The Characteristics of a Linear Antenna
With Tapered Resistive and Capacitive Loading

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I. Introduction

In 1965, Wu and King [1] theoretically analyzed a linear antenna with continuous complex impedance loading, and predicted that the current distribution on such an antenna could be represented by an outward traveling wave if the complex impedance loading on the antenna were properly chosen. The required loading given by Wu and King has the form of

$$z'(z) = \frac{60}{h} \frac{\psi}{|z|}$$

(1)

where

$$\psi = 2[sinh^{-1} \frac{h}{a} - C(2ka, 2kh) - jS(2ka, 2kh)] + \frac{j}{kh} (1 - e^{-j2kh}) \ .$$

(2)

2h and a are respectively the length and the radius of the linear antenna, and C and S are respectively the generalized cosine and sine integrals. As shown in Fig. 1, the real part of $\psi$ is rather frequency insensitive, whereas the imaginary part of $\psi$ is negative and proportional to frequency [5]. It is found that, at frequencies where $kh < \pi/2$, the imaginary part $\psi$ is generally smaller than its real part [5]. For this reason, Shen [2] and Lally and Rouch [3] as well as the author [4,5,6,7] have used only tapered resistive loading.

It should be noted, however, that the imaginary part of $\psi$ becomes comparable to its real part near frequencies where $kh = \pi/2$, and above frequencies where $kh > \pi/2$, the imaginary part of $\psi$ becomes even larger than its real part [5] as indicated in Fig. 1. Therefore, to create an outward traveling-wave current distribution on a linear antenna near and above frequencies where $kh > \pi/2$, the required impedance loading along the antenna is believed to be complex with resistance and capacitive reactance. For this reason, the characteristics of a linear antenna with resistive and capacitive loading are investigated theoretically and experimentally in this paper.

II. Theory

From the well-known, one-dimensional wave equation, the current distribution $I(z)$ on a linear antenna with tapered resistive and capacitive loading can be calculated by solving the following integral equation:

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\[
\frac{1}{4\pi} \int_{-t}^{t} I_z(z') \frac{e^{-jk\sqrt{(z-z')^2 + a^2}}}{\sqrt{(z-z')^2 + a^2}} \, dz' \\
= C \cos kx + \frac{jwC}{k} \int_{0}^{\infty} \left[ V_0 \delta(z-z') - z(z') I(z') \sin k(z-z') \right] dz.
\]

The constant C is to be determined from the boundary condition,
\[
I_z(t) = I_z(-t) = 0.
\]

For a linear antenna with a length of \(2h = 7.5\) cm and a radius of \(a = 0.3\) cm, the \(\psi\) in (2) becomes \(3.06 - j2.39\) at \(2\) GHz where \(kh = \pi/2\). To implement the real part of impedance loading given in (1), the antenna element was made by depositing a tapered thin-film alloy on a glass rod [5]. To achieve the imaginary part of impedance loading given in (1), the deposited thin-film alloy on the glass rod was cut by an argon laser to form a segmented antenna as shown in Fig. 2. The neighboring faces of a gap on this segmented antenna is considered as a lumped capacitance inserted in series with the antenna. Of course, as the number of segments per wavelength becomes large, the antenna can be considered as loaded with continuously distributed capacitance. The gap width, \(t\), between segments is chosen so that the linear antenna has a proper tapered capacitive reactive loading as given in (1).

\[
\sum_{m} z^i(z) = -\frac{60 \times 2.39}{h - |z|} = -\frac{3}{\omega d C_t},
\]

where the distance, \(d\), between two adjacent gaps is chosen to be \(5\) mm and the lumped capacitance of each gap \(C_t\) is given by \(\tau a^2 \varepsilon / t\). The required lumped capacitance of each gap and corresponding gap width so chosen are given in Table 1.

The integral equation given in (3) was solved using the method of moments. To permit a solution for the current distribution, suitable testing and basis functions are defined. In this study, piecewise-linear testing functions and piecewise-constant basis functions for expansion were chosen. In the method of moments, the current distributions are calculated by subdividing the linear antenna into 61 subsections.

III. Characteristics of a Linear Antenna with Tapered Resistive and Capacitive Loading

Fig. 3 shows the current distributions of a linear antenna with tapered resistive and capacitive loading. For comparison, the current distribution of a linear antenna with tapered capacitive loading only is also shown. Since the current distributions of the antenna are found to be basically traveling waves, the antenna is expected to have very broadband characteristics. The impedance of the antenna is shown in Fig. 4. The periodicities of the resistance and the reactance of
the antenna at the driving point were eliminated. The impedance of
the antenna varies, in general, monotonically for the broad frequency
range.

Using the current distributions of the antenna, the far-field
electric fields are calculated by

\[ E_\theta (r) = \frac{j\omega \sin \theta}{4\pi} \int_{-h}^{h} \frac{I (z')}{\sqrt{r^2 + z'^2 + 2rz' \cos \theta}} \, dz'. \]  

The far-field radiation patterns shown in Fig. 5 are found to be very
similar to those of an electrically short linear antenna. The
experimental results for the receiving transfer function of the linear
antenna with resistive and capacitive loading are shown in Fig. 6. 
Here, the receiving transfer function is defined as a ratio of the
output voltage of the antenna to the normal incident electric-field
strength. Fig. 6 indicates that the antenna does not exhibit any high
Q resonances at frequencies where \( k \theta = \pi/2 \).

References

California, June 1977.
[6] M. Kanda, International/AP-S Symposium, University of Maryland,
MD, May 1978.

![Fig. 1 (a) Real part of \( \psi \) (2\( h \)=7.5 cm, a=0.3 cm)](image1a)

![Fig. 1 (b) Real part of \( \psi \) (2\( h \)=7.5 cm, a=0.3 cm)](image1b)
Fig. 2 A linear antenna with tapered resistive and capacitive loading.

Fig. 3 Current distribution.

Fig. 4 (a) Antenna resistance.

Fig. 4 (b) Antenna reactance.

Table 1. Capacitive loading profile.

<table>
<thead>
<tr>
<th>Position Along Antenna (m)</th>
<th>Gap Width (m)</th>
<th>Lumped Capacitance (μF)</th>
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<tr>
<td>0.5</td>
<td>9.0 x 10⁻⁵</td>
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<tr>
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<td>3.8</td>
</tr>
<tr>
<td>2.5</td>
<td>1.3 x 10⁻⁴</td>
<td>2.7</td>
</tr>
<tr>
<td>3.0</td>
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