Johnson Noise Thermometry Measurements Using a Quantized Voltage Noise Source for Calibration

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***Abstract*—We describe a new approach to Johnson noise thermometry (JNT) that exploits recent advances in Josephson voltage standards and digital signal processing techniques. Currently, high-precision thermometry using Johnson noise is limited by the nonideal performance of electronic measurement systems. By using the perfectly quantized voltage pulses from a series array of Josephson junctions, any arbitrary broadband waveform can be synthesized and used as a calculable noise source for calibrating the cross-correlation electronics used in JNT systems. With our prototype JNT system, we have found agreement to two parts in 10^3 with a 1σ uncertainty of 1×10^{-3} between the voltage noise of a 100–Ω resistor in a triple-point Ga cell ($T_{90} = 302.916$ K) and a pseudo-noise waveform with the same average power that is synthesized by a quantized voltage noise source. We estimate the temperature of the resistor to be 302.5 K $\pm$ 0.3 K (1σ uncertainty based on the uncertainty from the cross-correlation). With better characterization of our JNT system, we expect to achieve relative accuracies of parts in 10^5 for arbitrary temperatures in the range between 270 and 1000 K.**

***Index Terms*—Correlation, digital–analog conversion, frequency control, Josephson array, noise, quantization, signal synthesis, standard, superconducting microwave devices, superconductor-normal-superconductor, synthesizer, temperature measurement, voltage.**

I. INTRODUCTION

IN Johnson noise thermometry, temperature is typically determined by measuring the mean-square Johnson noise voltage V_T^2 across a calibrated resistance $R(T)$. The Johnson noise voltage is given by the Nyquist formula [1]

$$\overline{V_T^2} = 4kTR(T)\Delta f \tag{1}$$

where Δf is the bandwidth of the measurement and k is the Boltzman constant.

Despite the simple thermodynamic relation between the measured signal and temperature, the accuracies achieved to date with the best JNT systems are not as good as that of currently used gas-based thermometry techniques ($<$ 50 ppm,). In previous JNT system designs, the accuracy (50–100 ppm) has been limited by the nonideal performance of the electronic measurement system. In the most successful JNT systems [2], [3], a switched-input digital correlator is used to compare the voltage of a resistor held at an unknown temperature with that of a resistor held at a known, calibrated temperature. In order to reduce systematic errors, the noise power to the correlator electronics is kept constant by keeping the RT product constant in the measurements.

We are developing a JNT measurement system [4] that also uses a switched input digital correlator, but in addition uses a quantized voltage noise source (QVNS) as the reference signal source. Details of the QVNS system are described elsewhere in these proceedings [5]. Use of a QVNS as a synthesized reference has several advantages, such as reduced measurement time, simultaneous matching of noise power and impedance to that of the temperature sensing resistance, and increased measurement bandwidth [4].

II. EXPERIMENTAL SETUP

A block diagram of the JNT system appears in Fig. 1. Each arm of the digital correlator has an analog gain of $\sim 10^6$ to amplify the small Johnson-noise voltage signals. Switched inputs allow the QVNS to be substituted for the passive resistor to calibrate the gain and frequency response of the electronic system. The QVNS generates a constant power spectral density that can be precisely calculated and set to any desired value to match the voltage noise of a known resistor at any arbitrary temperature. At any given time, the switches are configured so that both amplifier chains are measuring the same signal source. Both noise sources use five-wire interfaces that are schematically shown in Figs. 2 and 3. The QVNS has additional input connections for 10 Gb/s pulses in which the digital code represents the pseudo-noise voltage waveform that is created with a delta-sigma modulator [4]. There are 50-Ω resistors on each of the four voltage leads to match the output impedance of the QVNS to the impedance of the 100-Ω resistive sensors. Because there is a different 100-Ω resistance for each pair of leads, the Johnson noise from this resistance is not correlated and consequently does not appear in the cross correlation spectrum as a potential offset error.

A schematic of the preamplifier appears in Fig. 4. The inputs to the differential FET pair are ac coupled and use a ferrite bead inductor to block rf frequencies. The anti-alias filter, as shown in the block diagram (Fig. 1), is a four-pole Bessel filter with a cutoff frequency at 2 MHz. Amplified and filtered signals are digitized at 50 MHz by a 14-bit analog-to-digital converter. Field-programmable gate arrays (FPGAs) at the output of the digitizers then digitally filter the signal with a low-pass frequency of 100 kHz. The digitally filtered data are transmitted via

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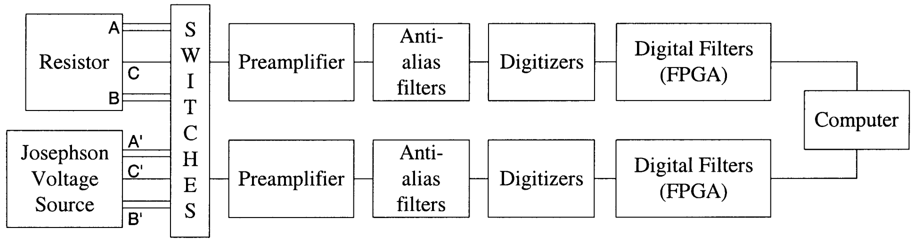

Fig. 1. Block diagram of the Johnson noise thermometry system. For both the resistor and the quantized (Josephson) voltage noise source, we use a five-wire (two signal pairs, A and B, and a common line, C) connection to the preamplifiers. The entire electronic system is battery-powered except for the computer. Connections to the computers are made with optical fibers.

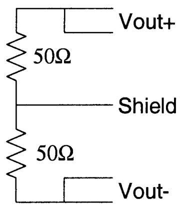

Fig. 2. Schematic of the five wire connection to a 100-Ω resistance.

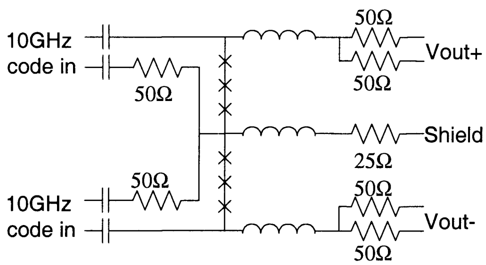

Fig. 3. Schematic of the QVNS. Xs represent Josephson junctions. The input capacitors are inner and outer dc blocks with cutoff frequencies from 10 to 250 MHz.

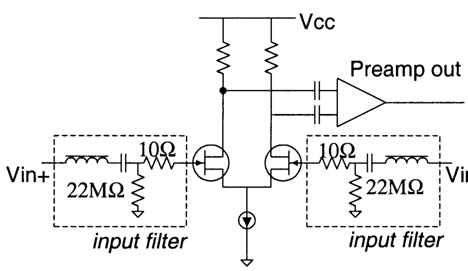

Fig. 4. Schematic of the preamplifier. An optional input filter is used to filter rf noise and remove dc offsets. The input inductor represents a ferrite bead impedance.

a 50 Mb/s optical link into a custom PCI card installed in a computer. Each channel transmits approximately 2.083 million samples/s, which is the effective sampling frequency of the signal. In the current system, a dual-CPU computer is used to calculate two 2^{21} point fast Fourier transforms (FFTs) in real-time (less than one second). The cross-correlation and auto-correlation power spectra are then calculated, accumulated, and stored for later analysis.

III. Results

A measurement spectrum with the correlation electronics using a QVNS-synthesized waveform appears in Fig. 5. We show three power spectra of a QVNS-generated single sine wave tone at approximately 3.2 kHz (second harmonic of the repetition frequency of the digital code [5]). Each spectra is averaged over 50 traces with the vertical axis in arbitrary units (digitizer bins). The upper two spectra are the auto-correlated spectra of each amplifier channel and overlap closely. The spectrum with the lowest noise floor is the cross-correlated power spectrum. The baseline noise of the cross-correlated spectrum continues to decrease as more spectra are accumulated. The frequency bins are spaced at $\sim$ 1.0 Hz. The QVNS was dc-coupled to the preamplifier (no input filters). The low-frequency knee at 100 Hz in the spectra is from the ac-coupling of the output stage of the preamplifier. The rolloff at 100 kHz is from the digital filters in the FPGAs. An important feature in these data is the set of points in the cross-correlation spectrum that are marked with solid circles. The circles indicate harmonics of the 1.6-kHz pattern repetition frequency of the pseudo-noise waveform synthesized by the QVNS. The presence of signals at harmonics of the fundamental is undesirable and would lead to systematic errors in the use of the QVNS as a calculable noise source. Further experimentation revealed that the amplitude of the additional harmonic power was inversely related to the desired amplitude of the tone being generated. This result is consistent with insufficient low pass filtering of the output of the QVNS for quantization harmonics above 10 MHz that mix down in the FET amplifiers. When input filters were inserted before the preamplifiers, the power at harmonics of the fundamental was significantly reduced. Fig. 6 shows the spectra from the QVNS when eight tones are synthesized. These data were taken by averaging 1000 spectra with input filters in the preamplifier stages. The hump at low frequencies is from the Johnson noise of the 22-MΩ resistor in the input filter not being

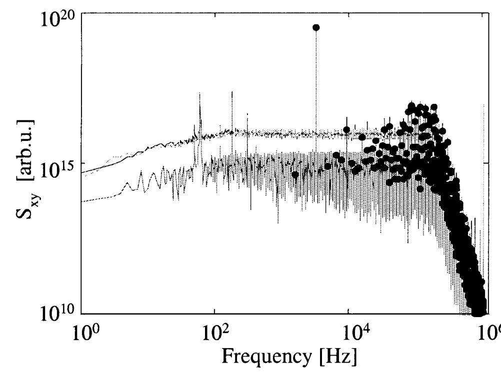

Fig. 5. Power spectra on a log-log scale of a QVNS synthesized sine wave. The upper two curves are noise spectra for each of the amplifier chains. The lower curve is the cross-correlated power spectrum. Black dots decorate the harmonics of the repetition frequency of the digital code.

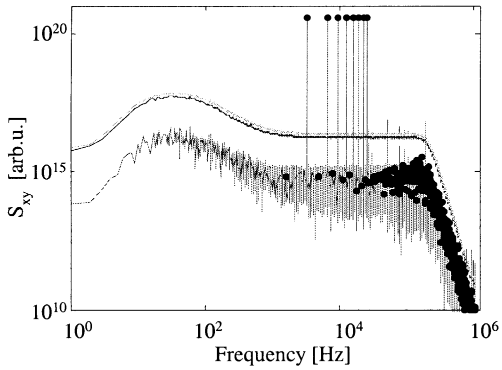

Fig. 6. Power spectra of a QVNS synthesized waveform of eight tones. The upper two curves are noise spectra for each of the amplifier chains. The lower curve is the cross-correlated power spectrum. Black dots decorate the harmonics of the repetition frequency of the digital code.

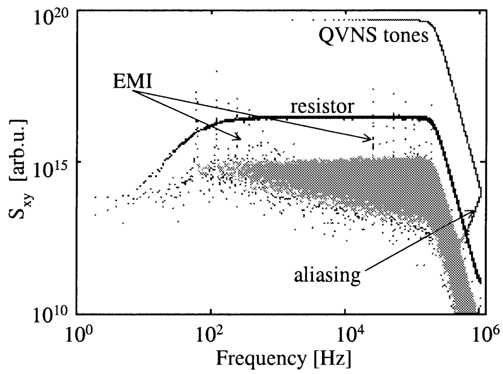

Fig. 7. Cross-correlation spectra of a QVNS pseudo-noise waveform and a 100-Ω resistor at the Ga triple-point. Each point represents a $\sim$ 1-Hz frequency bin. The electromagnetic interference (EMI) in this measurement is indicated by arrows.

filtered by the input ac-coupling capacitor at low frequencies. Clearly, the signals at harmonics of the repetition frequency have been significantly reduced.

We have used the QVNS in two different temperature measurement modes, absolute and relative. In the absolute measurement mode, the power spectral density of the QVNS signal is calculated from first principles and directly compared to the noise power from the resistor. Using (1), a thermodynamic temperature measurement of the resistor can thus be made without a fixed-point reference. In the relative measurement mode, the Johnson noise power at both a known temperature and an unknown temperature is balanced with two different QVNS synthesized noise powers. The ratio of the unknown to known temperature is then given by the ratio of the QVNS power spectral densities. This second method should be less sensitive to systematic errors. In this relative mode, we have achieved relative accuracy of a few hundred ppm and is described elsewhere [6]. In this paper, we present preliminary results while using the QVNS in the absolute mode.

Fig. 7 shows the cross-correlated spectrum from a 100-Ω resistor at the triple point of gallium ($T_{90} = 302.916$ K) and the spectrum of QVNS generated tones from 1.6 kHz to 2 MHz. The vertical axis is in arbitrary units (digitizer bins). The expected FFT and synthesis of this particular pattern is described in detail in [4]. Because of memory limitations in the digital code generator, it is not possible to generate tones at a significantly

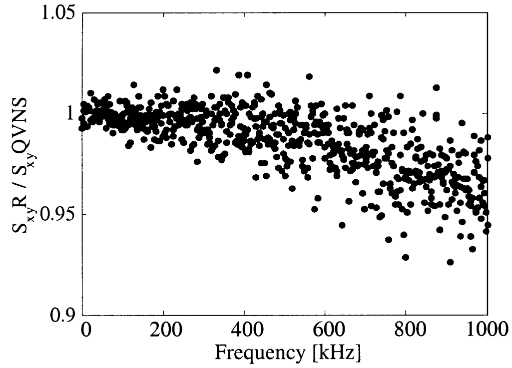

Fig. 8. Ratio of the cross-correlation spectra from the 100-Ω resistor at the Ga triple-point to the QVNS as a function of the tone frequencies in the QVNS waveform.

higher density (closer frequency spacing) and lower amplitude to more closely match the noise voltage of the resistor. To match power spectral densities, the amplitude of the QVNS at each harmonic is chosen so that the mean-squared voltage of each QVNS tone matches the resistor power spectral density integrated over a bandwidth equal to the 1.6-kHz repetition frequency around each tone. The rms amplitude of each QVNS tone is 51.55 nV. Consequently, the tones from the QVNS appear higher than the white noise ($\sim$ 1.293 nV/$\sqrt{\text{Hz}}$) of the resistor. An interesting artifact above the roll-off of the 100-kHz digital filters in this QVNS spectra is that the synthesized tones above 1 MHz are aliased into the spectrum below $\sim$ 1 MHz.

In the absolute temperature measurement mode, the temperature of the sensor is given by

$$T = \left(\frac{S_{xy}^{R}}{S_{xy}^{\text{QVNS}}} \right) \frac{S_{\text{calculated}}^{\text{QVNS}}}{4k_B R(T)} \tag{2}$$

where S_{xy}^{R} is the cross correlation measurement of the resistor noise, S_{xy}^{QVNS} is the measurement of the QVNS power, and $S_{\text{calculated}}^{\text{QVNS}}$ is the calculated power spectral density of the synthesized QVNS waveform. Fig. 8 shows the ratio of the power spectral density of the QVNS to that of a 100-Ω resistor at the Ga triple-point, plotted as a function of the QVNS tone frequencies. The data for this figure took a total of 10 min to acquire. Because the amplitude of the QVNS tones where chosen to match the noise power spectral density of a 100-Ω resistor at the Ga triple point, we expect the ratio to be close to unity for all frequencies. By averaging the ratio for frequencies in the 100 kHz passband of the digital filter we can infer that the temperature of the resistor is $T = 302.5\ \text{K} \pm 0.3\ \text{K}$ (1σ, based on the uncertainty from the cross-correlation). Possible sources of systematic error include transmission line differences between the QVNS cables and the resistor cables, nonohmic contacts in the circuit measuring the QVNS signals, mixing of QVNS-generated tones above 10 MHz, and poor rf grounding of the code generator input to the QVNS. It is also necessary to more carefully bound the errors arising from the digital signal processing, due to aliasing, windowing, quantization errors, and computing FFTs of nonperiodic QVNS signals (e.g., period of the fundamental is not a divisor of the number of time samples taken per spectrum). Another potential source of error is that the gain of the electronics system as a function of frequency may be changing more rapidly in frequency than the spacing of the QVNS tones.

IV. Conclusion

In summary, we are developing a new type of Johnson noise thermometer that uses a quantized voltage noise source as a calibration reference for the readout electronics. We have measured the noise spectra of both a sense resistor and a Josephson array waveform synthesizer using recently constructed cross-correlation electronics. Preliminary data indicate that the frequency response of the electronics as measured by the QVNS tones is different from that of the resistor noise leading to a systematic error in the temperature measurement on the order of two parts in 1000. Further work is being done to understand the potential systematic errors in the "absolute" mode of operation for our QVNS-based JNT system. With improvements, such as developing an arbitrary bit stream generator with more memory to generate waveforms more closely matched to resistor noise spectra, in the future we expect to measure temperatures with relative accuracies of one part in 10^5 for temperatures between 270 K and 1000 K.

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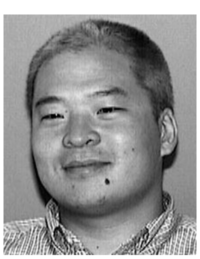

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Dr. Martinis is a Fellow of the American Physical Society and a Fellow of NIST.