefficient of the additional length of line.

\[ \gamma = \frac{1 - r_{22}}{1 - r_{22}e^{-2\gamma L}}, \]

\[ \text{where } L_1 \text{ and } L_2 \text{ are the lengths of the two lines, and } \Delta L = L_2 - L_1. \]

This method provides a solution for $\gamma$ accounting for all the transmission and reflection parameters ($S_{ij}$) from two or more lines of varying lengths. It does not make the port-match assumption, nor does it require instrument calibration, but it does need both forward and reverse transmission line measurements.

For this method we measure two CPW lines of different lengths without calibrating our instrument, as in the Ratio method. Then, using the NIST MultiCal software with an estimate of the effective dielectric constant ($\varepsilon_{eff}$), we solve for the propagation constant according to [1]. Any impedance mismatch is accounted for and does not generate systematic errors in $\gamma$. The accuracy is limited only by the random errors encountered in the connections to the two lines and by the accuracy of the length difference $\Delta L$.

### III. Mismatch Effects

Measurements from two sets of coplanar waveguide transmission lines (CPW1 and CPW2) are used to demonstrate each of the three methods. The CPW lines consist of 350 nm thick gold on semi-insulating GaAs. The center conductor width ($w$) and ground plane separation ($s$) are $w = 43 \mu$m and $s = 201 \mu$m for CPW1, and $w = 73 \mu$m and $s = 171 \mu$m for CPW2. Both sets include two lines of different lengths: $L_1 = 0.5 \text{ mm}$ and $L_2 = 20.195 \text{ mm}$, giving $\Delta L = 19.695 \text{ mm}$. Only the longer line is used for the Probe-Tip Method.

![Fig. 2. Frequency-dependent characteristic impedance of the CPW1 and CPW2 coplanar waveguide transmission lines.](image)

Frequency-dependent characteristic impedance data from both CPW geometries are shown in Fig. 2. These data were obtained from the Multiline propagation constant values [5] and an estimate of the capacitance per unit length ($C_0$) calculated from a model [6] based on a full-wave analysis of the CPW structure. For CPW1, $C_0 = 1.300 \text{ pF/cm}$, and for CPW2, $C_0 = 1.737 \text{ pF/cm}$. The high-frequency values of $|Z_0|$ for CPW1 and CPW2 are 68 $\Omega$ and 50 $\Omega$, respectively, but $|Z_0|$ increases rapidly for decreasing frequency as the resistance of the conductors dominates over the inductance. The significant differences in $|Z_0|$ from the nominal 50 $\Omega$ port impedance provides a means to observe mismatch effects in the propagation constant methods.

![Fig. 3. Effective dielectric constant and attenuation factors for CPW1 as measured by four techniques.](image)
Propagati on constants for CPW1 acquired with each of the four measurement techniques are plotted in Fig. 3 as the effective dielectric constant $\varepsilon_r$ and the loss factor $\alpha$. Above 3-4 GHz, where the impedance mismatch is relatively small, the Probe-Tip and Ratio methods approximate the fully-corrected calculation of the Multiline method. Below 3 GHz, these two methods deviate significantly as $|Z_0|$ increases above 70 $\Omega$. At 0.5 GHz, where $|Z_0| = 93$ $\Omega$, the Probe-Tip and Ratio methods report $\varepsilon_r$ = 7.4 and $\alpha$ = 2.0 dB/cm, while the actual values are closer to $\varepsilon_r$ = 10.5 and $\alpha$ = 1.5 dB/cm. This amount of error would be unacceptable for many high-speed interconnect applications, and the error increases even further for larger impedance mismatches.

The data in Fig. 4 are from measurements of the lower impedance lines (CPW2). Here, the Probe-Tip and Ratio data are in somewhat better agreement with the actual $\gamma$ values, and they follow the Multiline values to lower frequencies. The errors in the Probe-Tip and Ratio methods as predicted by (1) and (2) should decrease for improved impedance matching, and the data for CPW2 clearly demonstrate this to be the case.

Interestingly, the errors of the Probe-Tip and Calibrated Ratio Methods are nearly identical, emulating the uncalibrated Ratio Method for both CPW1 and CPW2. This observation and the results of Fig. 4 show that unaccounted impedance mismatches are a potential source of serious propagation constant error. The finding also reveals the mistake often made in assuming that a 50 $\Omega$ calibration will improve the accuracy of $\gamma$ measurements. In fact, unless the calibration uses the $Z_0$ of the lines as the reference impedance, significant errors will remain. For calibrations utilizing fixed $Z_{\text{ref}} \neq Z_0$ (for example, OSLT and LRM), the calibrated $S_{21}$ data will not describe the true transmission line propagation. This point is particularly notable since $Z_0$ depends strongly on frequency in many electronic interconnect applications, as seen in Fig. 2.

IV. Conclusions

In summary, this work explicitly demonstrates the significant errors that can be generated by three propagation constant measurement techniques when used to characterize the transmission lines of modern electronic packaging. Though this examination employed frequency-domain network analyzer measurements of coplanar waveguides, it serves as a model in identifying general conclusions applicable to time-domain instrumentation and to other types of transmission lines. We summarize our key findings here:

- Propagation constants derived from the calibrated $S_{21}$ parameters of a single line will contain systematic errors whenever the characteristic impedance of the line differs from the calibration reference impedance.
- Taking the ratio of uncalibrated $S_{21}$ parameters from two lines of different lengths does not solve the problem. The common transmission loss will cancel, but any impedance mismatch at the ports will create errors in $\gamma$.
- Propagation constants calculated from the ratio of *calibrated* $S_{21}$ parameters of two lines of different lengths will also include errors when the calibration $Z_{\text{ref}}$ does not describe the line $Z_0$.
- The systematic errors in the Probe-Tip, Ratio, and Calibrated Ratio methods increase for greater impedance mismatch.
- Fully-corrected solutions such as the Multiline Method provide the most accurate measurement of $\gamma$ whenever two or more lines of different length are available.

We are currently working on additional studies to better quantify the accuracy of propagation constant measurements and improve techniques for lines with fixed connectors.

References