Abstract
The output of a femtosecond fiber laser provides a comb of lines in frequency space that can be phase-locked to either a microwave or optical reference to form a stable frequency comb. We discuss the basic configuration of fiber laser-based frequency combs, the underlying physics behind the stabilization of the comb, and some applications of fiber-laser frequency combs to remote sensing and to measurements of the stability and phase noise of optical sources at 1550 nm.

Introduction
The development of frequency combs has led to a revolution in optical frequency metrology.\textsuperscript{1-4} Figure 1 shows the basic picture of a frequency comb. It is a set of well defined, evenly-spaced, lines in optical frequency space. The frequency of each line is defined directly in terms of the frequency of a reference oscillator that can be either a microwave oscillator or an optical oscillator. If a microwave reference oscillator is used, then the comb provides the basic, but previously extremely challenging, function of linking optical frequencies directly to microwave frequencies. If an optical reference oscillator is used, the comb provides the basic function of linking that optical frequency directly to other optical frequencies or to microwave frequencies. The importance of frequency combs to atomic clocks and precision spectroscopy is an important factor in the award of the Nobel prize to J. Hall and T. Hänsch in 2005.

The initial development of frequency combs was based on use of femtosecond, solid-state Ti:Sapphire lasers,\textsuperscript{1-4} and these systems continue to exhibit the highest levels of performance.\textsuperscript{5, 6} However, in recent years, frequency combs using femtosecond, fiber lasers have been developed.\textsuperscript{7-13} As they mature, their performance is approaching that of Ti:Sapphire based systems.\textsuperscript{13} The interest in fiber laser frequency combs stems from several aspects. First, they can be relatively inexpensive. As a result they are much more attractive for widespread use and there are several commercial products available. Second, they can be relatively compact and robust, since “all-fiber” systems are possible. Third, they cover a wavelength range of 1 to 2 μm, which covers the transparent window of fiber optics. As a result, fiber-laser frequency combs are compatible with the wide range of telecommunication components and Er-based amplifiers. Moreover, the output can be transmitted long distances over optical fiber (although not without impairment of the optical phase stability.)

\[ f_n = nf_{\text{rep}} + f_0 \]

Fig 1: Frequency comb output from a femtosecond fiber laser. The laser output spans ~80 nm directly or 1000 nm after broadening in nonlinear fiber. The comb teeth are separated by the repetition rate of \( f_{\text{rep}} \sim 50 \text{ - } 375 \text{ MHz} \), depending on the fiber laser. The position of the \( n \)th mode is given by \( f_n = nf_{\text{rep}} + f_0 \), where \( n \) is an integer that is ~4x10^6 for the comb line near 1550 nm.
The main challenge in stabilizing the frequency comb from a femtosecond fiber laser (or any femtosecond laser) is the detection of the offset frequency, $f_0$. (In contrast, the repetition frequency, $f_{\text{rep}}$, is easily detected by shining the output onto a photodetector.) The basic method for detecting the offset frequency is through “self-reference” detection with an f-to-2f interferometer, although other methods are possible. This self-referenced detection requires broadening the laser output to form a supercontinuum that spans an octave, i.e., factor of two, in frequency. The long-wavelength end is then doubled and beat against the short-wavelength end, yielding a beatnote at $f_0$. The difficulty lies in generating from the laser an octave of bandwidth that can provide a $f_0$ beat signal with sufficient signal-to-noise ratio. The first attempt at stabilizing the frequency comb output of a fiber laser essentially circumvented this problem by using a second Ti:Sapphire frequency comb. However, since the first observations of the offset beat, all current fiber laser frequency combs are based on this self-referencing technique.

Below, we first describe the fiber-laser frequency comb and then briefly discuss some of the applications, including basic frequency metrology in support of optical clocks, metrology of narrow-linewidth lasers, and remote sensing.

**Configuration**

Fiber laser frequency combs are based on passively mode-locked fiber lasers. A number of different laser designs have been successfully used including figure-8 laser, soliton or stretched-pulse fiber ring laser, and Fabry-Perot laser using a saturable absorber. The exact configuration of the laser does not appear to be important, although it may affect the feedback bandwidth. All these lasers put out ~100 fs pulses of laser light at ~0.1 nJ of energy and repetition rates of ~100 MHz. The spectral width of the pulses can range from 20 nm to 80 nm depending on the laser configuration. As discussed above, a full octave of bandwidth is needed to detect the offset frequency; therefore, this output must be externally spectrally broadened. Because the pulses are so short, the peak powers are significant and one would expect significant spectral broadening in optical fiber due to nonlinear effects. However, for current nonlinear fibers the pulse peak powers are not sufficient to generate a full octave of bandwidth. Therefore, the laser output is first amplified in a dispersion-managed erbium fiber amplifier. The output of the amplifier is then injected into a special highly nonlinear optical fiber that generates an octave-spanning supercontinuum. The development of this optical fiber was a critical step toward achieving fiber laser frequency combs.

![Fig 2: Basic schematic of a fiber laser frequency comb. The femtosecond fiber laser output is amplified and spectrally broadened in highly nonlinear fiber (HNLF). The offset frequency is phase-locked to a microwave reference. The remaining degree of freedom of the comb can be stabilized through (1) phase-locking the repetition rate to a microwave reference or (2) phase-locking one tooth of the comb to an optical reference (shown in gray). Solid lines represent fiber optic paths and dashed lines represent electronic signals.](image-url)
Once the supercontinuum is generated, the offset frequency \( f_0 \) is detected by doubling the \(~2 \mu m\) portion of the comb and heterodyning it against the \(~1 \mu m\) end of the comb to generate an rf beat signal at \( f_0 \). This signal is phase-locked to an rf synthesizer by feeding back to the pump laser power. In initial efforts, the offset beats signal, \( f_0 \), was extremely broad – on the order of 100 kHz or more. While the phase-locking ensured that the average value of \( f_0 \) remained fixed with respect to the reference, the broad linewidth indicated that on short time scales there was significant jitter on \( f_0 \). It turns out that this jitter arose from intrinsic amplitude noise on the diode lasers that pump the femtosecond fiber laser. In principle, the feedback to stabilize the offset frequency ought to remove this amplitude noise by applying the appropriate correction to the current supply driving the pump diodes. However, the amplitude noise was too large to be totally removed by the simple proportional-integral feedback. Recently we found that by operating the pump diode at high currents, its amplitude noise is considerably reduced. Moreover, by using phase-lead compensation (i.e., derivative feedback) in the feedback loop the feedback bandwidth can be extended considerably further. The end result is a tightly phase-locked offset frequency with \(~1\) radian phase excursions from the reference oscillator (corresponding to an instrument-limited linewidth). \(^{16}\)

The stabilization of the offset frequency holds the comb fixed in terms of translational motion (see Fig. 1); however the spacing between the comb teeth is still free to change. There are two alternatives to fully stabilizing the comb. First, one can phase-lock the repetition rate to a microwave reference by feeding back to the cavity length (see Fig. 2) through piezoelectric fiber stretchers. In that case, the comb frequencies are given by \( f_n(t) = n f_{\text{rep}}(t) + f_0(t) \), where we explicitly allow the frequencies to have some slow variation with time, \( t \). This allows one to define the optical frequencies in terms of the microwave reference and is therefore useful for frequency metrology. However, the instantaneous noise on the comb lines can then be quite large. The corresponding frequency noise power spectral density (PSD) as a function of Fourier frequency, \( \nu \) (conjugate to \( t \)), is \( S_n(\nu) = n^2 S_{\text{rep}}(\nu) + S_0(\nu) \). Here, the PSD, \( S_n(\nu) \), is defined as the Fourier transform of the slow time variations in the squared \( f_n(t) \). The frequency noise PSD \( S_{\text{rep}} \) and \( S_0 \) in terms of the variations in the squared \( f_{\text{rep}}(t) \) and \( f_0(t) \), respectively, are defined similarly. The frequency noise on the offset frequency and repetition frequency, \( S_{\text{rep}} \) and \( S_0 \), can include both the noise on the microwave reference and noise resulting from imperfect phase-locking of the comb to the reference. Since \( n \sim (200 \text{ THz/50 MHz}) \sim 4 \times 10^6 \), any frequency noise on the repetition frequency is “multiplied up” by \( n^2 \sim 10^{13} \), which is a very large number. As a result, the optical comb lines will be significantly broadened.

Alternatively, the second method of phase-locking the comb involves stabilizing one tooth of the comb to an optical reference, i.e., a narrow stabilized laser.\(^{6,20}\) This approach is shown as the grey line in Fig. 2. The optical reference laser might be stabilized to an optical cavity if only short-term stability is important. It might also be the output of an optical atomic clock if absolute stability is important. If the comb tooth \( n = n_{\text{ref}} \) is locked to the reference laser with frequency \( f_{\text{REF}} \) with an offset of \( f_0 \), the optical comb frequencies are given by \( f_n(t) = (n/n_{\text{ref}})(f_{\text{REF}}(t) - f_0(t)) + (1-n/n_{\text{ref}})f_0(t) \). The phase noise (again ignoring correlations between \( f_{\text{ref}} \) and \( f_0 \)) is now \( S_n(\nu) = (n/n_{\text{ref}})^2 S_{\text{REF}}(\nu) + (1-(n/n_{\text{ref}})^2)S_0(\nu) \), where \( S_{\text{REF}}(\nu) \) represents any frequency noise on the optical reference or on the phase-locking to the optical reference. The coefficients multiplying \( S_{\text{REF}}(\nu) \) and \( S_0(\nu) \) are now on the order of unity. For a sub-hertz stabilized optical reference, \( S_{\text{REF}}(\nu) \) is low, provided that a sufficiently high bandwidth piezo-electric fiber stretcher is used to phase-lock the fiber laser to the optical reference. In that case, very narrow lines can be observed from a fiber laser frequency comb. With IMRA America, we have...
demonstrated instrument-limited residual linewidths of less than 100 mHz across the comb spectrum. In essence, by phase-locking the comb to an optical reference, we can transfer the stability and low phase noise of a single optical reference all the way across an optical span of 1 to 2 microns. Additionally, the repetition frequency of the comb is now extremely stable with a projected residual timing jitter of ~ 1 fs or less. This high phase coherence and low timing jitter can be exploited in a number of applications.

**Metrology Applications:**

The basic application of the fiber-laser frequency comb to metrology involves heterodyning the laser-under-test with the frequency comb. The resulting beat signal is a measure of the frequency difference between the laser frequency, $f_{\text{laser}}$, and the frequency of the $n$th comb tooth, $f_n$. Assuming $n$ is known (and there are methods for determining $n$), the absolute frequency and stability of the laser can be determined with an uncertainty matching that of the comb tooth. If the comb is phase-locked to a microwave reference that is in turn stabilized to a time standard, then the absolute laser frequency can be determined to hertz level accuracy. This accuracy is seven orders of magnitude better than what can be achieved with a state-of-the-art wavelength meter. Experiments have demonstrated that very high levels of stability can be achieved with fiber laser frequency combs.$^{12, 13}$

In some cases, the absolute laser frequency is of less interest than the linewidth or phase noise on the laser under test. In that case, the frequency comb can be phase-locked to a narrow stable optical reference. The beat signal then gives a measure of the absolute linewidth of the laser under test over the given observation time. By demodulating the beat signal with a fast Fourier transform (FFT) device, one can quantify the phase (or frequency) noise spectrum of the laser under test. This approach, which is only just now being developed at NIST, would permit the characterization of the phase noise of optical sources in a quantitative way that is consistent with the current characterization of microwave sources.

**Remote Sensing Applications:**

One advantage of the fiber-laser frequency combs is that the output can be amplified with standard, commercially-available Er-doped fiber amplifiers. The possibility then arises of generating a high power, highly phase-coherent beam useful for remote sensing applications. The high bandwidth of the source implies a very high potential range resolution, while the narrow linewidths imply good Doppler, or vibrational, sensitivity at long ranges. In order to achieve the most sensitivity for ranging experiments, and to achieve any sensitivity for Doppler measurements, one is driven toward a coherent LIDAR system. In coherent LIDAR a strong
laser beam is scattered by a surface. A small fraction of the scattered signal is then collected and heterodyned with a local oscillator (LO), which is typically a frequency-shifted, delayed copy of the original transmitted pulse. For ranging applications with a pulsed transmitter, a range profile of the target is measured from the strength of the heterodyne signal as a function of the LO frequency. For vibrometry applications, the relative longitudinal velocity of the source and target is monitored from the frequency shift of the heterodyne signal.

The difficulties with developing a coherent LIDAR with the femtosecond fiber laser are threefold. First, as with any coherent LIDAR system, the return signal will suffer from speckle, which limits the signal-to-noise ratio for both ranging and vibrometry measurements. Second, the short pulse length of the transmitted signal will provide excellent range resolution, but only if the delay of the LO is precisely controlled. Third, any differential dispersion in the signal arm will degrade the range resolution accordingly. Recently, we devised a frequency-resolved coherent LIDAR (FReCL) system that effectively deals with these issues. The basic setup is shown in Fig. 4 below. By spectrally resolving the returning heterodyne signal we achieve three things. First, we can mitigate the effects of speckle on vibration measurements by incoherently averaging the signals from the N distinct detectors. This speckle-averaging reduces the variance in the Doppler measurement by a factor of N. Second, we can use a fixed delay since the effective delay of the LO can be scanned in post-detection software by applying the appropriate phase shift to the N signals. Third, we can remove any differential dispersion in the signal arm by applying the appropriate phase correction to the N signal channels.

![Fig 4: Frequency-Resolved Coherent LIDAR setup. For our laboratory setup, we used a rotating aluminum disk as the “target”. The frequency comb in this case was free-running. The return signal was spectrally resolved with an arrayed waveguide grating (AWG). We used six discrete detectors at a 4.8 nm spacing in order to span the 25 nm-wide output pulse.](image)

An example of the range images are given in Fig. 5 below. Fig. 5 compares the results obtained with a conventional setup (without spectrally resolving the return signal) to the results obtained with the FReCL approach. The images are identical, except that the FReCL images are acquired N-times faster, and without physically scanning the delay in the LO path. In these experiments, a single femtosecond fiber laser was used as the transmitter and local oscillator. As a result, the round-trip time to the target needed to be very close to an integer multiple of the laser repetition rate. (In other words, the software scanning of the delay can cover a range of only ~ 1.5 mm, corresponding to the bandwidth of the individual AWG channels.) However, with the recent results in phase-locking to fiber frequency combs to a single optical reference, one can envision a separate fiber laser frequency comb for a transmitter and local oscillator. In that case, arbitrary ranges could be accommodated.
Fig. 5. Range image of the wobbling, rotating disk for one full rotation (0.12 sec) with a 10 ms averaging time. (a) Truth data from conventional system at 20 LO delays. (b) Conventional data for a single delay. (c) FReCL data at the same LO delay. $T_A$ is the total acquisition time.

Conclusion

The output of femtosecond fiber lasers can be stabilized to either a microwave or an optical reference to form a frequency comb. The compatibility of fiber-laser frequency combs with telecommunication equipment should lead to a broad range of applications including frequency metrology, fiber transport of time and frequency, and remote sensing.

References: