CHALLENGES IN THE CALIBRATION OF POLARIMETRIC RADAR CROSS SECTION MEASUREMENT SYSTEMS

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ABSTRACT

We examine the calibration of polarimetric radar cross section (RCS) measurement systems using rotating dihedral and cylinder targets. Stringent requirements on the polarimetric system’s stability, dynamic range and sensitivity as functions of the cross-polarimetric parameters present several measurement challenges. We must consider the system sensitivity over the required dynamic range in the context of the overall system uncertainty, which might need to be improved to successfully complete a polarimetric calibration.

The minimum cross-polarimetric signal-to-noise and signal-to-drift ratios must be accurately characterized. In state-of-the-art radar cross section systems the signal-to-noise requirement is probably satisfied. However, the presence of very small drifts will invalidate a polarimetric calibration and must be removed from the data. A cylinder preserves the transmitted signal’s cross-polarization and can provide drift-free measurements. Hence, we propose a new polarimetric calibration procedure that combines calibration data from both a dihedral and a cylinder to obtain the system parameters.

Key words: drift, dynamic range, polarimetric calibration, radar cross section, sensitivity

1. Introduction

In polarimetric calibration we want to determine the system cross-polarimetric parameters $\epsilon_q$, where the subscript $q$ designates either the horizontal $h$ or vertical $v$ polarization of the transmitting channel [1–5]. For ideal systems with perfect channel isolations and which have been perfectly aligned, the $\epsilon_q$ vanish. Real systems will have finite cross-polarimetric parameters that, however, could be very small. This poses a measurement challenge: the overall system uncertainty [6] must be smaller than the the target-dependent system polarimetric factor, which we define as the ratio of the received signals measured by a real RCS system and an ideal RCS system. For example, when measuring a cylinder, the polarimetric factor (see Section 4 for the simple derivation) for $hh$ copolar measurements is

$$p^{v_h} = 1 + \epsilon_h \frac{C_{vv}}{C_{hh}},$$

where $C_{vv}$ are the cylinder’s copolar scattering matrix elements. We obtain the corresponding definition for $vv$ copolar measurements by interchanging $h$ and $v$. It is estimated that for RCS measurement systems currently in use, $0.01 \leq \epsilon_q \leq 0.15$. Hence, if $C_{hh} \approx C_{vv}$, the polarimetric factor in decibels is very small.

In Figure 1 we plot the ratios of the copolar scattering matrix elements as a function of frequency from $0.1–18$ GHz for two standard cylinders (usually designated as 900 and 1800 by the RCS community) with diameters 22.86 and 45.72 cm, and heights of 10.67 and 21.34 cm, respectively. Only small deviations from 1 are seen at most frequencies. In Figure 2 the polarimetric factors (dB) for the 900 cylinder are plotted for various values of $\epsilon$. We see that for $\epsilon = 0.15$, a possible upper bound for current RCS measurement systems, the polarimetric factor is less than 0.2 dB.

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Figure 2. The polarimetric factor in dB as a function of frequency from 0.1 – 18 GHz for the standard 900 cylinder. The curves, identified by the values of $\epsilon$, specify the required uncertainty limits of the polarimetric measurement system.

The curves in Figure 2 need to be interpreted in the context of RCS uncertainty analysis [6]. In the case of nonpolarimetric calibration with cylinders, these curves represent the polarimetric uncertainty as a function of frequency. To state an upper bound calibration uncertainty we need to assume an upper bound for $\epsilon$. A value of $\epsilon = 0.15$ is defensible based on known cross-polarimetric ratios obtained from near-field antenna measurements.

Alternatively, the curves in Figure 2 specify the total nonpolarimetric system uncertainty that must be met to perform polarimetric calibration successfully. For example, to determine a cross-polarimetric ratio $\epsilon = 0.15$, the combined nonpolarimetric system uncertainty from all sources of uncertainty [6] must be significantly less than 0.2 dB. In addition, the uncertainty limits for the polarimetric calibration must be small enough to stay outside the uncertainty bounds of the nonpolarimetric calibration. Only then can the polarimetric and nonpolarimetric calibrations be distinguished from each other. This is a serious measurement challenge even for state-of-the-art RCS measurement systems. Obviously, the smaller the system $\epsilon$, the more stringent the system uncertainty requirement becomes.

Polarimetric calibration can be implemented using either a cylinder, a rotating dihedral, or both. The uncertainty considerations just discussed apply to both cylinders and dihedrals. With a cylinder, the calibration occurs at a single signal level; however, with a dihedral we must accurately receive signals over a wide dynamic range estimated by (see Section 3)

$$20 \log_{10} \frac{2\epsilon}{1 - \epsilon^2}. \quad (2)$$

Figure 3 shows the dynamic range as a function of $\epsilon$ wherein signals must be accurately measured. The dynamic range increases as $\epsilon$ decreases and, as we have seen in Figure 2, the required system uncertainty decreases with $\epsilon$. Thus, the measurement challenge in calibration with a rotating dihedral is even more stringent: the system uncertainty must be small even at very low signal levels!

2. Basics of Polarimetric Calibration

The transmitted signal is scattered by a target in all directions in space and a signal is received at some location. In monostatic configurations, the transmitter and receiver are located at the same point. This scattering process is described by

$$s = rSt, \quad (3)$$

where $s$ is the polarimetric signal received by the radar, and $t$, $S$ and $r$ are the system transmit, target scattering and system receive matrices, respectively. In eq (3), $r$ is generally defined as

$$r = \begin{pmatrix} r_{hh} & r_{hv} \\ r_{vh} & r_{vv} \end{pmatrix}. \quad (4)$$

In the matrix elements in eq (4), the right index specifies the polarization of the incoming signal and the left index specifies the channel that is receiving the incoming signal. For reciprocal systems, $t$ is given by the transpose of $r$: $t = \bar{r}$, and

$$S = \begin{pmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{pmatrix}. \quad (5)$$

We can renormalize $r$ as

$$r = r_n\epsilon, \quad (6)$$
where
\[ r_n = \begin{pmatrix} r_{hh} & 0 \\ 0 & r_{vv} \end{pmatrix} \] (7)

and
\[ \epsilon = \begin{pmatrix} 1 & \epsilon_h \\ \epsilon_v & 1 \end{pmatrix}. \] (8)

Equations (4), (7) and (8) define the cross-polarization ratios as
\[ \epsilon_h \equiv \frac{r_{hv}}{r_{hh}} \] (9)

and
\[ \epsilon_v \equiv \frac{r_{vh}}{r_{vv}} \] (10)

Equation (3) can now be written as
\[ s = r_n \epsilon \hat{S} \hat{r}_n. \] (11)

In polarimetric calibration we want to determine \( \epsilon \). In general, \( r_n \) must be specified or eliminated from the measurements \( s \), before \( \epsilon \) can be found.

3. Polarimetric Calibration with a Dihedral

Polarimetric calibration using a rotating dihedral has been studied extensively [1 - 5]. Here, we review the fundamentals, and refine our understanding of the calibration process in light of the discussion presented in the introduction.

The scattering matrix \( \hat{S}_D \) for a dihedral (in the high-frequency limit) is
\[ \hat{S}_D = k_D \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}, \] (12)

where \( \theta \) is the angle of rotation from the vertical about the line-of-sight from the radar to the dihedral (see Figure 4), and \( k_D \) is a complex constant that depends on the location and size of the dihedral. Equations (11-12) give the polarimetric components of the received signals as
\[ s_{hh} = k_D r_{hh}^2 (e_h^2 - 1) \cos 2\theta + 2\epsilon_h \sin 2\theta, \] (13)
\[ s_{vv} = k_D r_{vv}^2 (1 - e_v^2) \cos 2\theta + 2\epsilon_v \sin 2\theta, \] (14)
\[ s_{hv} = k_D r_{hh} r_{vv} (e_h - e_v) \cos 2\theta + (1 + \epsilon_h \epsilon_v) \sin 2\theta, \] (15)
\[ s_{vh} = k_D r_{hv} r_{vv} (e_h - e_v) \cos 2\theta + (1 + \epsilon_h \epsilon_v) \sin 2\theta. \] (16)

After we measure \( s_{qq}(\theta) \), we obtain the \( n = 2 \) Fourier coefficients \( e_{2,qq} \) and \( s_{2,qq} \). The presence of a significant \( n = 0 \) Fourier component is the background signal; all other coefficients with \( n \neq 2 \) should be small, and are filtered. This Fourier filtering of the data is one of the benefits of using a rotating dihedral to obtain the system cross-polarimetric parameters.

From eq (13-14), we see that the ratio of the \( n = 2 \) coefficients
\[ \frac{s_{2,qq}}{c_{2,qq}} = \pm \frac{2\epsilon_q}{1 - \epsilon_q^2} \] (17)

can be solved for \( \epsilon_q \). We can readily verify that the two solutions of the quadratic in eq (17) are negative reciprocals of each other. The correct solution must be selected: we choose \(|\epsilon_q| < 1\).

In Figure 5 we show the dihedral response for a perfectly isolated system \((\epsilon_q = 0)\) and a real system \((\epsilon_q \neq 0)\) as seen by the \( hh \) channel. From eqs (13 - 14) we see that the dynamic range of the signals re-
ceived from a rotating dihedral, defined as the ratio
of the minimum and maximum measured signals, is
given by eq (2). An amplitude change $\alpha$ and a phase
shift $\delta$ as functions of $\epsilon_q$,

$$\alpha = \epsilon_q^2 - 1$$  (18)

and

$$\delta = \frac{1}{2} \tan^{-1} \frac{2\epsilon_q}{\epsilon_q^2 - 1},$$  (19)

characterize the polarimetric signals. The very small
amplitude and phase changes in eq (18 - 19) must be
accurately measured over a large dynamic range, as
seen in Figure 3. We note again: this is a difficult
measurement challenge!

If we ignore the phase shift shown in Figure 5, then $\alpha$
in eq (18) is the polarimetric factor in dihedral cali-
bration. As discussed in the introduction, $\alpha(\epsilon)$, shown
in Figure 6, gives the uncertainty within the dynamic
range given in eq (2) of a nonpolarimetric RCS system
that must be met to successfully perform polarimetric
 calibration.

A major difficulty, the presence of drift, encountered
on outdoor polarimetric measurement ranges has been
examined in [1]. It has been shown that after Fourier
analysis, only the $c_{4}^{\text{drift}}$ and $s_{4}^{\text{drift}}$ coefficients modify
the solutions to eq (17). The ratio of the $n = 2$ Fourier
coefficients, in the presence of drift, becomes [1]

$$\frac{s_{2}^{d_{0}^{\text{drift}}}}{c_{2}^{d_{0}^{\text{drift}}}} = \frac{s_{2}^{0}(1 - \frac{c_{4}^{\text{drift}}}{2}) + s_{2}^{d_{4}^{\text{drift}}}}{c_{2}^{0}(1 + \frac{c_{4}^{\text{drift}}}{2}) + c_{2}^{d_{4}^{\text{drift}}}}.$$  (20)

The way drift alters the final computation of the pol-
larimetric parameters $\epsilon_q$ will depend on the ampli-
tudes and phases of the $n = 4$ Fourier coefficients
present in the drift. The right side of eq (20) reduces
to the non-drift ratio if the $n = 4$ drift coefficients
vanish. Examination of eq (20) shows that even a
small drift can significantly alter the polarimetric solu-
tions. For example, let $s_{4}^{d_{4}^{\text{drift}}} > s_{2}^{0}/(k_D r_{qq}^2) << 1$
and $c_{4}^{\text{drift}} = 0$; the numerator in eq (20) can now be
larger by more than a factor of 2, since $c_{2}^{0}/(k_D r_{qq}^2) \approx 1$.
This example is easy to understand: the drift com-
ponent $s_{4}^{d_{4}^{\text{drift}}}$ substantially modifies the minimum signal
received as the dihedral rotates, thereby introducing
a large error into the computed $\epsilon_q$.

To obtain meaningful system parameters $\epsilon_q$, drift
must be severely limited during calibration or eliminated
from the data. Measurement of the system drift during
calibration has never been performed. Simulta-
neous measurements on a fixed target behind the di-
heled might be adequate, although the drift could
in fact be different even at a short distance from the
dihedral; phase and amplitude differences due to tar-
get separation could also be important. The challenge
here is to design a new calibration artifact that would
allow us to calibrate the rotating dihedral and simul-
taneously obtain drift data at the same location.

Alternatively, we need to perform polarimetric cali-
bration without drift. Rapid measurements on a cali-
boration cylinder might be useful here. In [1] we have
used the vanishing of the cross-polarization scattering
matrix element of a cylinder to eliminate the effect
of drift on the ratio in eq (20). This is possible be-
cause we can modify the drift component of the signal
without affecting the drift-free dihedral response.
We choose to modify the drift until we see no cross-
polarimetric response from the cylinder. For details
of this procedure, see [1].

4. Polarimetric Calibration with a Cylinder

We examine the possibility of polarimetric calibration
using a cylinder, since a cylinder can provide us with
drift-free calibration data. The polarimetric basics
presented in Section 2 apply. The scattering matrix
$S_C$ for a cylinder is

$$S_C = k_C \begin{pmatrix} C_{hh} & 0 \\ 0 & C_{vv} \end{pmatrix},$$  (21)

where $k_C$ is a complex constant dependent on the loca-
tion of the cylinder. We assume that the theoretical
(computed) values $C_{qq}$ are known. The off-diagonal
elements are 0: the cylinder only scatters but does not
de-polarize signals. Since the cross-polarimetric
components of the transmitted signals are preserved, we obtain information on the system parameters \( \epsilon_q \). The components of the received signals are given by

\[
\begin{align*}
    s_{hh} &= k_C r_{hh}^2 (C_{hh} + \epsilon_h^2 C_{vv}), \\
    s_{hv} &= k_C r_{hh} r_{vv} (\epsilon_v C_{hh} + \epsilon_h C_{vv}), \\
    s_{vh} &= k_C r_{hh} r_{vv} (\epsilon_v C_{hv} + \epsilon_h C_{vv}), \\
    s_{vv} &= k_C r_{vv}^2 (C_{vv} + \epsilon_v^2 C_{hh}).
\end{align*}
\]  

(22) - (25)

For perfectly isolated and aligned systems, \( \epsilon_q = 0 \), the cross-polar measurements vanish, and the copolar measurements are proportional to the cylinder's copolar response. The polarimetric factor in eq (1) is easily obtained from eq (22).

Equations (22-25) provide three independent conditions with the four unknowns \( r_{hh}, r_{vv}, \epsilon_h \) and \( \epsilon_v \). Unless independent data exist to obtain the receive coefficients \( r_{qq} \), we cannot use these equations to solve for \( \epsilon_q \). We can, however, eliminate \( r_{qq} \) by constructing

\[
    \chi^C = \frac{s_{hv} s_{vh}}{s_{hv} s_{vh}}.
\]

(26)

\( \chi^C \) is independent of the receive coefficients, and can be used to check on the integrity of the parameters \( \epsilon_q \) that were obtained independently. If we have used a dihedral to obtain \( \epsilon_q \), most likely the data have been contaminated with drift, and the \( \chi^C \) obtained from cylinder data and computed using eqs (22 - 25) will not be in agreement.

To use only cylinder data to determine \( \epsilon_q \) we must have independent knowledge of \( r_{qq} \). The measurement challenge here is to determine \( r_{qq} \) accurately prior to the polarimetric calibration. To our knowledge, this has never been accomplished. We note that we could use dihedral data to obtain \( r_{qq} \), but we must be cognizant of the real possibility of contamination by drift.

Given \( r_{qq} \), we can easily obtain \( \epsilon_q \) from eqs (22) and (25). To check the consistency of measurements and calculations, we form \( \epsilon_m \), as determined from the cylinder measurements. Let

\[
    s_0 = \epsilon_0 S_C \tilde{\epsilon}_0;
\]

(27)

then

\[
    (r_{qq} \epsilon_m)^{-1} s(r_{qq} \epsilon_m)^{-1} = \epsilon_m^{-1} s_0 \epsilon_m^{-1} = S_C
\]

(28)

if and only if \( \epsilon_m = \epsilon_0 \), where \( \epsilon_0 \) is the correct system cross-polarimetric matrix. If we have used dihedral data to determine \( r_{qq} \), most likely we cannot recover \( S_C \) in eq (28), because of drift.

We can try to relax the requirement that \( r_{qq} \) be known. We can determine \( \epsilon_m \), then evaluate

\[
    \epsilon_m^{-1} s_0^{-1} = \epsilon_m^{-1} (r_{qq} \epsilon_0 r_{qq}) \epsilon_m^{-1} \neq S_C,
\]

(29)

since, in general, \( r_{qq} \neq \mathbf{I} \), the unit matrix. The ratio of the copolar matrix elements of eq (29) is

\[
    \rho^2 \epsilon_h^2 s_{0vv} - 2 \rho \epsilon_h s_{0vh} s_{0hv} + s_{0hh} \epsilon_v^2,
\]

(30)

where \( \rho = r_{vv}/r_{hh} \) is the copolar channel imbalance. For \( \rho = 1 \) and \( \epsilon = \epsilon_0 \), the eq (30) ratio reduces to \( C_{hh}/C_{vv} \), and the cross-polar elements reduce to 0. Conversely, we can solve the simultaneous equations

\[
    \frac{(\epsilon_m^{-1} s_0^{-1})_{hh}}{(\epsilon_m^{-1} s_0^{-1})_{vv}} = C_{hh}/C_{vv},
\]

(31)

\[
    (\epsilon_m^{-1} s_0^{-1})_{hv} = 0
\]

for \( \epsilon_h \) and \( \epsilon_v \). We get two pairs of solutions:

\[
    \epsilon_h = \epsilon_{0h}, \quad \epsilon_v = \epsilon_{0v},
\]

(32)

and

\[
    \epsilon_h = \frac{\epsilon_{1h}^2 C_{vv}/C_{hh} - \epsilon_{0h} \epsilon_{0v} + 2}{\epsilon_{0v} + \epsilon_{0h} C_{vv}/C_{hh}},
\]

\[
    \epsilon_v = \frac{\epsilon_{1v}^2 + (2 - \epsilon_{0v} \epsilon_{0h}) C_{vv}/C_{hh}}{\epsilon_{0v} + \epsilon_{0h} C_{vv}/C_{hh}}.
\]

(33)

Since the \( C_{vv}/C_{hh} \approx 1 \) (see Figure 1), the amplitudes of the second pair of solutions exceed 1, and need to be rejected.

In theory, we may be able to configure a polarimetric RCS system with balanced copolar channels, although, to our knowledge, this has never been demonstrated in practice. The required accuracy of such a procedure needs to be understood. Let \( \rho = 1 + \delta \rho \), where \( \delta \rho << 1 \). Small parameter analysis (or a Taylor series expansion) shows that the solutions to eqs (31) will be

\[
    \epsilon_h = \epsilon_{0h} (1 - \frac{\delta \rho}{\rho} s_{0hh} \epsilon_{0h}),
\]

(34)

\[
    \epsilon_v = \epsilon_{0v} (1 + \frac{\delta \rho s_{0hv}}{\epsilon_{0h} s_{0hv}})
\]

(35)

For \( \epsilon_h \) and \( \epsilon_v \) to be acceptable estimates, the second terms in the above solutions have to be small; we obtain the condition

\[
    \delta \rho << \epsilon_{0h} s_{0hv} \epsilon_{0hv} \approx O(\epsilon_{0h}^2),
\]

(36)
where \( s_{0qq} \) and \( s_{0pq} \) designate the copolar and cross-polar elements of \( s_0 \) in eq (27). The copolar channel imbalance must be very small: to achieve an accuracy to second order in \( \epsilon_{0q} \) is a very difficult measurement challenge.

5. Calibration with a Cylinder and a Dihedral

When we measure both a cylinder and a dihedral, we can develop a polarimetric calibration procedure that is satisfactory theoretically. We are then left with the important challenge to reduce the measurement uncertainty to levels low enough to reduce polarimetric effects. It even state-of-the-art RCS systems need to be carefully assessed to achieve the low level of uncertainties required by polarimetric calibration.

In eq (26), the quantity \( \chi \) is independent of \( r_n \); \( \chi \) can be expressed theoretically in terms of the unknown parameters \( \epsilon_q \), and can also be obtained from the data. We use eqs (13 - 16) with \( \theta = 0 \) and assume that \( \epsilon_{0h} \neq \epsilon_0 \), to express \( \chi^D \) for a dihedral, and use eqs (22 - 25) to express \( \chi^C \) for a cylinder; then we equate these to corresponding \( \chi \) obtained from measurements:

\[
\chi^D(0) = \chi^D_0(0), \quad \chi^C = \chi^C_0. \tag{37}
\]

These two equations will be drift free, and can be solved for the two unknowns \( \epsilon_q \) and \( \epsilon_v \). Note, however, that \( \chi \) contains a 4th-order term \( \epsilon^2_q \epsilon^2_v \), hence, eight pairs of solutions will be obtained. We will accept solutions that satisfy \( \epsilon_q < 1 \). To distinguish and select the correct solution from the acceptable pairs, we must turn to the dihedral data as a function of the angle of rotation. A phase ambiguity of \( \pi \) might be easily resolved; small differences between solution pairs might be more difficult to resolve because of drift. This needs closer examination, and will be the subject of a future study.

Small parameter expansions (or Taylor series) show that solutions to eq (37) are approached very steeply. Let \( \epsilon_q = \epsilon_{0q} + \delta \epsilon_q \). Then, to first order in \( \delta \epsilon_q \), when \( \epsilon_{0h} \neq \epsilon_0 \),

\[
\frac{\chi^D(0)}{\chi^D_0(0)} = 1 + \frac{2}{\epsilon_0 - \epsilon_{0h}}(\delta \epsilon_h - \delta \epsilon_v) + O(\delta \epsilon^2_q) + O(\delta \epsilon^2_v) \tag{38}
\]

and

\[
\frac{\chi^C}{\chi^C_0} = 1 + 2 \frac{C_{vv}}{s_{0hh}} \delta \epsilon_h + 2 \frac{C_{hh}}{s_{0hv}} \delta \epsilon_v + O(\delta \epsilon^2_q) + O(\delta \epsilon^2_v). \tag{39}
\]

Here all coefficients of \( \delta \epsilon_q \) are large: small deviations from \( \epsilon_{0q} \) will produce large deviations from the data. We will see a deep null in the computed \( \chi \) as solutions to eq (37) are approached. For a one-dimensional example, see [1], where we exhibited the deep null as the vanishing of the cylinder’s cross-polar response is recovered when the solution \( \epsilon_{0h} \) is approached.

6. Summary and Future Efforts

We have recognized three major obstacles to performing polarimetric calibration successfully: (1) the small polarimetric effect demands that the overall RCS system uncertainty be very small, (2) the presence of drift, and (3) the lack of data on the radar’s receive matrix \( r_n \). Consequently, (1) the currently accepted RCS measurement uncertainties will need to be reduced to make polarimetric calibration a reality, and (2) we recommend a new calibration procedure using a cylinder and a rotating dihedral. We combine the polarimetric matrix elements to obtain an expression independent of drift and \( r_n \). The method yields multiple sets of solutions, and we can use the data obtained with the rotating dihedral to select the correct pair. Some details of this technique will be developed with real data in the near future.

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