First-principles phase diagram calculations for the HfC–TiC, ZrC–TiC, and HfC–ZrC solid solutions

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We report first-principles phase diagram calculations for the binary systems HfC–TiC, TiC–ZrC, and HfC–ZrC. Formation energies for superstructures of various bulk compositions were computed with a plane-wave pseudopotential method. They in turn were used as a basis for fitting cluster expansion Hamiltonians, both with and without approximations for excess vibrational free energies. Significant miscibility gaps are predicted for the systems TiC–ZrC and HfC–TiC, with consolute temperatures in excess of 2000 K. The HfC–ZrC system is predicted to be completely miscible down to 185 K. Reductions in consolute temperature due to excess vibrational free energy are estimated to be ~7%, ~20%, and ~0%, for HfC–TiC, TiC–ZrC, and HfC–ZrC, respectively. Predicted miscibility gaps are symmetric for HfC–ZrC, almost symmetric for HfC–TiC and asymmetric for TiC–ZrC.

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I. INTRODUCTION

Transition metal carbides, including the NaCl-structured group IV (Ti, Zr, and Hf) carbides, have extremely high melting points and are therefore referred to collectively as the “refractory carbides.” In addition to their high-temperature stabilities, these compounds exhibit interesting physical properties such as high hardnesses, high electrical conductivity, and superconductivity.1 These properties make them suitable as bulk or thin-film materials in many technological applications: they are used as first-wall coatings for fusion reactors, protective coatings for cutting tools,2 and low-friction coatings for bearings.3 Their hardness is retained to very high temperature, and they have low chemical reactivity.1,4 At room temperature, they are susceptible to attack only by concentrated acid or base in the presence of oxidizing agents, and retain good corrosion resistance to high temperature.1,5,6 The transition-metal carbides have also been explored for a potential application as diffusion barriers in electronic devices.7 Aside from the pure end members, solid solutions formed by these carbides are potentially of significant importance as properties can be optimized by varying the compositions in the binary—and higher order—systems.

Systematic experimental investigations of the phase diagrams and physical properties of such nonstoichiometric compounds are extremely challenging as many compositions need to be explored. The high melting points and limited solubilities of group IV carbides complicate experimental studies.8–10 Phase diagram calculations allow one to establish the shape of the phase diagram with semi-quantitative accuracy. Several semi-empirical methods exist to construct phase diagrams based on approximate free-energy functions such as the model of subregular solutions4,11,12 fitted to existing experimental data. Such thermodynamic models have also been applied to group IV carbides.13–15

Although these methods are useful to fit phase diagram data and approximate thermodynamic functions, it is desirable to compute phase diagrams strictly from knowledge of the constituent atoms without empirical parameters. To calculate phase equilibria the Gibbs free energy must be known for competing structures over a wide range of concentrations, and consequently the formation energy (ΔEy) must be computed for a large number of phases along the solid solution.

At present, the three most widely employed approaches for this purpose can be summarized as follows. The most direct approach is the supercell method (e.g., Refs. 13 and 16–20), which is commonly employed to analyze local atomic structures and variations in bond lengths with chemical composition. The supercell method is based on the principle of spatial ergodicity, according to which all possible finite atomic arrangements are realized in a single infinite sample. This approach is computationally prohibitive when the size of the supercell is on the order of hundreds of atoms. In contrast to the supercell approach, methods based on perturbation theory perform the configurational average analytically. The most well-known approximation within alloy theory is the coherent-potential approximation (CPA) (e.g., Refs. 21–23). Recently it has also been demonstrated how short-range order effects can be incorporated in total-energy calculations using a nonlocal modification of the CPA.33 Within the CPA-based methods no self-consistent calculation of elastic relaxation energies has been demonstrated to date, and such effects are generally treated using separate theoretical frameworks.24–26 A third option is the cluster expansion (CE) approach.27–31 It allows incorporating contributions to solid-solution energies arising from short-range order as well as elastic relaxations. In the CE method effective cluster interactions (ECIs) are fit to an extensive set of ΔEy for supercells in the binary system, which is then explored to compute the phase diagram from Monte Carlo simulations. This approach is implemented in the alloy theoretic automated tool-kit (ATAT) code,32–34 which has successfully been applied to the study of phase stabilities in intermetallics, alloys, and pseudobinary systems (e.g., Refs. 35–40).

Here we explore by means of the CE method the subsolidus phase diagram of the binary refractory carbides HfC–
TiC, ZrC–TiC, and HfC–ZrC, by combining the ATAT package with *ab initio* total-energy calculations. We look in detail at the extent of miscibility between the phases and the consolute temperature \((T_C)\) and composition \((X_C)\). As experimental data and thermodynamic assessments of these systems are available we compare predictions from the computations to measurements.

## II. METHODOLOGY

Formation energies, \(\Delta E_f\), were calculated for transition-metal carbides HfC, TiC, and ZrC and many \(M_{m}M'_{n}C_{m+n}\) supercells, in which \(M\) and \(M'\) are Hf, Ti, or Zr. All electronic-structure calculations were performed with the Vienna *ab initio* simulation program (VASP) using ultrasoft Vanderbilt-type pseudopotentials with the generalized gradient approximation (GGA) for exchange and correlation. Valence electron configurations for the pseudopotentials are Hf\(=5p^66s^25d^2\), Ti\(=3d^34s^1\), Zr\(=4s^24p^65s^24d^2\), and C \(=2s^22p^2\). Total energy calculations were converged with respect to \(k\)-point sampling, and a plane-wave energy cutoff of 500 eV was used which yields \(\Delta E_f\) values that are converged to within a few meV per cation (Hf, Ti, and Zr). Cell constant and ionic positions were fully relaxed in all supercell computations.

Based on these results the first-principles phase diagram (FPFD) calculations were performed with the ATAT software package. The ECI that define the CE are obtained by a least-squares fit of a subset of results for \(\Delta E_f\) that are in turn used to predict \(\Delta E_f\) of supercells computed in VASP but not included in the fit. The quality of the prediction is measured by the cross-validation score (CVS); unlike the traditional root-mean-square error, the CVS avoids overfitting the data, because it is able to detect the loss of predictive power resulting from an excessively large number of adjustable parameters in the fit. The optimal cluster sets were determined by minimizing the CVS between the *ab initio* computations and the CE prediction.

Contributions of lattice vibrations \(\left(\frac{\partial^2E}{\partial a^2}\right)\) to the free energies were approximated to reduce the computational burden of calculating phonon densities of states for a large set of superstructures, the bond-length-dependent transferable force-constant scheme was employed. As discussed in Ref. 36 nearest-neighbor force constants were obtained for the end-members HfC, TiC, and ZrC as functions of imposed lattice parameters: in each system the lattice constants are varied between the end-member equilibrium, and forces are computed for five to six values. Depending on the variations in lattice constant this can lead to slightly different force constants for the same end member, e.g., HfC in the TiC–HfC and the HfC–ZrC systems.

The resulting bond stiffness versus bond-length relationships were applied to predict force constants for all superstructures, using the relaxed bond lengths and the chemical identities of bonds as predictors of their stiffness. The quantum-mechanical expression for the free energy was used, rather than the typical high-temperature (classical) limit. The resulting free energies were fit to temperature-dependent CE, which served as input for grand-canonical

![Figure 1](134112-a.png)
The CEs were fit to a moderate number of structures to obtain a good CVS (Table I). They include pair interactions only, or pair plus one triplet interaction (Table I and Fig. 2). Convergence of the effective cluster interactions Fig. 2 is relatively fast, with pair interactions of less than the fifth-nearest-neighbor sufficient to obtain converged CEs. This contrasts with studies of metallic alloys, e.g., Al-TM (TM = Ti, Zr, and Hf) where interactions beyond the 10th neighbor shell are required, and a large number of many-body (triplet and four-body) clusters need to be included. As expected, the ECIs exhibit a general trend of decreasing in amplitude with distance except for a significant negative value at four times the nearest-neighbor distance. This corresponds to ordering of cations of the same type on second-neighbor sites along the cell edges. For all three systems, the resulting CVS is significantly below 10 meV/atom, i.e., a small fraction of the formation energies.32 We have explored different CEs with additional structures to explore the ECI behavior at four times the nearest-neighbor distance and found the feature to be robust. The presence of three-body terms Table I in the CE Hamiltonians for HfC–TiC and TiC–ZrC implies asymmetric immiscibility in these systems.

Figure 3 shows volumes as functions of bulk composition from the cluster expansion. Trends for all three systems are close to linear—Vegard’s law—with a fit for HfC–ZrC that is within computational error of ideal mixing. Positive deviations from ideality are predicted for HfC–TiC and TiC–ZrC. Excess volumes of mixing in these systems imply immiscibilities that would persist under pressure. In contrast, immiscibility with negative volumes of mixing, as in the wurtzite-structured nitrides,37 implies decreased immiscibility under pressure.

The dependence of the stretching and bending force constants on bond length are shown in Fig. 4. Stretching constants decrease monotonically with increasing bond length; bending terms, by contrast, are relatively insensitive. The Ti-C stretching and bending constants in HfC–TiC and TiC–ZrC plot as a single curve. Thus, Ti-C force constants are transferable between the HfC–TiC and TiC–ZrC systems. This effect was also predicted for Au-Cu, Au-Pd, and Cu-Pd systems.45 In the Al-TM systems, however, the Al-Al bond force constants exhibited significant differences in the Al-Ti, Al-Zr, and Al-Hf systems.39

Subsolidus phase diagrams of the binary systems are computed with and without $F_{\text{vib}}$(Fig. 5). As expected from the presence of three-body terms in the ECIs for HfC–TiC and TiC–ZrC (Table I and Fig. 2) the corresponding phase diagrams are asymmetric.

The system HfC–TiC exhibits almost complete immiscibility below ~750 K and there is slight asymmetry with

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**Table I. Characteristics of the calculated cluster expansions.**

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>HfC–TiC</th>
<th>TiC–ZrC</th>
<th>HfC–ZrC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of structures</td>
<td>45</td>
<td>41</td>
<td>49</td>
</tr>
<tr>
<td>Number of clusters</td>
<td>2+11+1</td>
<td>2+10+1</td>
<td>2+12</td>
</tr>
<tr>
<td>Cross-validation scores (meV/atom)</td>
<td>7.1</td>
<td>5.2</td>
<td>0.9</td>
</tr>
</tbody>
</table>
The mismatch is largest in TiC-ZrC and almost nonexistent in HfC–ZrC, corresponding to the largest and smallest $T_C$, respectively. This suggests that immiscibility in group IV carbides is caused by ionic size effects. Similarly, larger values for $\%\Delta R_{ij}$ yield larger reductions in $T_C (\Delta T_C)$ induced by $F_{\text{vib}}$. As a consequence, the highest $\Delta T_C$ occurs in the system with the largest $T_C$. This is in contrast to the wurtzite-structured nitrides in which $T_C$ reductions due to $F_{\text{vib}}$ are less significant.

Considerations of elastic energy provide an alternative way to assess the energetics of cation substitutions and rationalizing the asymmetries of phase diagrams. Here we compute the elastic energy by the $e-G$ approximation for the three binary systems: The equations of state of end-members HfC, TiC, and ZrC are computed, i.e., $E(V)$ where the volume of the cell for the end members are varied and then transformed to $E(X)$ via the CE volumes. Finally, $\Delta E(X)$ is approximated by a linear combination of end-member equations of state (e.g., for HfC–TiC, $\Delta E(X) = (1 - X)E_{\text{HfC}}(X) + XE_{\text{TiC}}(X)$). The resulting $\Delta E(X)$ for HfC–ZrC...
is symmetric (Fig. 7), for HfC–TiC and TiC–ZrC it is asymmetric with maxima closer to the end member with the smaller ion\textsuperscript{46} (Fig. 7). This implies that more energy is required to replace a smaller ion with a larger one than vice versa and that the deviation of $X_C$ from 0.5 is directly correlated with the difference in cation radii\textsuperscript{49} Fig. 6. This is consistent with the wurtzite-structured nitrides,\textsuperscript{37} and the quasi-binary system TiC–ZrC.\textsuperscript{50}

IV. CONCLUSIONS

First-principles phase diagram calculations for the binary systems HfC–TiC, TiC–ZrC, and HfC–ZrC predict miscibility gaps with consolute temperatures, $T_C = 2120$, 2695, and 185 K, respectively. Including $F_{\text{vib}}$ reduces $T_C$ by $\sim 7\%$.

![Fig. 6](image-url) (a) (Color online) Variation in the consolute temperature $T_C$ as a function of the percentage difference in the ionic radii of exchangeable ions from the calculations with (triangles) and without $F_{\text{vib}}$ (squares). Experimental data are included from Refs. 8 and 9. (b) Related percentage reduction in $T_C$ from including $F_{\text{vib}}$ in the cluster expansion Hamiltonian.

![Fig. 7](image-url) FIG. 7. (Color online) Elastic energy estimated by the $\epsilon$–$G$ approximation for the systems HfC–TiC, TiC–ZrC, and HfC–ZrC. The dotted line connects the maxima.

### TABLE II. Calculated consolute points.

<table>
<thead>
<tr>
<th>System method</th>
<th>Without $F_{\text{vib}} {X_C, T_C(K)}$</th>
<th>With $F_{\text{vib}} {X_C, T_C(K)}$</th>
<th>Method references</th>
</tr>
</thead>
<tbody>
<tr>
<td>HfC–TiC</td>
<td>0.50, 2275</td>
<td>0.50, 2120</td>
<td>FPPD\textsuperscript{a}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.55, 2133</td>
<td>Exp.\textsuperscript{b}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.55, 2053</td>
<td>Exp.\textsuperscript{c}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.60, 2173</td>
<td>SE\textsuperscript{d}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.56, 2075</td>
<td>SR\textsuperscript{e}</td>
</tr>
<tr>
<td>TiC–ZrC</td>
<td>0.50, 3350</td>
<td>0.41, 2695</td>
<td>FPPD\textsuperscript{a}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.45, 2273</td>
<td>Exp.\textsuperscript{b}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.45, 2373</td>
<td>SE\textsuperscript{d}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.35, 2281</td>
<td>SR\textsuperscript{e}</td>
</tr>
<tr>
<td>HfC–ZrC</td>
<td>0.50, 200</td>
<td>0.50, 185</td>
<td>FPPD\textsuperscript{a}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\sim$, &lt;300</td>
<td>SR\textsuperscript{e}</td>
</tr>
</tbody>
</table>

\textsuperscript{a}FPPD=First-principles phase diagram; this work.
\textsuperscript{b}Experimental results from Kieffer et al. (Ref. 8).
\textsuperscript{c}Experimental results from Rogl et al. (Ref. 9).
\textsuperscript{d}SE=Combined CALPHAD modeling with experimental data and \textit{ab initio} results from Markström et al. (Ref. 13).
\textsuperscript{e}SR=Subregular model from Gusev (Ref. 14).
\( \tilde{\text{\~{A}}} \text{DJAOU\text{D}} \) et al. reported for NaCl-KCl. 36 Miscibility gaps are predicted for ZrC system the \( \text{F} \) isive immiscibility in the systems with high consolute tem-
zuC, and an asymmetric phase diagram for TiC–ZrC. Exten-
diagram for HfC–TiC, a symmetric phase diagram for HfC–
material properties through ionic substitutions. For the car-
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are directly correlated with differences in ionic radii of the
exchangeable cations, with larger ions more readily replaced
for these systems, respectively. In the TiC-
limits optimization of the exchangeable cations, with larger ions more readily replaced
by smaller ones than vice a versa.

Acknowledgments

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