Finite coplanar waveguide width effects in pulsed inductive microwave magnetometry

M. L. Schneider, A. B. Kos, and T. J. Silva
National Institute of Standards and Technology; Electromagnetics Division, 325 Broadway, Boulder, Colorado 80305

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The effect of finite coplanar waveguide (CPW) width on the measurement of the resonance frequency in thin ferromagnetic films has been characterized for pulsed inductive microwave magnetometry. A shift in resonant frequency is a linear function of the ratio of sample thickness to CPW width. The proportionality constant is experimentally determined to be 0.74±0.1 times the saturation magnetization of the film. The frequency shift may be modeled as arising from an effective magnetic-anisotropy field. [DOI: 10.1063/1.1769084]

The switching speed of fundamental magnetic components in disk drives will need to greatly increase if the current trend in hard disk data transfer rates continues. This increase in required bandwidth has stimulated recent research on the high frequency susceptibility and magnetic anisotropy of thin magnetic films. One promising method of investigating magnetic anisotropy of thin magnetic films is the use of a pulsed inductive microwave magnetometer (PIMM). This method has the advantage of yielding time-domain data on the precessional motion of the magnetization. In combination with classic magnetostatic measurements, a PIMM can provide information on the uniaxial anisotropy, the effective magnetization, the Landé g factor, and the Gilbert damping parameter.

One complication in using the PIMM is the influence of the finite width of the coplanar waveguide (CPW). In this letter, we experimentally determine the influence of the finite CPW width on the resonance frequency of magnetic precession. A frequency shift is observed that may be explained by the excitation of a distribution of Damon-Eshbach magnetic modes. A model for the correction factor agrees within 6% of our experimentally measured value.

The samples used in this study were polycrystalline Ni$_{81}$Fe$_{19}$ Permalloy films with thicknesses of 25, 50, 75, and 100 nm. The Permalloy was deposited via dc magnetron sputtering in a 0.533 Pa pressure Ar atmosphere. The sputtering system had a base pressure of 10$^{-6}$ Pa. The substrates were 1 cm × 1 cm × 100 μm coupons of (0001)-oriented sapphire and 1 cm × 1 cm × 150 μm glass coupons, and were prepared in situ Ar/O$_2$ ion milling prior to deposition. A 5 nm seed layer of Ta was deposited onto the cleaned substrates to enhance the adhesion of the Permalloy films. An external magnetic field of 20 kA/m was applied along the easy axis of the sample, and another bias field ranging from 1592 to 7958 A/m was applied parallel to the easy axis of the sample, and another set of data was acquired. After this, a bias field ranging from 1592 to 7958 A/m was applied parallel to the easy axis of the sample, and another set of data was acquired. The two data sets were subtracted, and the resonance frequency, as determined from the zero crossings of the real part of a fast Fourier transform of the acquired data, was found. The magnetic resonance frequency ranged from 1.5–3 GHz and was well within the measurable bandwidth of the system.

The effective resonance frequency $\omega_{\text{eff}}$ of the magnetic excitation is the result of a distribution of surface magnetostatic modes for a slab of thickness $\delta$. In the long wavelength limit $k\delta \ll 1$ and weak magnetic fields $H_b + H_k \ll M_s$, the dispersion relation for the characteristic frequency $\omega_0$ of such modes is given by

$$\omega_0(k) = \gamma \mu_0 \sqrt{M_s \left( H_b + H_k + \frac{M_s}{2} k \delta \right)},$$

where $|\gamma| = (g \mu_B) / h$ is the gyromagnetic ratio, $g$ is the spectroscopic splitting factor, $\mu_B$ is the Bohr magneton, $\mu_0 = 4\pi \times 10^{-7} (H/m)$ is the permeability of free space, $M_s$ is the saturation magnetization, $H_b$ is the applied field, and $H_k$ is the in-plane anisotropy field.

Figure 1 shows resonance frequency squared, $f_0^2 = (\omega_0 / 2 \pi)^2$, versus applied longitudinal bias field $H_b$ measured on a 100 nm thick Permalloy film for the three different CPW widths $w$. We estimate the error in our data to be 5 MHz, which, on the graph, is smaller than the size of the data points. We extrapolated the linear fit of the data to $f_0^2 = 0$ to find $H_k^0$ for the Permalloy thin films.
CPWs. Note that the value of $H_k$ different samples, each measured on the three different geometries in the limit of a purely linear response. Assuming CPWs are inhomogeneous.

Figure 2 shows the measured values for $H_k^{\text{eff}}$ for four different samples, each measured on the three different CPW widths. Note that the value of $H_k^{\text{eff}}$ increases as the ratio of sample thickness $d$ to CPW width $w$ is increased. The change in the value of $H_k^{\text{eff}}$, when using a narrow CPW with a wide sample, arises because the excitation fields generated by the CPW are inhomogeneous.

It is important to quantify the effect of field inhomogeneity in the limit of a purely linear response. Assuming $M$ is initially oriented perpendicular to the direction of the excitation field, and assuming the small angle motion of $M$, the magnetic excitation precession frequency is given by Eq. (1).

The susceptibility spectrum for each mode is found by linearizing the Landau-Lifshitz equation for damped gyromagnetic precession in a ferromagnetic film. In the limit of small-angle motion of $M$ and large $M$, the resultant equations of motion are those of a harmonic oscillator. Using Fourier analysis, the susceptibility for a given mode with characteristic frequency $\omega_0$ is

$$\chi(\omega, \omega_0) = \chi_0(k) = \frac{\omega_0^2}{\omega_0^2 - \omega^2 - i(2\alpha\omega)}.$$

where $\tau \sim 1/(2\alpha\gamma \mu_0 M)$. $\alpha$ is the damping parameter where $\alpha \approx 0.01$ for all samples, and $\chi_0(k)$ is the dc susceptibility for a given mode of wave number $k$. The susceptibility for a mode may be found by inspecting Eq. (1), where the dipole fields produced by the sinusoidal magnetization distribution act as an additional anisotropy term that increases the resonance frequency. Therefore,

$$\chi_0(k) = \frac{M_s}{H_k + H_k + (M_s/2)k^2}.$$

The spacing between the sample and CPW surface is $d$. For a thin magnetic film ($d \ll L$) of dimension $L$ that is large compared to the CPW width ($L \gg w$), the detected flux per mode is

$$d\Phi(\omega) = \frac{\mu_0 L d \delta}{4\pi} \sin^2 \left(\frac{k w}{2}\right) \sin \left(\frac{k d}{2}\right) \chi(k, \omega) dk,$$

where we have used reciprocity and the Karlovqvist equation to determine the spectral sensitivity function of the CPW and the inductive voltage is $V(\omega) = i\omega\Phi(\omega)$.

So far, we have ignored the temporal frequency spectrum of the exciting field pulse. For excitation with a perfect step function, the spectral distribution of the excitation is weighted by a factor of $1/\omega$. Thus, the final result for the measured inductive voltage in the case of a step excitation is

$$V(\omega) = \frac{\mu_0 L d \delta}{8\pi} \int_0^{\pi/2} \sin^2 \left(\frac{k w}{2}\right) e^{-2kd} \chi(k, \omega) dk.$$

Here, we have divided by an additional factor of 2 in order to account for the CPW symmetry, which divides the inductive voltage between two counterpropagating signals that radiate away from the sample.

The ramifications of the multimode excitation with a narrow CPW are twofold. First, the peak in the measured susceptibility spectrum is shifted toward higher wavelength magnetostatic surface modes. Second, the resonance peak is broadened due to the superposition of a distribution of resonance lines. There is evidence that this line broadening due to the finite CPW width may, in general, have an effect on the resonance frequency shift by approximating the integral in Eq. (4), since $\sin^2(x)$ falls off rapidly for $x > \pi/2$. Furthermore, we are operating in the long wavelength limit, where $kd \ll 1$. Thus we are left with

$$V(\omega) = \frac{\mu_0 L d \delta}{8\pi} \int_0^{\pi/2} \chi(k, \omega) dk.$$

The most important factor in determining the effective peak in the resonance spectrum for the collection of modes is the distribution of the dc susceptibilities in Eq. (2). This results from the fact that the functional shape of the spectrum in Eq. (1) does not strongly depend on the resonance frequency, with the exception of the position of the resonance. Thus, the average resonance frequency $\langle \omega_0 \rangle$ for an ensemble of oscillators with relative strength $\chi_0(k)$ is given by
which is 94% of the calculated value of $p$ all known values; thus we define experimentally check the value of the constant thickness over CPW width. Equation (\ref{eq:HL}) used the plots of Fig. 2. In Fig. 2 we plot the measured To check our model with experimental measurements, we that arising from an additional anisotropy component $H_k$ for the uniform mode $s$ depends on the CPW width. We thus use the slope $C_w M_s$ to experimentally determine the constant shift in the value of $H_k$ as a function of sample thickness over CPW width. We found good agreement between our experimental results and a model based on a linear superposition of magnetostatic surface modes in the small-angle limit.

In summary, CPW width has an effect on the measure-ment of $H_k$ when using a PIMM. This change in the measured $H_k$ is due to the nonuniform field created by a thin CPW structure. The effect of the nonuniform field on a thin film has been modeled and it was found that the change in $H_k$ is a linear function of the ratio of sample thickness to CPW width. We performed several measurements on thin Permalloy films using a PIMM in the small-angle regime, and experimentally determined the constant shift in the value of $H_k$ as a function of sample thickness over CPW width. We found good agreement between our experimental results and a model based on a linear superposition of magnetostatic surface modes in the small-angle limit.

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\begin{equation}
\tilde{\chi} = \frac{\pi \delta \bar{\chi}(k=0)}{2w}.
\end{equation}

where $\bar{\chi} = \frac{\omega (k=\pi/w)}{\omega (k=0)}$. (\ref{eq:HL}) establishes a way to experimentally measure the additional anisotropy component $H_k^\text{MSSW}$ for the uniform mode $(k=0)$. This limiting behavior for the frequency shift has been verified to be valid up to $2\tilde{\chi}/\pi \approx \pi$ by numerical integration of Eq. (\ref{eq:HL}).

Equation (\ref{eq:HL}) allows to use experimentally determined the constant shift in the value of $H_k$ as a function of sample thickness over CPW width. Equation (\ref{eq:HL}) is an additional anisotropy term that is added to the $H_k$ term in Eq. (1). Note that this additional anisotropy term is the only one that depends on the CPW width. We thus use the slope $C_w M_s$ to experimentally check the value of $C_w$.

Figure 3 shows our experimentally measured values for the constant $C_w$ as a function of film thickness. The data are shown for three different excitation voltages. There is no apparent dependence of $C_w$ on the excitation voltage. This helps affirm that we are in the small angle limit in all of our measurements, as assumed in the model. There is no clear trend in $C_w$ as a function of sample thickness, as expected. Averaging the data at all voltages, and for all values of sample thickness, yields a mean value for $C_w=0.74\pm0.1$, which is 94% of the calculated value of $\pi/4$.

In summary, CPW width has an effect on the measurement of $H_k$ when using a PIMM. This change in the measured $H_k$ is due to the nonuniform field created by a thin CPW structure. The effect of the nonuniform field on a thin film has been modeled and it was found that the change in $H_k$ is a linear function of the ratio of sample thickness to CPW width. We performed several measurements on thin Permalloy films using a PIMM in the small-angle regime, and experimentally determined the constant shift in the value of $H_k$ as a function of sample thickness over CPW width. We found good agreement between our experimental results and a model based on a linear superposition of magnetostatic surface modes in the small-angle limit.

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