Capacitors with Very Low Loss: Cryogenic Vacuum-Gap Capacitors

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Abstract—We report on measurements of capacitors with about 1 pF of capacitance, which have unmeasurably small leakage at very low frequencies, placing a lower bound of about $10^{-15}$ Ω on the parallel resistance at an effective frequency of 1 mHz. These measurements are made possible by two themes: the use of vacuum-gap capacitors (i.e., no dielectric material, operated in vacuum), and detection of leakage using single electron tunneling (SET) electrometers, which have very high input impedance. We also report on good achieved results in time stability and lack of frequency and voltage dependence.

I. INTRODUCTION AND MOTIVATION

There are several motivations for the search for capacitors with very high stability, very low loss, and very low frequency and voltage dependence.

For metrologists, one general motivation is that such capacitors allow a better reference to define the absolute value of phase angle. Put another way, they provide a more “pure” capacitance, with less resistive reactance. Using measurements at nonzero frequency, it is very difficult to absolutely define the zero of phase angle, since one can never be sure of the complete absence of any resistive reactance.

There has been previous work on defining an absolute loss angle, $\theta$, and dissipation factor, $D$, such that $\tan \theta = D = \frac{1}{\omega RC}$, where $R$ and $C$ are in parallel. This work, performed at audio frequencies ($2\pi f = 10^4$ s$^{-1}$), concentrated on vacuum-gap (room temperature) cross capacitors. In particular, the effects of surface dielectric films were deduced from measurements of electrode separation [1], and by cross-comparison of the capacitance of various combinations of electrodes [2]. These authors assigned uncertainties of about $2 \times 10^{-8}$ to their measurements of the loss angle. These measurements are difficult and time-consuming, and rely on knowing and accounting for all possible sources of loss.

In contrast, by performing measurements at arbitrarily low frequencies, it is possible to put an arbitrarily large lower limit on the parallel leakage resistance. Thus, an alternative way to define an absolute scale for phase angle or loss, is to work up from such low-frequency measurements. One difficulty in such a procedure is avoiding the finite input impedance of the electrometer. In this paper, we show that extraordinarily high limits on the parallel leakage resistance can be achieved by using SET detectors at low temperatures and low frequencies.

A second motivation for studying these capacitors is their potential use as the reference capacitor in temperature dependence measurements of the dielectric constant of various materials. Measurements of the dielectric constant and loss as a function of temperature in many materials yield valuable structural information, such as about two-level systems in glasses. However, the very small amplitude of the temperature dependence sets sharp requirements on the stability, frequency, and voltage dependence, and to a lesser extent on the loss of the reference (standard) capacitor [3]. One study [3] in particular developed low-temperature reference capacitors with either excellent stability but poor voltage dependence, or the inverse; the authors could not find a single capacitor satisfying both criteria. Here, acceptable stability and voltage dependence were defined as a relative drift of less than about $10^{-5}$/h and dependence of less than $10^{-6}$/V, respectively [3]. In this paper, we show that the capacitors described herein come close to or meet both of these criteria.

A final motivation, and the one which is the particular cause for our work in this area, is the recent efforts [4] at NIST to measure absolute capacitance standards using SET pumps [5] and electrometers [6]. These experiments will consist of two different stages: first, we will use a SET pump to deliver a measured number of electrons onto the plate of a stable, low-loss transfer capacitor. Second, we will hold the number of electrons fixed while measuring the voltage (of order 1 V) developed across the capacitor. These two stages will require seconds or tens of seconds to accomplish, and during this time we require that the number of electrons on the capacitor plate remains fixed (within the metrological uncertainty of $10^{-8}$). This requirement sets the desired loss, corresponding to a leakage resistance at this “frequency” (real time period) of about $10^{20}$ Ω. We plan to then compare this transfer capacitance to the unknown or reference standard which is held at room temperature. We thus also require the capacitance of the low-temperature transfer capacitor to be stable over the course of minutes or hours. Since the comparison to the room-temperature capacitor will likely be done at audio frequencies, we also require a very low frequency dependence (or one that is stable and well-characterized) for the transfer capacitance. These requirements are the crucial ones for the transfer capacitor, and here we report our current progress, with lower bound results which are within about a factor of 10 of the requirements for both loss and stability at audio frequencies (with much larger uncertainties at low frequency).

In particular, with the vacuum-gap capacitors, we have observed no drift within our uncertainties, leading to an upper
bound in the change of capacitance ratios of about $10^{-6}$ over the course of days. We have also observed no leakage across the capacitor plates, with a best case lower bound of about $10^{10}$ Ω. This lack of leakage is the main new result of this paper; in contrast, the limits on time stability, and voltage and frequency dependence are useful and important for our work, but not close to those achieved for capacitors used to maintain national standards. These results are achieved with three-terminal capacitors made with copper plates held off from each other by sapphire balls. For comparison, at an effective frequency of about 1 mHz, this corresponds to a loss $D \leq 3 \times 10^{-3}$; if, as we believe, the leakage resistance is independent of frequency, at the standard audio frequency of $2\pi f = 10^4$ s$^{-1}$, the loss $D \leq 1 \times 10^{-11}$. These results can be contrasted to earlier measurements made by our group on capacitors made using silica disks coated with metallic electrodes, which had parallel leakage resistances of order $10^{13}$ Ω [7].

II. PHILOSOPHY AND DETAILS OF FABRICATION AND ASSEMBLY

One of the major difficulties of performing capacitance measurements is the rejection of the stray capacitance to ground from both the leads running to the capacitors, as well as from the capacitor plates themselves. The classic way to avoid this difficulty is to perform a three-terminal capacitance measurement [8]. This scheme consists of two parts. First, all cables and equipment, as well as the plates of the capacitors, are electrostatically shielded from the environment by continuous shields; in the case of the cables, this generally means that they must be coaxial. This continuous shield prevents any change in the measured capacitance owing to, for example, movements of wires near the cables. Secondly, the measurement circuit consists of a bridge configuration, with one plate of the each capacitor driven by a low-impedance source, and the other plate connected to the null detector, and thus at a virtual ground. The result of the physical and circuit configurations is to reject any component of the stray capacitance to ground from entering into the measured capacitance; as noted, this includes changes, for example, due to the motion of nearby wires.

In addition to rejecting the effects of capacitance to ground in the leads and plates, the three-terminal measurement also rejects the effects of leakage resistance in these same places. Examples include leakage current through the insulator in coaxial cables and leakage from the capacitor plates to ground. It does not reject any leakage directly from one plate to the other.

We can thus see that the objectives of the three-terminal measurement are a clean definition of the capacitance, and a rejection of parallel resistance. These objectives motivated the design philosophy we have used: we have fabricated three-terminal capacitors, where each of the plates is mechanically supported only by contact with the grounded shell. Thus any leakage current through the insulating supports goes only to the grounded shell and is thus rejected in the three-terminal measurement. In addition, the use in vacuum at low temperatures minimizes both the leakage currents and also dielectric loss, which is not intrinsically rejected by this configuration.

An exploded sketch of the capacitors is shown in Fig. 1. The three pieces of metal were machined from OFHC copper. The two plates (left and right pieces) have three triangular grooves which run approximately radially outwards from the center, and the shell (center piece) has three conical holes on each side. This arrangement, with spherical balls placed between the
III. Electrical Results

A. Frequency and Voltage Dependences; AC Stability

In this section, we will discuss the results of AC capacitance bridge ratio measurements. The circuit is shown in Fig. 2, and formed a three-terminal bridge, with the two capacitors both being vacuum-gap capacitors as described in Section II. There was one significant difference, however, from conventional bridges: the first stage of the null detector was the SET electrometer, whose bias current, \( I_{S-D} \), was converted to a voltage, and then amplified and demodulated by the lock-in amplifier (LIA), which is part of the detector \( D \). Since an imbalance at point \( n \) modulated \( I_{S-D} \) through the transconductance of the SET electrometer, the signal out of the LIA was a direct measure of the imbalance in the bridge (i.e., the amount by which \( C_1/C_2 \) differs from \( \frac{V_2}{V_1} \)). The nominal ratio of the capacitors \( C_1/C_2 \) was determined by the ratio of voltages output by the IVD (inductive voltage divider) when point \( n \) was at null (i.e., when the null detector reading was lowest, limited by its noise floor).

We note here that we are reporting measurements of the ratio of capacitances. Thus, the lack of frequency and voltage dependence, and the time stability, should be viewed only with reference to an imbalance to the two vacuum-gap capacitors, rather than an individual measurement on either. This fact has an important consequence: It is conceivable that these capacitors could, in general, have the identical unwanted behavior (e.g., a drift in time) which would thus cancel in the ratio measurement. For the time stability, we believe such an accidental cancellation to be unlikely, and thus the results reported here are indicative of the performance of each capacitor separately. For the voltage and frequency dependences, such an accidental cancellation is possible (to the level at which the two capacitors are identically constructed), and the results should be viewed in this context.

Three-terminal bridge measurements performed at various voltage source frequencies and amplitudes yield the dependences of the bridge balance on these parameters. We have plotted the results of such measurements in Fig. 3. Here \( V \) is the amplitude of the sinusoidal voltage from the source, and is about twice the amplitude of either \( V_1 \) or \( V_2 \). This plot shows clearly that there was no systematic dependence on frequency or amplitude of applied voltage, within the uncertainties. From these measurements, we can place an upper bound of about \( 1 \times 10^{-5}/V \) on the voltage dependence of the ratio measurement.

The smallest uncertainty in the ratio bridge measurements occurred at the largest applied voltage of 10 V peak-peak, with a spread of about \( 5 \times 10^{-6} \) in the ratio of \( C_1/C_2 \), over the audio frequency range from 100 to 1000 Hz. We note that this spread is simply an upper bound on the true frequency dependence.

In addition, crude measurements using DC voltages indicated that the ratio \( C_1/C_2 \) was \( 1.0221 \pm 0.0001 \) at an effective frequency of about 0.01 Hz; thus these very low frequency...
measurements also indicate no frequency dependence over the entire frequency range within the much larger uncertainty of $10^{-4}$. We will continue to decrease uncertainties in the frequency and voltage dependences in the next round of measurements.

To demonstrate the stability in time of these capacitors at audio frequencies, we fixed the IVD setting, and monitored the imbalance signal from the null detector (LIA output). For short times, we then digitized the observed signal as a function of time. An example is shown in Fig. 4. Here we have four sections of approximate duration 300 s each, with different settings on the IVD corresponding to the ratios shown. We can see that, over the course of the 300 s for a single section, as well as the total duration of 1200 s, the balance point (and thus...
the ratio \( C_1/C_2 \) is unchanged (note that the first and last IVD ratios are the same). With an upper bound for the change in \( C_1/C_2 \) of about \( 4 \times 10^{-6} \), we can thus assign an upper bound for the short-term stability of \( d(C_1/C_2)/dt \approx 1.0 \times 10^{-4} \) h\(^{-1}\). For longer times, we observed the balance point and any changes at fixed frequency and voltage, over a period of two weeks. We observed a spread of values of \( 2 \times 10^{-6} \), with no systematic drift, over a period of 12 days. These measurements were performed at 280 Hz and 10 V peak-peak. We thus assign an upper bound of about \( 1 \times 10^{-3} \) h\(^{-1}\) to the long-term drift in \( C_1/C_2 \), at audio frequencies.

### B. Leakage Resistance; DC Stability

In this section, we will examine the leakage (dissipation) at very low frequencies; these results will also yield bounds on the drift in \( C_1 + C_2 \) at low frequencies. To measure the leakage (or put a lower bound on the parallel leakage resistance), we used a circuit as shown in Fig. 5. The source is either an electrochemical battery whose output is manually switched, or a digital source. The two vacuum-gap capacitors are connected in parallel, as shown, with possible leakage resistances denoted as \( R_1 \) and \( R_2 \). After the imposition of an abrupt change (a “switch”) in \( V_{1,2} \), a finite value for \( R_1 \) or \( R_2 \) or equivalently a drift in \( dC/dt \) (\( C = C_1 + C_2 \)) will result in a drift in \( V_{1,2} \), and thus a drift in the electrometer output, \( I_{S-D} \). We have also indicated the stray capacitance to ground between the capacitors and electrometer, \( C_{\text{stray}} \), since it is important for the analysis.

We will now do the calculation necessary to estimate \( R_1 \), \( R_2 \), and \( dC/dt \) from changes in \( I_{S-D} \). For the data discussed below, we have estimated (based on other measurements) \( C_{\text{stray}} \) to be approximately 40 pF. This value is much greater than in our previous work [7], and is due to the close spacing of the plates to the ground shell. This large value is undesirable, since a larger \( C_{\text{stray}} \) decreases the sensitivity of the bridge balance measurement. In the next design, we will rectify this weakness, while maintaining adequate isolation between the two plates.

Thus \( C_{\text{stray}} \) is much greater than \( C_1, C_2, C_{\text{g}} \), or the other capacitances in the electrometer, which implies that \( V_{1,2} \) \( \ll V_{1,2} \). Thus we can set \( V_{1,2} = 0 \) initially. Then, after a switch to a value \( V \) for \( V_{1,2} \), the effect of parallel resistance is that a leakage current \( I \) develops: \( I = V/R \), where we have assumed \( R_1 = R_2 = 2R \). Since \( C_{\text{stray}} \) is much greater than the capacitances in the electrometer, we have

\[
\frac{dV_{1,2}}{dt} = I/C_{\text{stray}} = V/(R C_{\text{stray}})
\]

or

\[
R \approx V/C_{\text{stray}} \left( \frac{dV_{1,2}}{dt} \right)^{-1}.
\]  \hspace{1cm} (1)

Similarly, a drift \( dC/dt \) in \( C = C_1 + C_2 \) will manifest itself as follows

\[
V_{1,2} \approx (C/C_{\text{stray}}) V \Rightarrow dV_{1,2}/dt \approx V (dC/dt)/C_{\text{stray}}
\]

or

\[
dC/dt \approx C_{\text{stray}} (dV_{1,2}/dt)/V. \hspace{1cm} (2)
\]

To use this analysis, we examine data of the type shown in Fig. 6. The upper panel shows the time dependence of \( V_{1,2} \): A switch, followed by a long constant period, and then a small ramp (barely visible in the figure) at the end. The lower panel shows the corresponding time dependence of \( I_{S-D} \): A switch at the same time as \( V_{1,2} \), a long mostly constant period, and the oscillations at the end, due to the ramp in \( V_{1,2} \). These oscillations show the behavior of the electrometer transconductance, \( I_{S-D}(V_{1,2}) \) or \( I_{S-D}(V_{1,2}) \), and allow us to calibrate any changes in \( I_{S-D} \) over the time period. We note that we have made many measurements of the type shown in Fig. 6; some of these show as little drift in \( I_{S-D} \). Others show substantial time dependence, but not the
linear time dependence which would be exhibited by a parallel resistance, as shown in (1). Instead, the time dependence is the random noise or discrete switching behavior commonly observed in the SET electrometer [9]. We have never observed the monotonic time dependence of $V_2$ which would result from parallel leakage resistance.

Thus, we can examine the data in Fig. 6 to set lower and upper bounds on $R$ and $dC/dt$, respectively. A detailed analysis of the data in Fig. 6 indicates that most of the small amount of drift observable in $I_{S-D}$ between 50 s and 950 s is due to drift in the battery-imposed $V_{1,2}$ as measured by a precision voltmeter. We can set an upper bound on the intrinsic change in $I_{S-D}$ over this time period as corresponding to a change in the gate voltage of $0.03 \Delta V_G$ ($\Delta V_G$ is the period in voltage of one oscillation), or $\delta V_2 \lesssim 0.010$ mV over 900 s. If we attribute this to a linear drift in $V_2$, we can assign an upper bound of $dV_2/dt \lesssim 0.010$ mV/900 s. From (1), this puts a lower bound on the resistance of $R \gtrsim 5 \times 10^{18}$ $\Omega$, or a lower bound on the parallel leakage resistance of one capacitor $R_1$ or $R_2 \gtrsim 10^{18}$ $\Omega$. Similarly, from (2), we can put an upper bound of $dC_1/dt \lesssim 8 \times 10^{-4}$ pF/h, or $dC_1/dt \lesssim 4 \times 10^{-4}$ pF/h. This corresponds to a time stability $1/(C_1 \cdot dC_1/dt) \lesssim 8 \times 10^{-4}$ h$^{-1}$ at very low (order 1 mHz) frequencies. In addition, similar to the audio frequency measurements, we also performed multiple low-frequency bridge ratio measurements, with a spread in the values of $C_1/C_2$ of $3 \times 10^{-4}$ over the course of 11 days, corresponding to an upper bound on long-term drift of $\delta C_1/dt \lesssim 1.1 \times 10^{-6}$ h$^{-1}$.

**IV. CONCLUSION**

The various parameters we have been able to place on these simple custom-fabricated vacuum-gap capacitors are listed in Table I. We can see that, in general, the time stability and lack of frequency and voltage dependence makes these capacitors attractive candidates for the reference capacitors in low-temperature dielectric constant and loss measurements [3].

We can also see that, although the lower bound on the parallel resistance is about one million times larger than the previous attempt [7], we have not yet demonstrated that the loss in these capacitors is sufficiently small to make them acceptable for the SET pump-charging experiment [4], which will require resistances in excess of $10^{20}$ $\Omega$.

Our next steps will focus on two important goals. The first is to demonstrate bridge ratio measurements with uncertainties of no more than $10^{-8}$ in $C_1/C_2$, and resistances of at least $10^{20}$ $\Omega$. The second goal is to design these capacitors with lower capacitance to ground, $C_{stray}$, and with the ability to tune $C_1$ to within $10^{-4}$ pF of a nominal value. This latter goal may require the tuning to be performed at operating (cryogenic) temperatures, because the change in $C_1$ from room temperature may not be predictable to this level.

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**REFERENCES**


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