Abstract

By means of concrete examples, we discuss the determination of an uncertainty budget for calibration of thermocouples by comparison techniques. In the first example, the calibration of a type T thermocouple (copper versus copper-nickel alloy) in a stirred liquid bath with a platinum resistance thermometer as a reference is considered. The second example is the calibration of a type K (a nickel-chromium versus nickel-aluminum alloy) thermocouple at temperatures up to 1200 °C in a tube furnace, using a platinum-rhodium alloy thermocouple as a reference. Methods for determining the magnitudes of these uncertainties are given for each uncertainty component.

1. Introduction

This paper is intended as a tutorial in the determination of uncertainty budgets for the calibration of thermocouples by comparison techniques. The focus is on obtaining the standard uncertainty, \( u_c \), where the coverage factor is \( k=1 \), or the expanded uncertainty \( U_c \) where the coverage factor is fixed at \( k=2 \). For a discussion of obtaining expanded uncertainties at stated confidence limits, the reader is referred to references [1-3].

Thermocouples appear to be simple: two dissimilar wires are joined at one end in a small, compact measuring junction, and the thermoelectric electromotive force (emf) is measured across the two opposite ends. However, the emf is actually created along the length of the two wires, wherever the wires pass through a temperature gradient. This fact, coupled with the small magnitude of the generated emfs and some inherent metallurgical limitations of thermocouples, complicates the calibration process. Some of the uncertainty components are common to calibration of all types of thermometers: the thermal uniformity and stability of the calibration environment; the uncertainty of the thermometer used as a reference standard; and the uncertainty of the thermometer readouts. For these components, the discussion focuses on aspects of particular interest for thermocouples, such as the contributions of extraneous thermal electromotive force on the readout uncertainty. Several uncertainty components are unique to thermocouples: the need to characterize the temperature of the reference junctions; the effects of thermocouple immersion on the calibration results; and the drift of the thermocouple response that occurs during the calibration process itself.

2. Uncertainty components

Table 1 provides a list of the possible uncertainty components for the calibration of thermocouples at specified test temperatures. This paper does not address uncertainties
associated with either interpolation between test temperatures or modeling the thermocouple response function.

Table 1. Uncertainty components for thermocouple calibration.

1. Reference thermometer calibration
2. Reference thermometer drift
3. Reference thermometer repeatability
4. Reference thermometer readout
5. Test thermocouple repeatability
6. Test thermocouple readout
7. Test thermocouple inhomogeneity
8. Test thermocouple stability
9. Reference junction temperature uncertainty
10. Bath or furnace temperature stability
11. Bath or furnace temperature non-uniformity
12. Extraneous emf of wiring, scanners, etc.

The uncertainties for the items labeled repeatability (3 and 5) are typically evaluated as Type A uncertainties, using statistical methods. The other items are primarily evaluated using Type B methods.

Instead of determining uncertainties for all the individual components, it is often convenient to repeat the calibration process multiple times (ideally at least 10 times, and preferably 20 times), analyze the data to obtain the emf of the test thermocouple at the nominal test temperature, and to take the standard deviation of the set of measured emf values. The standard deviation is a Type A measure of both bath stability and short-term repeatability of the measurement systems for the test and reference thermometer. This alternative approach is best suited for temperatures where drift of the test thermocouple is negligible.

In Table 1, a number of terms, such as the bath non-uniformity, are naturally expressed in units of temperature. Other terms, such as the extraneous emf, are conveniently expressed in units of voltage. The general conversion between an uncertainty in emf and the equivalent temperature uncertainty is given by $u(E) = u(t)/S_n(t)$, where $S_n(t)$ is the Seebeck coefficient of a thermocouple of type $n$, at temperature $t$. Care is needed in choosing the Seebeck coefficient if the reference temperature is being measured by a thermocouple of a different type than the test thermocouple. The variation of $S_n$ with temperature may be significant. For almost all of the terms in Table 1, $t$ is taken as the bath or furnace temperature, but to convert an uncertainty of reference junction temperature into equivalent emf, the temperature is taken as the reference junction temperature, typically 0 °C.

There are no significant correlations in the components listed in Table 1, with one minor exception discussed in Section 4.3. As a result, once all of the components are expressed in consistent units of either voltage or equivalent temperature, all components may be added together in quadrature to obtain the combined standard uncertainty. In the sections below, these components are discussed for two examples.
3. Example 1: type T thermocouple in a stirred liquid bath

In the first example, a type T thermocouple is calibrated in a stirred liquid bath, using a standard platinum resistance thermometer (SPRT) as a reference thermometer. The thermocouple emf is measured through a scanner by a digital multimeter. The SPRT is measured with an AC bridge, in conjunction with a temperature-controlled reference resistor [4].

To cover the range of calibration temperatures, several baths or bath fluids may be necessary. Bath stability and uniformity may vary substantially over the operating range of the bath, so it is appropriate to evaluate these terms at the upper, mid-point, and lower operating limits of each bath. On the other hand, the reference junction temperature and extraneous emf terms are independent of the bath temperature. All the other terms vary smoothly with temperature, and these terms may be evaluated in 100 °C intervals, with uncertainties at intermediate temperatures obtained by linear interpolation or fitting of a smooth curve.

3.1. Repeatability

A number of the entries in Table 1 either may be evaluated as a set, or may be evaluated in separate experiments. At NIST, independent tests of the repeatability of the SPRT in a triple-point-of-water cell and of the repeatability of the emf measurement systems in measuring stable 10 mV and 0 V sources demonstrated that these terms are negligible compared to bath stability. Bath stability is discussed in detail below.

Repeatability of the complete calibration process may be evaluated by taking the standard deviation, at each test temperature, of a set of emf values measured on check-standard thermocouples (check standards are artifacts that undergo repeated calibration cycles, for the purposes of statistical process control) [5,6]. Because calibration of thermocouples causes inhomogeneity in the tested thermocouples, it is absolutely necessary that any check standards subjected to repeated calibration should either be limited to use below approximately 150 °C or should be used at exactly the same immersion in the test apparatus. Alternatively, a single lot of thermocouple wire may be used as a source of single-use check standards (see Section 4.1).

3.2. Reference thermometer calibration and drift

The calibration uncertainty of the reference thermometer can generally be obtained from the calibration certificate. For an SPRT, this uncertainty is typically stated as the uncertainty of the resistance ratio \( W = R(t)/R(0.01 \, ^\circ C) \), where \( R(t) \) is the SPRT resistance at temperature \( t \). The uncertainty of the calibration certificate does not include:

- a. drift of the resistance-temperature relation with time,
- b. uncertainty of the user’s \( R(0.01 \, ^\circ C) \) determination, or
- c. uncertainty of measuring the resistance ratio.

If the reference thermometer is an industrial variety of PRT, the calibration certificate may state the calibration uncertainty directly in temperature. For this case, the certificate uncertainty does not include:

- a. drift of the resistance temperature relation with time, or
- b. uncertainty of measuring resistance in true ohms.

In addition, the calibration uncertainty may not include:

- c. reference thermometer hysteresis, which may be found from manufacturer’s literature.
All of these components should be converted to standard uncertainty, in units of equivalent temperature. In the evaluation of drift, it is useful to measure the SPRT regularly at the ice point or the triple point of water (TPW). Drift is often observed as a systematic increase in resistance with time. A simple calculation of the standard deviation of the ice point or TPW measurements is not an appropriate measure of the uncertainty. Instead, one should measure the ice point or TPW immediately after receipt of a newly calibrated thermometer. The uncertainty due to drift may be calculated by analyzing the deviation of subsequent readings from the initial TPW or ice point reading (e.g., model drift as a rectangular distribution with limits ±α equal to the maximum observed deviation). Drift in an SPRT can be compensated by routinely adjusting the value of R(0.01 °C) used in the W determination. For industrial PRTs, drift at elevated temperatures may or may not result in proportional drift at the ice point or TPW, and corrections cannot be made reliably.

3.3. Reference thermometer readout

Most choices of a properly functioning AC bridge and temperature-controlled reference resistor will have uncertainties that are small relative to other uncertainties for thermocouple calibrations. The standard uncertainty in the measurement of W may be computed by taking the manufacturer’s specification for the bridge as the limit of a rectangular distribution. The reference resistor will vary slightly due to a temperature change δt. Taking ±δt as the limits of a rectangular distribution for the maximum temperature deviations possible, the uncertainty in W from the combination of bridge and reference resistor uncertainty is

$$ u(W) = \left[ \left( \alpha \delta t / \sqrt{3} \right)^2 + \left( \delta W / \sqrt{3} \right)^2 \right]^{1/2} $$

where α is the temperature coefficient of the reference resistor. The corresponding uncertainty in units of indicated temperature is obtained by dividing by the derivative of the SPRT reference function, W_r: $u(t_{SPRT}) = u(W) / dW_r/dt$.

3.4. Test thermocouple readout

Manufacturers typically state the specifications for DC voltage measurements with a digital voltmeter as a fraction of the voltage E that is measured, plus a fraction of the range. For example, on a 100 mV range, the specifications may be quoted as 4×10^-6 of reading plus 3×10^-6 of range, which is mathematically expressed as a tolerance $a_V = (\beta E + \delta)$, where $\beta = 4 \times 10^{-6}$ and $\delta = 0.3 \, \mu V$. If the user does no further characterization of the voltmeter, then the standard uncertainty contributed by the voltmeter may be estimated by assuming that the manufacturer’s tolerance sets bounds ±$a_V$ for a rectangular distribution of measurement errors. The standard uncertainty of such a distribution is $a_V / \sqrt{3}$, or for the example above, $u_{dvm}(E) = (4 \times 10^{-6} E + 0.3 \, \mu V) / \sqrt{3} = (\beta E + \delta) / \sqrt{3}$.

In practice, we have found that it is possible with proper measurement techniques to greatly reduce the component of $u_{dvm}(E)$ that is independent of E, $u_{dvm,0}$, by measuring the thermal emfs of the voltmeter and scanner wiring (See Ref. [7]). If such a method is undertaken, the uncertainty in E becomes:

$$ u(E)^2 = u_{dvm,0}^2 + (\beta E / \sqrt{3})^2. $$
A simple method to determine \( u_{dvm,0} \) is to place both the measuring and reference junctions of a test thermocouple into an ice point, suitably protected from the water, and to measure the thermocouple emf at the ice point. Repeat this process for several times during a day, and for several days. The standard deviation of the measured emf can be taken as \( u_{dvm,0} \). Alternatively, the thermocouple may be removed and the copper leads in the reference-junction ice bath may be shorted with a single length of untinned copper wire, free of kinks and mechanical strain.

3.5. Thermocouple Inhomogeneity

Very little data has been published on the inhomogeneity of new, as-received thermocouples. The data of Hust et al. [8] reports an extensive set of measurements of thermocouple wire spanning the temperature range from room temperature to liquid nitrogen (−196 °C) and to liquid helium (−269 °C). Unfortunately, Hust et al. report for each manufacturer and each wire type only the maximum observed inhomogeneity, and give no additional statistical information. A reasonable approximation to thermocouple inhomogeneity is to take the average short-range inhomogeneity at liquid nitrogen reported in Table III of Ref. [8] as the standard uncertainty \( u(E(−196 °C)) \), and to scale this value to the measuring temperature to obtain the standard uncertainty:

\[
  u_{inh} = \left( \frac{u(E(−196 °C))}{E(−196 °C)} \right) E(t) = u_{r,inh} E(t) .
\]  

(3)

Table 2 gives values of \( u_{r,inh} \) obtained from combining results for the different manufacturers listed in Ref. [8].

There is a small body of evidence that either modern fabrication produces thermocouple wire substantially more homogeneous than typical wire produced at the time of the Hust work, or that Eq. 3 gives values that are too large. For example, Table 5 (see Section 4.1) gives the standard deviation in measured emf of samples of type K wire taken from a total length of 45 m, as obtained recently at NIST. At 1100 °C, the standard deviation of the readings was 2.5 µV, which includes repeatability of the comparison process as well as thermocouple inhomogeneity. For comparison, the scaled inhomogeneity values from Ref. [8] for type K thermocouples at 1100 °C range from 7 µV to 25 µV, which is considerably larger. More data on recently produced wire needs to be obtained before a definitive uncertainty can be assigned to thermocouple inhomogeneity of as-received wire. Thermocouple drift, discussed in the next section, causes additional inhomogeneity of the thermoelements.

<table>
<thead>
<tr>
<th>( u_{r,inh} )</th>
<th>type E</th>
<th>type J</th>
<th>type K</th>
<th>type T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>1.2×10^{-4}</td>
<td>3.0×10^{-4}</td>
<td>1.5×10^{-4}</td>
<td>1.7×10^{-4}</td>
</tr>
<tr>
<td>Max</td>
<td>4.6×10^{-4}</td>
<td>1.0×10^{-3}</td>
<td>5.6×10^{-4}</td>
<td>5.5×10^{-4}</td>
</tr>
<tr>
<td>Average</td>
<td>3.0×10^{-4}</td>
<td>6.8×10^{-4}</td>
<td>3.5×10^{-4}</td>
<td>4.1×10^{-4}</td>
</tr>
</tbody>
</table>

3.6. Thermocouple Drift

Thermocouple materials undergo significant drift at temperatures as low as 200 °C [9]. Thermocouple drift at elevated temperatures is well documented. Much less data are available at
temperatures in the range significantly below the upper usage temperature, but above 200 °C. Much of the available date focuses on drift during use. In fact, the act of calibration itself may cause measurable drift in the few hours of exposure necessary to complete a calibration. Table 3 briefly summarizes the results of tests at NIST on type T and type K thermocouple wire. In one method (the “heat” method) the test thermocouple is inserted into the furnace and remains at fixed immersion throughout the test. In the second method (the “plunge” method), the furnace is adjusted to the desired temperature, the test and reference thermocouple are then plunged into the furnace, emf values are read as soon as thermal steady state is attained, and then the thermocouples are removed from the furnace prior to the next test point. The average time of heating at each test point is 20 min for the plunge method and 80 min for the heat method. A full description of this work will be published elsewhere. For bare-wire thermocouples, the values in Table 3 are only applicable for temperatures below the ASTM recommended upper temperature limit. Table 4 gives the results of some more precise measurements for the single temperature of 200 °C. Reasonable choices for the standard uncertainty due to thermocouple drift would be 0 for temperatures below 100 °C, average values from Table 4 for 200 °C, and average values from Table 3 for temperatures above 200 °C.

Table 3. Difference in measured emf between “plunge” and “heat” calibrations.

<table>
<thead>
<tr>
<th>Temperature °C</th>
<th>Type T (µV)</th>
<th>Type K (µV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>4 to 10</td>
<td>1.4 to 6</td>
</tr>
<tr>
<td>400</td>
<td>4 to 10</td>
<td>-</td>
</tr>
<tr>
<td>500</td>
<td>-</td>
<td>5 to 15</td>
</tr>
<tr>
<td>800</td>
<td>-</td>
<td>10 to 15</td>
</tr>
<tr>
<td>1000</td>
<td>-</td>
<td>Highly variable</td>
</tr>
</tbody>
</table>

Table 4. Typical cumulative drift for first 2 h of exposure at 200 °C.

<table>
<thead>
<tr>
<th>Temperature °C</th>
<th>Type E (µV)</th>
<th>Type J (µV)</th>
<th>Type K (µV)</th>
<th>Type N (µV)</th>
<th>Type T (µV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>+2 to +4</td>
<td>+1 to +2</td>
<td>−0.6 to +0.6</td>
<td>−1 to −2</td>
<td>+1 to +2</td>
</tr>
</tbody>
</table>

Thermocouple drift is directly related to the accumulation of additional thermocouple inhomogeneity. As a thermocouple is exposed to elevated temperatures, only the portion at the elevated temperatures is subject to metallurgical changes (either physical or chemical). A scan of the Seebeck coefficient along the length of the wire will demonstrate that the portion of the wire maintained at room temperature will have the same value as the original specimen, but the wire section exposed to elevated temperature will be altered. If a used thermocouple is shifted in position or if it is calibrated in an apparatus that has a different thermal profile than the usage apparatus, the emf will vary relative to the emf response in the original position. Figure 1 gives an example of the emf variations of type T and type K thermocouples that have been exposed to 200 °C for one year at a fixed immersion. In the figure, the in situ drift appears as a vertical displacement of the data at 0 cm deviation from normal immersion. At reduced immersion depths, the cumulative emf drift of the aged thermocouples can be three times larger than the
drift at a fixed immersion depth. Because the measured emf of an inhomogeneous thermocouple is very dependent on the depth of immersion, recalibration of used, inhomogeneous thermocouples is subject to large errors not covered by the uncertainty budgets described in this paper. Noble metal thermocouples can be reannealed to remove certain types of inhomogeneity, but base-metal thermocouples should be calibrated in situ [10]. As an alternative, base metal thermocouples may be replaced periodically, at an interval established from the results of in situ calibrations, literature values for drift rate, or other process measurements.

Figure 1. Change in emf of representative type T (squares) and type K (circles) thermocouples after 400 days of exposure at 200 °C. Different shades indicate different thermocouples

3.7. Reference Junction Temperature
The standard uncertainty of a well-prepared ice bath is 0.001 °C [11], which is negligible for thermocouple calibrations. However, verification of proper immersion in the ice bath is often neglected. Large diameter thermocouples and copper wires (as with a type T thermocouple) may conduct a significant amount of heat. The measuring junction end of the test thermocouple, inserted into a glass tube to keep it dry, may be placed at a fixed depth of immersion into a second ice bath. After equilibrium of both reference junctions and measuring junctions is established, the reference junctions are raised in 1 cm steps and the emf measured. Systematic variations in the emf with depth of immersion are an indication of insufficient immersion. If this is observed, the problem is best fixed rather than adding a component to the uncertainty. See Fig. 2 for an example immersion profile.

For calibrations of long duration and for ice baths located near hot baths, care should be taken that the ice bath remains sufficiently full of ice over the course of the calibration. A test similar
to the test of immersion depth can be made, where the measuring junction is kept in a second ice bath, and the emf is measured first with an ice bath at the end of a calibration and then with the ice in the reference bath refreshed.

Figure 2. Temperature $t_{\text{bath}}$ read by a copper versus platinum thermocouple, as a function of immersion depth into the ice bath, taking full immersion (20 cm) as 0 °C. Within the measurement uncertainty, the measuring junction is in thermal equilibrium with the ice bath for immersions greater than 15 cm, for this particular setup.

3.8. Bath stability
Most often, bath stability is characterized by measuring the temperature fluctuations of the stirred liquid bath with the reference thermometer over a time span long enough to capture typical fluctuations in the bath temperature (30 min to 60 min). A slow drift of the bath temperature does not affect the calibration process; what matters most is the variation of the bath temperature on time scales shorter than maximum of a. the time needed to measure both the reference and test thermometers, and b. the response times of each thermometer. A typical window span is approximately 10 min. If slow drift is observed, the bath variation can be computed by calculating the standard deviation over a 10 min window, and then computing an average standard deviation by sweeping the window over the full measurement span. Ref. [12] gives an example of bath stability measurements.

An alternative method is to repeat the calibration process multiple times as described in Section 2.

3.9. Bath uniformity
Bath uniformity may be measured by placing a reference thermometer in the position typically used for the reference thermometer to measure $t_{\text{fix}}$, and then measuring the bath at other locations
within the useable volume of the bath with a second thermometer to measure $t_{\text{move}}$. Taking the
difference between the two thermometers results in a measurement that is largely insensitive to
temporal fluctuations of the bath temperature. The standard uncertainty can be computed by one
of two methods:
a. if only a few locations are probed near the extremities of the useable volumes, the maximum
of $|t_{\text{move}} - t_{\text{ref}}|$ can be taken as the limit $a$ of a rectangular distribution, and

$$u(\text{bath uniformity}) = |t_{\text{move}} - t_{\text{ref}}|_{\text{MAX}} / \sqrt{3}.$$  

b. if approximately 10 or more locations are measured, the standard uncertainty may be
obtained from the rms average of $t_{\text{move}} - t_{\text{ref}}$:

$$u(\text{bath uniformity}) = \left(\frac{(t_{\text{move}} - t_{\text{ref}})^2}{\text{MAX}}\right)^{1/2}.$$  

See Ref. [12] for more details of this method and some sample results.

If two thermometers are not available, it is possible to vary the position of a single thermometer
within the useable measuring volume. The standard deviation of the single thermometer over all
positions gives a combined measurement of both bath stability and uniformity.

The “useable volume” may be intentionally restricted in size in order to give better uniformity,
but in all cases the sensors in actual calibrations should reside within the defined volume. Bath
uniformity may degrade with time, due to several possible mechanisms: water absorption by
fluids used below ambient temperature; evaporation of volatile components or chemical
degradation of high-temperature fluids; or partial failure of bath heating elements. Thus, periodic
validation of the bath uniformity or liquid properties is needed.

3.10. **Extraneous emf**

Techniques for minimizing extraneous emf are given in Ref. [7]. A useful method for measuring
extraneous emf is to simultaneously short out each of the various measurement channels at the
ice bath. For the shorting wire, a single length of untinned copper wire, free of kinks and
mechanical strain, may be used. All of the measurement channels are measured, using the same
software as used for acquiring calibration data, and with the wiring in the same thermal
environment as experience during calibration. At NIST, one of our measurement channels is
always shorted with a copper wire at the junction box used to connect thermocouples to the
scanner, and this channel is designated as our zero-voltage reference. If the measured emf of
other channels relative to this reference zero is repeatable, as measured at several times of the
day and on different days, then corrections may be applied, and the uncertainty for any channel is
taken as the standard deviation of the emf of that channel minus the measured emf of the
reference channel. These measurements are very similar to those described in Section 3.4.

4. **Example 2. Type S thermocouple in a tube furnace**

In the second example, a type S thermocouple is calibrated in a tube furnace, using a calibrated
type S thermocouple as a reference thermometer. The test and reference thermocouple are
welded together to form a common measuring junction. The thermocouple emf is measured
through a scanner and digital multimeter, as with the type T example. The reference
thermocouple and test thermocouple are measured with the same voltmeter.
The methods for determining the repeatability, test thermocouple readout, test thermocouple stability, reference junction temperature, and extraneous emf of the wiring are all the same as for the type T example. At NIST, a number of the uncertainty components were determined by examining the differences between results of comparison calibrations and calibrations done with thermometric fixed points. See Ref. [5,6] for a discussion of this method.

Below, the remaining components are discussed assuming that fixed-point cells are not available.

4.1. **Repeatability**
The general considerations discussed in Section 3.1 apply. As a particular example of calibration of a type K thermocouple in a tube furnace, Table 5 gives the results of measuring five cuts of the same lot of type K thermocouple wire in a tube furnace. The standard deviation at each temperature gives the combined repeatability of the calibration process, including the inhomogeneity of the particular lot of tested thermocouple wire.

<table>
<thead>
<tr>
<th>Temperature / °C</th>
<th>Cut A</th>
<th>Cut B</th>
<th>Cut C</th>
<th>Cut D</th>
<th>Cut E</th>
<th>SD / µV</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>4117.8</td>
<td>4116.3</td>
<td>4115.6</td>
<td>4115.4</td>
<td>4116.0</td>
<td>1.0</td>
</tr>
<tr>
<td>200</td>
<td>8143.8</td>
<td>8141.2</td>
<td>8139.7</td>
<td>8138.7</td>
<td>8141.0</td>
<td>1.9</td>
</tr>
<tr>
<td>400</td>
<td>16413.2</td>
<td>16409.8</td>
<td>16406.5</td>
<td>16406.9</td>
<td>16411.0</td>
<td>2.8</td>
</tr>
<tr>
<td>500</td>
<td>20687.7</td>
<td>20685.4</td>
<td>20681.5</td>
<td>20680.3</td>
<td>20686.2</td>
<td>3.2</td>
</tr>
<tr>
<td>600</td>
<td>24954.8</td>
<td>24954.2</td>
<td>24948.2</td>
<td>24950.6</td>
<td>24954.4</td>
<td>2.9</td>
</tr>
<tr>
<td>800</td>
<td>33322.5</td>
<td>33321.6</td>
<td>33319.5</td>
<td>33319.7</td>
<td>33323.9</td>
<td>1.8</td>
</tr>
<tr>
<td>1000</td>
<td>41321.5</td>
<td>41319.2</td>
<td>41316.5</td>
<td>41316.2</td>
<td>41318.5</td>
<td>2.2</td>
</tr>
<tr>
<td>1100</td>
<td>45150.2</td>
<td>45146.7</td>
<td>45144.7</td>
<td>45144.7</td>
<td>45148.9</td>
<td>2.5</td>
</tr>
</tbody>
</table>

4.2. **Reference thermometer calibration and drift**
The calibration results of the type S reference thermometer may be taken directly from the calibration report.

The drift of the type S reference thermocouple will depend on the time of exposure to elevated temperatures, degree of chemical contamination, and possibly work hardening of sections of the wires not exposed to temperatures above about 300 °C. A useful method for measuring drift is to use a newly calibrated reference thermometer to calibrate a used reference thermometer that has a documented number of calibration cycles. Differences in the emf versus temperature response of the used reference between the original and recent calibration can be ascribed to drift in the thermocouple. Knowledge of the typical drift rate can then be used to set a maximum number of calibration cycles for a reference thermometer. As in Section 3.2, the uncertainty due to drift may then be modeled as a rectangular distribution with limit ±a₅ equal to the maximum allowable deviation.
4.3. Reference thermometer readout
When the same voltmeter is used to read both the test and reference thermometer, measurement errors are correlated, and in large part cancel out. When the test and reference thermometer are the same type of thermocouple, the errors cancel out nearly completely, and the uncertainty in the emf of the test thermocouple is dominated by other terms, particularly the repeatability of the readings and extraneous emf contributions.

4.4. Furnace temperature stability
The effect of furnace stability can be studied by taking a series of calibration points at approximately the same temperature, but at varying rates of temperature change with time. A plot of the apparent test thermocouple emf versus rate of temperature change will indicate the sensitivity of the measured emf to the rate. Data acquisition software can be designed to insure that the rate of change of furnace temperature does not exceed preselected limits \( \pm a_{\text{stab}} \), and the standard uncertainty is then \( a_{\text{stab}}/\sqrt{3} \).

4.5. Furnace temperature non-uniformity
As a rule of thumb, we have found that for test and reference thermocouples welded together, furnace uniformity is much more important than furnace stability in obtaining good quality calibrations. When a comparison apparatus with good thermal uniformity is available, comparison of calibration results from the furnace and this other apparatus are useful in assigning an uncertainty to the furnace. Figure 3 shows the difference in emf readings, expressed in units of equivalent temperature, between calibrations in a tube furnace and in stirred liquid baths. Since the stirred liquid baths are known to be highly uniform, the difference between the two calibration results is attributed to thermal non-uniformity of the furnace. The fit, extrapolated to higher temperatures, is a reasonable estimate of the standard uncertainty due to furnace nonuniformity.

In the absence of tests in fixed points or stirred liquid baths, there are no good methods that we are aware of for the accurate determination of calibration uncertainties due to non-uniformity of tube furnaces. Qualitatively, it is useful to perform calibrations at several depths of immersion and then examine the variation of the test thermocouple emf as a function of immersion depth. Quantitatively, there is not yet an accepted algorithm to convert results for test emf versus immersion depth into a standard uncertainty. Care must be taken with the measurements so that the results are not complicated by thermocouple inhomogeneity induced during the test. The test should be conducted relatively quickly, and preferably in the direction of increasing depth of immersion into the furnace. After one immersion into the furnace, the base-metal test thermocouple should be replaced, or noble-metal thermocouples given an anneal to restore thermoelectric uniformity [7].
Figure 3. Difference in calibration results for a lot of type K thermocouple wire, expressed as the apparent difference in indicated temperature $t_i$, between a calibration in a tube furnace and a calibration in stirred liquid baths.

5. Conclusions
The examples given here are not exhaustive: there are numerous combinations of equipment and procedures that will require slightly different methods of determining uncertainty. For each calibration method, a systematic approach that considers each entry in Table 1 will ensure that no significant contributions to the uncertainty are ignored.

6. References