Absolute optical ranging using low coherence interferometry

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We describe a method for measuring submicrometer distances with an asymmetric fiber Michelson interferometer having an LED as a source of radiation. By measuring the phase slope of the Fourier components in the frequency domain, it is possible to locate the position of reflections with nanometer precision even in the presence of sample dispersion. The method is compatible with time domain sampling at the Nyquist rate which assures efficiency in data acquisition and processing.

I. Introduction

Low coherence or white light interferometry is a well-established method for measuring range in a precise and unambiguous way in sensors and in the diagnostics of small-scale optical devices. Reported applications include estimation of trench depth in integrated circuit devices, position and strain sensing, determination of the thickness of thin films, and location of scattering centers in lightguides. We describe here a technique that was evaluated as part of a program to develop optical diagnostic probes for testing the guiding characteristics of semiconductor laser sources. The ranging information was obtained from an analysis of the Fourier transform of the reflection signatures using an extension of techniques familiar in dispersive Fourier transform spectroscopy.

The essential components of an unbalanced fiber interferometer are shown in Fig. 1. The device under test is located in one arm of an otherwise symmetric scanning Michelson interferometer. Radiation is constrained to propagate in single-mode fibers and the directional coupler except at the ends of the two arms where bulk components collimate the respective beams. A suitable optical source is a light emitting diode—most of the present work was done with LEDs operating at 1300 or 1550 nm having FWHM spectral widths from 60 to 120 nm. The object of this paper is to demonstrate how range in transparent materials can be determined from a Fourier analysis of the interferograms arising from surface reflections. The interferograms are generated by translating mirror M in the reference arm at a constant velocity. In practical situations all solid samples will exhibit dispersion, which has important effects on the signatures. As we shall see, this need not be a problem in identifying the exact location of reflections.

Although other means have been suggested, the usual way of estimating distance with a low coherence Michelson interferometers is to note the travel of mirror M between maxima on the appropriate visibility envelopes. However, it is difficult to locate accurately the position of an extremum to much less than a wavelength of light, and this is especially true with dispersive materials. The effect of dispersion is to reduce fringe contrast, broaden and usually distort the interferogram. Francois and co-workers gave examples in which dispersion introduces ambiguities as much as ten wavelengths in the choice of a proper reference feature on the signature. We now show that, although the signals are acquired as a function of time, a more accurate and unambiguous measure of distance can be achieved by processing the data in the frequency domain.

II. Theory

The interferogram in time domain $M(t)$, and its Fourier transform in the frequency domain $m(\omega)$, are related by the expression

$$m(\omega) = \int_{-\infty}^{\infty} M(t) \exp(i\omega t) dt,$$

where $m(\omega)$ is complex in general and can be written in polar form as

$$m(\omega) = B(\omega) \exp[-i\phi(\omega)].$$

In these expressions, $\omega$ is the angular frequency of the optical radiation. The usual assumptions are made that, for the modulus, $B(-\omega) = B(\omega)$ and, for the phase factor, $\phi(-\omega) = -\phi(\omega)$. Also, we consider only double-sided interferograms in what follows. If we take the
front surface of the sample in Fig. 1 to be located at the position of zero path difference, the resulting interferogram from the Fresnel reflection will be symmetric. The amplitude of the Fresnel reflection is actually slightly dependent on wavelength, but this dependence will be neglected in the treatment that follows. With lossless test materials, \( B(\omega) \) then represents the source spectrum. At \( t = 0 \) the fringe contrast will be a maximum since all the Fourier components arrive in phase at the photodetector. If radiation travels a distance \( z \) through a dispersive sample, there will be a phase shift introduced into the back surface interferogram [Fig. 2(b)] given by\(^{10}\)

\[
\phi(\omega) = \frac{\omega}{c} z n(\omega),
\]

(3)

with \( c \) the velocity of light, and \( n(\omega) \) the index of refraction of the transmission medium. It is convenient to expand this phase in a Taylor series about a frequency near the center of the power spectrum \( B(\omega) \) (which for most LED sources will be approximately Gaussian):

\[
\phi(\omega) = \phi(\omega_0) + \frac{d\phi}{d\omega} \omega_0 (\omega - \omega_0) + \frac{1}{2} \frac{d^2\phi}{d\omega^2} (\omega - \omega_0)^2
\]

\[+ \frac{1}{6} \frac{d^3\phi}{d\omega^3} (\omega - \omega_0)^3 + \ldots \]  

(4)

The derivatives are taken with respect to angular frequency and evaluated at \( \omega_0 \). The constant in Eq. (4) is unimportant in the present context. We can identify phase slope \( \frac{d\phi}{d\omega} \) with group delay \( T(\omega) \) as follows:

\[
T(\omega) = \frac{d\phi}{d\omega} = \frac{\omega}{c} N_1(\omega_0) + \frac{dN_1}{d\omega}(\omega_0)(\omega - \omega_0)
\]

\[+ \frac{1}{2} \frac{d^2N_1}{d\omega^2} (\omega - \omega_0)^2 + \ldots \]  

(5)

and we have used the definition of group velocity

\[
N_g(\omega) = n(\omega) + \alpha \frac{dn}{d\omega} \omega.
\]

(6)

It is clear from the form of Eq. (5) that the group delay times will vary slightly for Fourier components in the neighborhood of \( \omega_0 \). The higher-order terms \( \frac{dN_1}{d\omega} \) and \( \frac{d^2N_1}{d\omega^2} \) describe the effects of dispersion.\(^{11}\) For our purposes we need consider only the first phase derivative. Now \( z \) can be written, when \( \omega = \omega_0 \),

\[
z = \frac{c}{N_g(\omega_0)} \frac{d\phi}{d\omega} \omega.
\]

(7)

Equation (7) provides us with the desired relation between the experimentally derived phase slope and distance. The group index is assumed to be known. The experimental implementation of this relation is discussed in the next section.

III. Experiment

The fiber interferometer used in this work is similar to the one shown schematically in Fig. 1. Further experimental details are given elsewhere.\(^{11}\) The dispersive sample used for the data in Figs. 2 and 3 was a 1.9-cm thick silica etalon. The LED source operated at 1560 nm and had a FWHM of \( \sim 130 \) nm. Before using Eq. (7) to estimate range with this apparatus, we describe some complications which must be addressed in most experimental systems. These are data sampling and shifting the computational origin.

A. Data Sampling

The most efficient experimental method of acquiring and processing data for Fourier transformation involves sampling the interferogram. Each data point in the time domain is separated by a constant time interval \( \Delta T \). For optimal efficiency, the data must be sampled at approximately the Nyquist rate, which amounts to two samples per cycle at the cutoff frequency \( \omega_{\text{max}} \). Sampling can be done with a stepper motor drive on movable mirror \( M \) or with an electronic digitizer and a continuous scan. Schemes that employ continuous scans usually average the signal with a time constant approximately equal to the sampling interval to avoid wasting the time spent traveling between data acquisition points. Although an optimal sampling process does not destroy any signal information, the appearance of the interferogram is altered. Figures 2 and 3 show signatures which are first oversampled and then sampled at the Nyquist rate. Obviously it is easier to estimate the location of the visibility maximum in the oversampled case, but this fact is of no importance if we process the data in the frequency domain.

B. Origin Shifts

One difficulty encountered in the direct application of Eq. (7) is that large phase values can accumulate rapidly as \( z \) increases. In fact it is easy to show that the phase differences between discrete points in the fre-
The frequency domain will exceed $2\pi$ when $z > \frac{\pi c k}{2N_\omega \omega_{\text{max}}}$. These phase ambiguity problems can be avoided by shifting the computational origin from $t = 0$ to time $m\Delta T$ near the extremum of the backreflection signature [Fig. 2(b)]. Here $m$ is an integer and $\Delta T$ is the same quantized time interval as before. According to the shift theorem the phase relative to the new origin is

$$\phi_s(\omega) = \frac{z}{c} \omega n(\omega) - \omega m\Delta T,$$

and, following the same arguments that led up to Eq. (7), $z$ is now given by

$$z = -\frac{c}{N_\omega(\omega_0)} \left[ \frac{d\phi_s(\omega_0)}{d\omega} + m\Delta T \right].$$

If we had chosen the new origin at exactly $t = N_\omega(\omega_0)z/c$, then $d\phi_s(\omega_0)/d\omega = 0$, which defines a point of stationary phase. Because of the quantized nature of the sampling, in general there will be no such coincidence. Then the zero crossing occurs at a displaced time interval $d\phi_s(\omega_0)/d\omega$, provided that $m\Delta T$ represents the nearest integer sampled point.

The point of stationary phase exists at a time corresponding to the approximate visibility maximum. This is plausible if we consider a smooth symmetrical source spectrum where $\omega_0$ is the mean optical frequency. The dispersive medium will introduce a frequency dependence on delay, but most of the frequency components around $\omega_0$ will have nearly the same phase shift and consequently these fringes will add constructively. One then expects that the fringe contrast will be a maximum near the point where the phase is independent of frequency.

Figure 4 shows the effect on the calculated phase slope when the origin is shifted in units of $\Delta T$ around the point of stationary phase. The origin closest to the zero crossing is designated by $m = 0$. These figures use the transform data of the signatures shown in Fig. 3. The phase slope is calculated from Eq. (4) using a least-squares routine. For each point corresponding to a time shift, the phase slope is recalculated using the same procedure. The data demonstrate a linear dependence of $d\phi(\omega_0)/d\omega$ on delay as predicted by Eq. (9). In these figures the data have been normalized so that the quantities plotted on both axes are dimensionless. In conventional fast Fourier transform (FFT) algorithms the discrete frequency intervals $\Delta \omega$ are related to $\Delta T$ by $\Delta \omega = 2\pi/k\Delta T$ with $k$ the FFT array dimension. Then the phase slope can be written in terms of a frequency step as $d\phi/dq = m/k$ ($q$ is an integer) and time can be expressed in units of $m$.

IV. Discussion

A. Signature Distortion

It is of interest to examine some qualitative features of the signatures which result from dispersion. The signature distortion can be obtained directly from the inverse Fourier transform of Eq. (1):

$$M(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} B(\omega) \exp[-i\phi(\omega)] \exp(-iwt) \, d\omega.$$

This equation can be solved exactly for the first-order dispersion term in Eq. (4) if $B(\omega)$ has a Gaussian shape given by

$$B(\omega) = \exp[-(\omega - \omega_0)^2/2\sigma^2].$$
Fig. 4. Experimental values for the phase slope at various time-shifted computational origins around the position of stationary phase. The zero crossing occurs near the maximum of the interferogram in Fig. 3(b). The sample number is arbitrarily labeled so that \( m = 0 \) is nearest the zero crossing. The phase slope is plotted in dimensionless units of \( k^{-1} \), where \( k \) is the FFT array dimension (see text).

\[
M(t) = \exp(-t^2/2\sigma_t^2)g(t). \tag{12}
\]

Here \( g(t) \) incorporates the sinusoidal fringe features of the signature. The envelope function is Gaussian with a \( \sigma_t \), rms width of

\[
\sigma_t = \sigma_0 \left( 1 + \left[ \frac{d^2\phi(\omega_0)}{d\omega^2} \right] \omega_0^2 \right)^{1/2}, \tag{13}
\]

with the first-order dispersion defined by the usual relation

\[
D(\omega_0) = -\frac{\omega_0^2}{2\pi \nu_c} \frac{d^2\phi(\omega)}{d\omega^2}. \tag{14}
\]

In this case the signature is symmetric about \( t = 0 \). Near the zero dispersion wavelength (1273 nm in silica) the second-order dispersion will be significant, and this will result in an asymmetric envelope function.

**B. Measurement Precision**

An estimate of the precision of distance measurement may be obtained by observing that increasing \( z \) slightly is completely equivalent, from a data processing point of view, to shifting the FFT origin an amount \( t = c/N_g(\omega_0) \) in the opposite direction. The only error which appears in a computation of this sort, using Eq. (7) or (9), is that associated with the fit to the phase function. This ignores a lot of other problems which affect the overall accuracy, such as dimensional stability, linearity, \( \Delta T \) calibration, and the small residual phase arising from instrumental imperfections, but these need not be considered here. The variation in slope can be estimated from standard regression analysis, and for the data in Fig. 3(b) implies an uncertain-

ty in location of the point of stationary phase of \( \sim 0.01 \) fringe. The precision is therefore quite good.

In the derivation of Eq. (9) we assumed that \( \omega_0 \) was the central frequency in a symmetrical source spectrum. Since the frequency intervals (along with the time intervals) are discrete in a FFT, the sampled frequency \( \omega_0 \) will not as a rule be equal to the mean frequency. However, the conditions we originally assumed are overly restrictive. Selecting an off-center \( \omega_0 \) only means that \( N_g(\omega_0) \) must apply to that new frequency, and the accuracy of the method is unaffected. To give an idea of the magnitude of this effect, we note that, in silica, \( N_g \) changes by \( \sim 0.05\% \) over a bandwidth of 130 nm at 1550 nm.

**C. Group Index Measurements**

Implementation of Eqs. (7) and (9) requires that the group index be known. For bulk materials \( N_g \) can be calculated from Eq. (6) if the phase index is known as a function of frequency. Such information is available for many materials. Single-mode guided wave structures will also exhibit waveguide dispersion which must be taken into account. In this case \( N_g \) can be replaced with an effective group index and determined experimentally. Several interferometric methods have been reported for making measurements of this sort in microoptic devices and short (centimeter) lengths of optical fibers.

**D. Resolution and Source Power Considerations**

Precision in distance measurement should not be confused with resolution, which usually refers to the ability to distinguish between reflections in close proximity. The Fourier transform methods require that the signatures be distinct. To avoid possible overlap from adjacent reflecting surfaces, the individual signatures should have as short a duration as possible. This implies maximizing the source spectral width. However, in many practical cases of interest, greater optical bandwidth is accompanied by less optical power transmitted in the lightguide. For example, some commercially available semiconductor emitters at 1300 nm, the FWHM and power (coupled into a single-mode fiber) are: superluminescent diodes (500 \( \mu \)W, 70 nm), edge emitting LEDs (100 \( \mu \)W, 60 nm), surface emitting LEDs (0.1 \( \mu \)W, 130 nm). For a halogen lamp source, \( \sim 0.1 \mu W \) can be injected into a single-mode fiber in a bandwidth of 300 nm. There is therefore something of a trade-off between spatial resolution and signal dynamic range. It should be noted that, with the technique described here, there is no particular accuracy advantage in using broadband sources as long as the reflections are separated by a distance much greater than the source coherence length.

**V. Summary**

We have described a simple technique for obtaining absolute distance in microoptic structures by analysis of the interferograms generated by a fiber Michelson interferometer and low coherence source. The ranging information is obtained directly from the Fourier
transform of the appropriate time domain signatures. We have shown that reflections arising from refractive index discontinuities can be located with a precision of \( \sim 0.02 \mu m \).

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References


10. Ref. 8, p. 22.


