A NEW APPROACH TO JOHNSON NOISE THERMOMETRY USING A
JOSEPHSON QUANTIZED VOLTAGE SOURCE FOR CALIBRATION

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ABSTRACT

We describe a new approach to Johnson Noise Thermometry (JNT) that addresses certain limitations found in
the conventional approach. The concept takes advantage of recent advances in digital synthesis and signal
processing techniques together with advances in Josephson voltage standards. By using the perfect quantization
of voltages from the Josephson effect, a synthesized broadband waveform can be used as a calculable noise
source for calibrations. A collaboration between NIST and MSL explores this approach with the initial goal of
creating a JNT measurement system capable of achieving relative accuracies of 0.001\% in the range of
temperatures between 84 K and 430 K. In this paper, we discuss the use of a broadband Josephson waveform
generator to produce a calculable reference noise source, the related metrological challenges, the technical
advantages conveyed by this approach, and the commensurate opportunities to advance the state of the field.

1. INTRODUCTION

The determination of thermodynamic temperature is limited to those physical systems having a known
thermodynamic relation to a measurand and whose non-ideal characteristics are well understood [1].
Hence the gas-based thermodynamic techniques, which best satisfy this condition, have been
predominant in state-of-the-art temperature metrology. In contrast, the accuracies achieved in Johnson
noise thermometry (JNT) have not been comparable to those of gas-based techniques due to
limitations from the non-ideal performance of electronic detection systems. In almost all JNT system
designs until now, however, there has been a notable absence of a synthesized ac voltage reference. A
reference is a key component in any electronic measurement system in order to characterize properly
any effects of non-ideal circuit performance and enable in-situ calibration of the instrument.

The various technical approaches used in JNT systems over the past 45 years have been well
documented in several review articles [2-4]. Typically, the temperature $T$ is inferred by measuring the
mean-square of the fluctuating Johnson noise voltage, $V_T$, across a calibrated resistance $R(T)$. The
noise power is given by the Nyquist formula [5],

$$\overline{V_T^2} = 4kT R(T) \Delta f,$$

where $\Delta f$ is the equivalent noise bandwidth for the measurement and $k$ is Boltzmann’s constant.
Because the noise signals are small, of the order of 1 nV/Hz\textsuperscript{1/2}, the signal must be amplified by $\sim 10^6$ in
order to be measurable. In addition, the bandwidth must be made as large as possible to minimize the
necessary integration time. Despite the simple thermodynamic relation (Equation (1)), and a sensor
that is believed to be ideal, difficulties with the electronic instrumentation and interference from
extraneous noise sources pose severe limits on the performance of noise thermometers.

The most successful technique developed to date is the switched-input digital correlator pioneered by
Brixy, et. al. [6-8]. The correlator is implemented by digitizing the signals from the two correlator
channels and carrying out the multiplication and averaging function of the correlator in software. The
correlator eliminates the amplifier and transmission line noise superimposed on the thermal noise signal. In use, the thermometer switches between a reference noise source, with a Johnson noise voltage $V_0$, source resistance $R(T_0)$ at a reference temperature $T_0$, and another Johnson noise source $V_T$, source resistance $R(T)$ at an unknown temperature $T$. The switching removes the effects of drifts in the gain and bandwidth of the amplifiers and filters. The unknown temperature is then inferred as

$$T = \frac{V_T^2}{V_0^2} \frac{R(T)}{R(T_0)} T_0,$$

so requires only measurements of resistance ratio and noise power ratio. However, there are three main factors limiting the accuracy of the conventional switched-input correlation thermometer.

**Measurement time:** Because the noise signal is random, the signal must be averaged for long periods to eliminate statistical fluctuations. The statistical uncertainty for a single noise power measurement is

$$\sigma_{V_T}^2 = \frac{1}{2\tau\Delta f_c} \left( \frac{V_T^2 + V_n^2}{V_T^2} \right)^2 + 1,$$

where $\tau$ is the measurement time, $\Delta f_c$ is the correlation bandwidth [9] and $V_n$ is the uncorrelated noise due to the amplifiers and transmission lines. With the conventional switched correlator, two noise measurements are made, which quadruples the variance for a given total measurement time. Thus, to obtain a temperature measurement with a statistical uncertainty of 0.001% in a system with $V_n^2/V_T^2 = 0.4$ and a 100 kHz bandwidth, a measurement time of 167 hours (~7 days) is required.

**Nonlinearity:** With the high amplification and wide bandwidths employed in JNT, the thermometer must be operated as a comparator, comparing the noise powers from the reference arm at $T_0$ and the sensing arm at $T$. We assume the amplitude transfer function of the first channel of the correlator is

$$V_1 = \sum a_{ii}(V_T + V_n)^i, \quad a_{11} = 1,$$

where all of the coefficients $a_{ii}$ (except $a_{11}$) represent the nonlinearities and are assumed to be small, and the second channel is represented similarly. It follows that the lowest-order products at the output of the correlator yielding a non-zero mean are

$$V_1V_2 = V_T^2 + 3(a_{13} + a_{23}) V_T^2 (V_n^2 + V_T^2) + \ldots.$$  

The nonlinearities therefore lead to an error in the noise power ratio of Equation (2) of

$$\left( \frac{V_T^2}{V_0^2} \right)_{\text{meas}} = \left( \frac{V_T^2}{V_0^2} \right) \left[ 1 + 3(a_{13} + a_{23}) \frac{V_T^2 - V_0^2 + V_n^2 - V_{n,0}^2}{V_T^2 - V_0^2 + V_n^2 - V_{n,0}^2} \right],$$

so that the error in the ratio depends on the differences in the noise powers. Since the nonlinearity arises in a variety of different components, it is not calculable, and is difficult to measure, and probably varies with time and ambient temperature. Thus, the nonlinearity makes it necessary to operate the noise thermometer with the same correlated and uncorrelated noise power in the two arms of the comparator [10]. The sensing resistances must therefore be chosen so that $R(T_0)T_0 = R(T)T$. 
Transmission-line effects: In Equation (1) it is assumed that the sensor’s full signal is presented to the input of the preamplifiers of the noise thermometer. Because the preamplifiers must be located some distance from the sensors, the transfer function of the transmission line enters into the measurement:

\[
\overline{V_T^2} = 4kTR(T) \int_0^\infty |X(f)|^2 |H(f)|^2 df,
\]

where \(X(f)\) is the frequency response of the transmission line and \(H(f)\) is the frequency response of the band-pass filters defining the bandwidth of the noise thermometer. While the response of the filters is the same for both measurements, the interaction of the sensor resistance with the transmission line means that \(X(f)\) depends on the sensor resistance. For a short transmission line, which can be characterized by lumped parameters, the error in the measured noise power ratio is [11]

\[
\frac{\overline{V_T^2}}{\overline{V_0^2}} = \frac{\overline{V_T^2}}{\overline{V_0^2}} \left(1 + \omega_m^2 (C_t + 2C_{in})^2 \left(R(T_0)^2 - R(T)^2\right)\right),
\]

where \(\omega_m\) is the rms operating frequency of the thermometer, \(C_t\) is the transmission line’s capacitance and \(C_{in}\) is the input capacitance of the two preamplifiers. The error is most simply eliminated by operating the noise thermometer with a constant sensing resistance \(R(T_0) = R(T)\). However, this requirement conflicts directly with the condition \(R(T_0)T_0 = R(T)T\) required to reject the nonlinearity.

In practice, the operating frequency is limited to reduce the magnitude of the transmission-line error, and in some cases a correction may be applied. The selection of \(R(T)\) to satisfy an impedance matching condition with the line impedance can improve the flatness of \(X(f)\) [12], but the error represented by Equation (8) will remain unaffected. The transmission-line error thus limits the noise power ratio accuracy as well as the statistical uncertainty achievable for a constant measurement time (Equation (3)) because \(\omega_m\), and hence the bandwidth, must be limited in order to minimize the effect.

2. PRINCIPLES OF THE NEW APPROACH

The conventional approach does not contain sufficient free parameters to resolve the contradictory requirements for matching both the sensing resistances and the noise powers. This limitation originates from the fact that the reference measurement is performing two functions: calibrating the noise power gain of the thermometer, and normalizing to the reference temperature (see Figure 1(a)). The new approach presented here keeps the proven elements of the switched correlator, but separates the roles of the temperature reference and the voltage reference (see Figure 1(b)). In this approach the sensing resistor \(R(T_0)\), in the reference arm of the comparator, is replaced by an ac Josephson Voltage Standard (JVS). The separation of functions and use of a synthesized reference has several advantages:

- Reduced measurement time. The use of a deterministic (non-random) synthesized reference signal reduces the statistical uncertainty and measurement time associated with the measurement of the reference noise power.
- Simultaneous matching of noise power and sensing resistance. The reference noise source, being fully programmable, is adjusted to inject any required ac power to match the noise power from the sensor. The output impedance of the source can also be chosen at will to match that of the sensor.
- Increased bandwidth. With equal terminating resistances on the end of each transmission line and matched to the characteristic impedance of the line, the bandwidth of the noise thermometer can be extended well beyond that of the conventional noise thermometer. This leads to a corresponding reduction in the measurement time.
Figure 1: (a) The block diagram for the conventional switched-input noise-correlator instrument, S: switching, A: amplification, filtering, and digitization. (b) The block diagram for the new approach.

In Figure 1(b), the role of the JVS reference $V_r$ is to serve as a stable link between measurements of $V_0$ at $T_0$ and measurements of $V_T$ at $T$ performed at a later time. The inputs are still switched between $V_T$ and $V_r$, to measure the ratio $V_T^2 / V_r^2$, which is still constrained to near unity for every $T$ measured. The temperature is then inferred from a modification of Equation (2):

$$T = \frac{V_T^2}{V_r^2} \cdot \frac{V_r^2 R(T)}{V_0^2 R(T)} \cdot \frac{R(T_0)}{R(T)} \cdot T_0,$$

where the ratio $\frac{V_T^2}{V_r^2}$ is a numerical factor calculated purely from the operating conditions of the Josephson voltage source. In this scheme, the nature of the measurement of resistance ratio has changed from the ratio of two different resistances to the ratio of a single resistance at two different temperatures. Note particularly that all of the measured ratios in Equation (9) are held close to unity, so simultaneously satisfying the matching criteria for sensing resistance and noise power.

The temperature measurement represented by Equation (9) now involves four measurements of noise power, whereas Equation (2) implies only two. In the new scheme, the two measurements of the thermal noise are subject to uncertainty according to Equation (3), thus doubling the variance and measurement time as before. While the reference signal may be deterministic, the inevitable uncorrelated noise that is superimposed leads to a finite uncertainty in each of the measurements of the reference noise power. The statistical uncertainty in each of these measurements is given by

$$\frac{\sigma^2_{V_T}}{V_T^2} = \frac{1}{2\tau_r \Delta f} \left( \frac{V_T^2 + V_r^2}{\sigma^2_{V_r}} \right) \left( \frac{V_T^2 + V_r^2}{\sigma^2_{V_r}} - 1 \right),$$

which has a form very similar to that of Equation (3) except that the variance tends to zero as the uncorrelated noise goes to zero. Although at first sight the statistical uncertainty seems greater with the new scheme, the increased bandwidth, and the fact that only one measurement of noise power at the reference temperature need be made, means that the total measurement time is less. This improvement plus the increased bandwidth allowed by matching the sensing resistors offers, perhaps, a 10-times reduction in the measurement time.

Equation (6) established that the mean noise powers for the reference signal and the noise signal must be the same to avoid errors caused by the nonlinearity in the thermometer electronics. The correlator’s nonlinearity imposes the same and additional requirements on the synthesized reference signal. Ideally, the correlator produces an output signal that is proportional to the second moment of the noise distribution (the variance, since the noise has zero mean). All of the distortion products caused by the
nonlinearity are proportional to other higher-order moments of the total combined \( (V_T \text{ and } V_n) \) noise distribution. Thus, it is clear that the reference signal must have the same statistical distribution of amplitudes as does thermal noise of the same mean noise power. The simplest solution is to compose the reference signal of a series of many sinusoids distributed in the frequency domain with constant amplitude and random phase. In this way, the distribution of the instantaneous amplitudes will be Gaussian, yet the average noise power is constant (non-random). By following the line of argument used to prove the central limit theorem \([13]\) it can be shown that as the number \( N \) of sinusoids increases, the moments of the synthesized distribution all converge to the values for the Gaussian as \( \text{O}(1/N) \). Thus for a correlator with 0.1 \% distortion, about 1000 sinusoids are sufficient to ensure an accuracy of the order of 0.000 1\% in the measurements of noise-power ratio.

The most compelling feature of the new approach is the capability of the ac JVS to produce intrinsically calculable voltages of any waveform. If the JVS is used to generate noise of a calculated and constant power spectral density \( V_{\text{cal},T}^2 \), then the temperature can be inferred absolutely as

\[
T = \frac{V_T^2}{V_{\text{cal},T}^2} \cdot \frac{V_{\text{cal},T}^2}{4kR(T)}.
\]

In this case, it is not necessary to measure the Johnson noise voltage at the reference temperature \( T_0 \), because the definition of the kelvin is contained in the value of \( k \) \([14]\). The use of a reference of constant power spectral density also eliminates the need to know the frequency response of the filters. The uncertainty from the resistor’s ac-dc difference at 100 kHz now becomes a significant factor. Note that the application of Equation (11) to measurements at the triple point of water provides a determination of Boltzmann’s constant.

The proposed scheme is similar to the system that Storm \([15]\) attempted to use for a determination of Boltzmann’s constant. This approach, however, has clear advantages. By using a reference signal of the same amplitude as the thermal noise signal rather than a high-level signal and attenuators, it eliminates the need for attenuators and the accompanying calibration problems. In addition, because it uses a reference signal that matches the statistics of the thermal noise it eliminates the need to produce a sinusoidal reference of very low distortion, and impractically linear amplifiers, filters and correlator.

### 3. AC JOSEPHSON VOLTAGE STANDARD

The JVS has a relatively broad bandwidth and produces a signal with a calculable magnitude based on the perfectly quantized voltage pulses of Josephson junctions \([14,16]\). The time-integrated area of every Josephson pulse is precisely equal to the flux quantum, \( h/2e \), the ratio of Planck’s constant to twice the electron charge. Knowledge of the number of pulses and their position in time is sufficient to precisely determine the voltage of any synthesized waveform. Digital synthesis using perfectly quantized pulses enables the generation of waveforms with amplitudes that are dependent only on fundamental constants and a time standard.

NIST researchers have demonstrated a number of techniques and Josephson circuits that can synthesize arbitrary waveforms with low harmonic distortion and stable, calculable, and reproducible voltage amplitude and phase at frequencies up to a few megahertz \([17-20]\). The maximum output voltage of a series array of junctions is \( V_p = nNfh/2e \), where \( n \) is the number of quantized output pulses per input pulse, \( N \) is the number of series junctions in the array, and \( f \) is the sine-wave frequency. The area \( h/2e \) of each quantized pulse is very small, approximately 2 \( \mu \text{V GHz} \). In order to generate a waveform with a 1 V peak amplitude, approximately \( 5 \times 10^{14} \) pulses/s must be synthesized and controlled. Thus, the development of an ac Josephson voltage standard that produces signal amplitudes
Figure 2: Block diagram of the ac-coupled input bias technique for generating bipolar waveforms with the Josephson arbitrary waveform synthesizer. High-frequency signals from the digital code generator are ac-coupled to the array through a dc blocking capacitor (HPF) while the low-frequency bias is applied separately through the low-pass filter (LPF) and amplifier.

sufficient for high-performance audio and radio-frequency applications, including ac voltage standards, low-noise radar, and electronic instrument calibration, has been a major challenge.

The need for higher voltage output led to the development of a bipolar waveform technique [17-19], followed by the ac-coupled technique [20]. The bipolar method adds a sine wave to the digital input signal, while the ac coupled method divides the broadband digital input signal into low- and high-frequency components that are applied to arrays through separate low- and high-speed transmission lines. A block diagram of the ac-coupled technique is shown in Figure 2. The digital code signal $D$ is ac-coupled to the array using a “dc block” high-pass filter with a 10 MHz to 18 GHz pass band. All signals with frequencies below 10 MHz are effectively removed from the broadband input bias. However, the low-frequency part of the original digital code signal is necessary for biasing the array. This low-frequency bias is reconstructed from the $D$-bar signal with a low-pass filter (LPF) and inverting amplifier and then applied to the array through a separate low-speed transmission line. In order to calibrate the correlation electronics for the noise thermometer, we must synthesize a noise signal with a power spectrum that is flat up to a desired bandwidth on the order of 1 MHz. The synthesizer uses a digital code that is $M$ bits long and is repeatedly cycled through the circulating memory of the digital code generator. The synthesized waveform thus has a minimum frequency called the pattern repetition frequency, $f_1 = f_s/M$, where $f_s$ is the clock frequency of the code generator, which is also the sampling frequency of the analog-to-digital conversion of the modulator algorithm [21].

The synthesized noise waveform is constructed by combining harmonics of the pattern repetition frequency. The amplitude of each harmonic is defined to be the same, while their relative phases are randomized. The time-dependent voltage of a noise waveform with 1 048 576 bits clocked at 3 GHz is shown in Figure 3(a). It has a pattern repetition frequency of 2.86 kHz and 128 harmonics. The pattern has a period of about 350 µs as indicated by the arrow bar. The output voltage is measured across an array of 4096 junctions that are biased with a 7.5 GHz sine wave using the bipolar technique. The junctions are Nb-PdAu-Nb superconductor-normal metal-superconductor junctions 2 µm in diameter. The array output is amplified by a differential preamp with a gain of 50. Although the amplitudes of all the harmonics were designed to be identical, the spectrum analyzer measures a slight decrease in amplitude for the tones of highest frequency as shown in Figure 3(b). This decrease in amplitude is due to the limited frequency response of the preamplifier, and demonstrates the utility of the synthesized reference signal for evaluating instrument performance. With the large output voltage from a large number of junctions, it is relatively easy to ensure that the Josephson array is working within operating margins. However, a smaller number of junctions is required for the Johnson noise thermometer because only a microvolt or so is needed to emulate the thermal noise for typical resistances and temperatures. A small voltage amplitude creates another challenge as any undesired voltage induced on the transmission line inductance between the series junctions or anywhere between the output voltage taps can be a source of error. This is a serious problem for the bipolar and ac-coupled bias techniques because the digital input signal current that drives the junctions has harmonic components at exactly the same frequencies that are synthesized by the junctions.
4. PROGRESS TO DATE

The long-term goal of the project is to build a noise thermometer with an uncertainty of 0.001 % over the temperature range 83 K to 430 K. The first step is to build a ‘proof of concept’ JVS/JNT system capable of achieving 0.001 % statistical precision at the triple point of Ga. This will be used to clarify the design constraints and evaluate which method, the relative scaling mode (Equation (9)) or the absolute scaling mode (Equation (11)), is potentially more accurate.

Work has commenced on the implementation of the JVS. We have identified three different techniques for reducing the undesirable drive currents in the JVS. All three methods use ac coupling through a dc block (or other high-pass filter with sufficient stop band at low frequencies) in order to remove the low-frequency components of the digital input signal. All three methods also require the use of waveforms with small voltage amplitudes. This allows us to eliminate the low-frequency input bias of the ac-coupled technique that would result in undesirable inductive voltage signals. The use of intentionally low-amplitude waveforms keeps the array within its operating-current margins without any low-frequency compensation current.

The ac-coupled method described above and shown in Figure 2 can be used with this low-amplitude waveform technique to yield bipolar output voltage waveforms. We have demonstrated this first technique and found adequate operating margins for the bipolar waveforms. However, since only low voltages are needed for the Johnson noise thermometer application, there is no need to use bipolar waveforms that were originally developed to achieve higher voltages. We can simplify the system design by removing the sinusoidal drive of these methods and return to using unipolar waveforms where the Josephson pulses have a single polarity [15, 16]. This second technique avoids the challenge of ensuring that the sinusoidal drive remains phase-locked with the clock frequency for the long integration times necessary for JNT applications. This second technique has been demonstrated and found to have adequate operating margins. However, a disadvantage of this second technique is that an extra set of low-speed bias leads is required in order to bias the array on its operating margins. In a third technique, we include a negative dc offset of about $0.75V_p$ in the input waveform of the modulator algorithm. For appropriate pulse amplitudes, no leads for a dc offset bias are then required.

Work has also commenced on the correlator, which comprises both analog and digital signal processing blocks. The preamplifiers have fully differential inputs and an input noise voltage of less than $1 \text{nV/Hz}^{1/2}$. The anti-aliasing filters are 4-pole Bessel filters with a cutoff frequency of 100 kHz. The digital signal processing begins with a custom digitizer board capable of digitizing at 14 bits and
20 M samples per second. The board also includes a field-programmable gate array for simple signal processing and data reduction. The data is transmitted to a computer via a fiber-optic interface to electrically isolate the front-end electronics from the computer. The receiver end of the fiber-optic cable is a custom interface card capable of bus-mastered addressing for efficient transfer of data into the computer memory with little processing overhead. With high-performance processors, it is now possible to compute million-point FFTs and cross correlations in real time. With a high-speed network of 2 to 3 computers, the JNT system will acquire, process, and display the data as it is acquired.

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REFERENCES


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