ABSTRACT
This paper is a continuation of a recent ASME Conference paper entitled "Design of a Python-Based Plug-in for Benchmarking Probabilistic Fracture Mechanics Computer Codes with Failure Event Data" (PVP2009-77974). In that paper, which was co-authored by Fong, deWit, Marcal, Filliben, Heckert, and Gosselin, we designed a probability-uncertainty plug-in to automate the estimation of leakage probability with uncertainty bounds due to variability in a large number of factors. The estimation algorithm was based on a two-level full or fractional factorial design of experiments such that the total number of simulations will be small as compared to a Monte-Carlo method. This feature is attractive if the simulations were based on finite element analysis with a large number of nodes and elements. In this paper, we go one step further to derive a risk-uncertainty formula by computing separately the probability-uncertainty and the consequence-uncertainty of a given failure event, and then using the classical theory of error propagation to compute the risk-uncertainty within the domain of validity of that theory. The estimation of the consequence-uncertainty is accomplished by using a public-domain software package entitled "Cost-Effectiveness Tool for Capital Asset Protection, version 4.0, 2008" (http://www.bfrl.nist.gov/oae/ or NIST Report NISTIR-7524), and is more fully described in a companion paper entitled "An Economics-based Intelligence (EI) Tool for Pressure Vessels & Piping (PVP) Failure Consequence Estimation," (PVP2010-25226, Session MF-23.4 of this conference). A numerical example of an application of the risk-uncertainty formula using a 16-year historical database of probability and consequence of main steam and hot reheat piping systems is presented. Implication of this risk-uncertainty estimation tool to the design of a risk-informed in-service inspection program is discussed.
Keywords: ASME B&PV Code; Error propagation; failure consequence; failure probability; nuclear power plant; probabilistic risk assessment; risk uncertainty formula.

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I. INTRODUCTION

It has been almost forty years since the U.S. Atomic Energy Commission initiated in 1972 a study of the safety of civilian nuclear reactors. This study, known as the WASH-1400 report [1], most likely gave birth to the method of probabilistic risk assessment (PRA) as we know it today, by using reliability analysis tools available in the defense and space industries to predict the effect of failures of small components in large, complex nuclear systems.

As shown in Figure 1, one of the results of this study compared the public risk from nuclear power with risks from other natural phenomena and industrial accidents. What was astonishing to the first author (Fong), who was then serving a one-year Commerce Science and Technology Fellowship at the Office of Mr. W. A. Anders, the first Chairman of the newly established U.S. Nuclear Regulatory Commission (NRC), was the fact that in the WASH-1400 report, the existence of uncertainty in component failure probabilities and consequence estimation was documented, and yet in the summary of the report, the results were plotted (see Figure 1) as if the numerical data were deterministic (i.e., without uncertainty).

In September 1978, an independent review group appointed by USNRC issued a report [2] that questioned many of the assumptions and conclusions of the WASH-1400 study. This report and other public controversies surrounding the use of nuclear power caused the NRC commissioners to withdraw endorsement of the “Executive Summary” of WASH-1400, and consequently dealt a major set-back to PRA. It took a reactor accident at Three Mile Island (TMI) in the following year (March 28, 1979) and many studies thereafter (see, e.g., [3, 4, 5]) for engineers to regain the appreciation of PRA as an indispensable tool for identifying vulnerabilities and the relative safety importance of reactor systems and components. A sobering remark in the Kemany report [3, p.15] is worth quoting to emphasize the need for a tool such as PRA:

“While throughout this entire document, we emphasize that fundamental changes are necessary to prevent accidents as serious as TMI, we must not assume that an accident of this or greater seriousness cannot happen again, even if the changes we recommend are made. Therefore, in addition to doing everything to prevent such accidents, we must be fully prepared to minimize the potential impact of such an accident on public health and safety, should one occur in the future.” [Bold by authors of the present paper.]

The purpose of the present paper is two-fold: (a) To briefly review the history since 1972 and the state of the art of PRA as it is practiced today, and (b) to derive a risk-uncertainty formula that will enable engineers to treat risk as a stochastic rather than a deterministic quantity, the latter of which was on clear display in 1975 (see Fig. 1).

Table 1. Perception of Risk of Nuclear vs. Electric (non-nuclear) Power by a sample of 3 groups of diverse people (after Ayyub [9])

<table>
<thead>
<tr>
<th>Activity or Technology</th>
<th>League of Women</th>
<th>College Students</th>
<th>Experts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nuclear power</td>
<td>1</td>
<td>1</td>
<td>20</td>
</tr>
<tr>
<td>Motor vehicles</td>
<td>2</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>Hand guns</td>
<td>3</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Smoking</td>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Motorcycles</td>
<td>5</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>Alcoholic beverages</td>
<td>6</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>General aviation</td>
<td>7</td>
<td>15</td>
<td>12</td>
</tr>
<tr>
<td>Police work</td>
<td>8</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>Pesticides</td>
<td>9</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Surgery</td>
<td>10</td>
<td>11</td>
<td>5</td>
</tr>
<tr>
<td>Firefighting</td>
<td>11</td>
<td>10</td>
<td>18</td>
</tr>
<tr>
<td>Heavy construction</td>
<td>12</td>
<td>14</td>
<td>13</td>
</tr>
<tr>
<td>Hunting</td>
<td>13</td>
<td>18</td>
<td>23</td>
</tr>
<tr>
<td>Spray cans</td>
<td>14</td>
<td>13</td>
<td>25</td>
</tr>
<tr>
<td>Mountain climbing</td>
<td>15</td>
<td>22</td>
<td>28</td>
</tr>
<tr>
<td>Bicycles</td>
<td>16</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Commercial aviation</td>
<td>17</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>Electric (non-nuclear) power</td>
<td>18</td>
<td>19</td>
<td>9</td>
</tr>
</tbody>
</table>

Before reviewing the history of a risk-based engineering practice in the nuclear power industry, it is necessary to understand the public perception of risk vs. that of a nuclear engineer. The timing of that difference, as documented in 1996 by a National Research Council report [8] and further elaborated in a study by Ayyub [9], coincided with the cancellation of 74 civilian nuclear reactors after 1979 (TMI) with a combined design capacity of 83,815 MWe [6], leaving the United States today with only 104 nuclear power reactors and a combined capacity of 100,266 MWe [7].

As shown in Table 1, Ayyub [9] released an interesting survey in his 2003 book (page 42, Table 2.2) on the perception of risk of nuclear vs. electric (non-nuclear) power by a sample of 3 groups of people of background as diverse as “League of Women Voters,” “College Students,” and “Experts.” That survey result implied that the public rejected the conclusion of the WASH-1400 report [1] as shown in Fig. 1.

In Section II of this paper, we will give a brief review of the risk-based engineering practice in the nuclear power industry through the activities of the past forty years (1971-2010) by ASME, the American Nuclear Society, the Nuclear Regulatory Commission, and researchers in industry and academia. We will also introduce a simple formula for risk, namely, the risk of a failure event is estimated as the product of the estimated failure probability and the failure consequence.

As mentioned in Section 1, PRA came into being in 1972 when the work on WASH-1400 was initiated. Acceptance of the PRA methodology was slow until the 1979 accident at TMI, which practically halted the entire U.S. nuclear power industry including methodology development.

In 1990, NRC published a landmark report [5] by evaluating five light water reactor (LWR) designs for the explicit purpose of assessing the public health risks of those reactors, if built, from not only internal initiators but also from earthquakes and fires.

Two conclusions were new: (1) The estimated risks of the those five LWR designs to the public were smaller than had been predicted in WASH-1400. (2) Of the approximately 100,000 active safety-related components in such a nuclear power plant, only a relatively “small” number (50 to 500 or less than 0.5 %) of the total determined about 90 % of the so-called Core Damage Frequency and were prominently identified as significant contributors to risk.

The first conclusion did not do much good because the public had already been turned off by the TMI accident, but the second was a major finding for the utilities, because it allowed the operators to allocate maintenance resources more wisely by concentrating inspection and repair dollars on those \( \approx 500 \) components identified as “critical” and paying less attention to the other \( \approx 99,500 \) with the confidence that plant safety would not be compromised. Thus was born a need to add risk information into three “libraries” of ASME documents:

1. ASME Guidelines for Risk-Based In-service Inspection (ISI) [11-14], Guidelines for Risk-Based In-Service Testing (IST) [15-16], Standards for PRA [17-19], and Reliability-Based Load and Resistance Factor Design Methods for Piping [20].

2. ASME Boiler and Pressure Vessel (B&PV) Code, Section XI Rules for ISI of Nuclear Power Plant Components [21], a Non-mandatory Appendix R [22], and 7 Code Cases [23-33] that required the existence of a suitable plant-specific PRA. Fig. 2 is an example of an ASME-recommended risk-informed ISI process as developed in Code Case N-577 [26].

3. ASME Code on Operation and Maintenance of Nuclear Power Plants [34], and 6 Code Cases [35-40] to categorize safety-significant components.
Figure 2. Overall Risk-Informed (RI) In-Service Inspection (ISI) Process as developed in Code Case N-577 [26], where the risk evaluation based on the consequence and failure probability calculations is presented to an expert panel for review and selection of structural elements for inspection and implementation of the ISI program (after Fig. 45.2 of Reference [10]).

Figure 3. A two-dimensional risk matrix using qualitative “numeric range” metric for the “likelihood probability” (from extremely unlikely to likely) and purely qualitative (non-numeric) metric for the “consequence” (from none to catastrophic) (after Table 2 of Ref. [60]).
II. RISK-BASED ENGINEERING PRACTICE IN THE NUCLEAR POWER INDUSTRY: A REVIEW (CONT’D)

During the same period (1991-2009), the U.S. Nuclear Regulatory Commission issued ten regulatory documents [41-50] for plant-specific, risk-informed ISI and IST decision-making, but decided not to generically endorse some of the ASME Code Cases as listed in Refs. [23-27] and Ref. [38], because they were judged to be not “plant specific enough.” (see Sect. 45.7.1 of Ref. [10]). Comparable documents on risk-informed ISI and PRA [51-61], and risk-informed decision-making [62] were also published by industry and academia for the 3-prong (ASME-NRC-Utilities) effort of securing a “sound” foundation for a risk-based engineering practice. Ayyub, Prassinos, and Etherton [62] summarized this practice in a “risk matrix” as shown in Fig. 3, and a commonly-accepted definition of risk as shown in Equation (1):

\[ Risk = Probability \text{ of event } \times \text{ its Consequence} \]  

(1)

This simple-minded formula, \( r = p \times c \), where \( r = \text{risk, } p = \text{probability of a failure scenario, and } c = \text{consequence of that failure, can be denoted by a pair symbol, } r = (p, c) \), as shown in Ayyub [57] and Ayyub, Prassinos, and Etherton [62]. In Fig. 2 [62], the pairing of a single failure-consequence can be generalized for a plant with \( n \) number of possible failures:

\[ Risk = f \left[ (p_1, \xi_1), (p_2, \xi_2), \ldots, (p_i, \xi_i), \ldots, (p_n, \xi_n) \right] \]  

(2)

where \( p_i \) is the occurrence probability of an outcome or scenario \( i \) out of \( n \) possible scenarios, and \( \xi \) is the occurrence of consequences or outcomes of each scenario.

As shown in Fig. 4, Cohn, Fong and Besuner [63] reported an example of a multiple-failure scenario for a hot reheat piping system with three types of welds, each with its own qualitative measures of probability and consequence as ranges rather than numeric values with error bounds. In short, the state of the art of the risk-based engineering practice in the nuclear power industry is hardly quantitative, and the practice of reporting probability and consequence estimates is still “deterministic.”
II. RISK-BASED ENGINEERING PRACTICE IN THE NUCLEAR POWER INDUSTRY: A REVIEW (CONT’D)

Figure 4. A purely qualitative Risk Matrix for three types of Welds in a Hot Reheat Piping System with “likelihood probability” varying from very low to very high, and “consequence” from median high to high and very high (after Fig. 2 of Ref. [63]).

Note: In the left diagram showing a hot reheat piping system, the numbers, L1 through L18 denote longitudinal seam welds; E1 through E9, clamshell welds; #1, #2, 17E, 23E, and 23W, girth welds.
III. UNCERTAINTY IN MECHANICS-BASED FAILURE PROBABILITY ESTIMATION

In 2000, Khaleel, Simonen, Phan, Harris, and Dedhia published a landmark report [64], in which they summarized the state of the art of the methodology for estimating the probability of failure of a component based on a fracture-mechanics model, or PFM, for short, as follows (see page 10.1 of [64]):

“. . . The (PFM) calculations gave a wide range of failure probabilities for the selected components, with some components having end-of-life probabilities of through-wall cracks of nearly 100 percent and others with probabilities of less than 10\(^{-6}\).

“. . . It is recognized that there are uncertainties in these calculated failure probabilities and core damage frequencies. Sources of the uncertainties come from assumptions made in the fracture mechanics and probabilistic risk analysis (PRA) models themselves and from the inputs to the models.”

In other words, engineers who attempt to estimate failure probability or to predict time-to-failure need to characterize their models with “stochastic” rather than “deterministic” variables. Furthermore, engineers must include as many source uncertainties as possible before making PFM calculations to produce a result (failure probability) with result uncertainty.

In 2009, Fong, Marcal, Hedden, Chao, and Lam [65] responded to the suggestion in the Khaleel, et al. report [64] by formulating an uncertainty equation for the life prediction model of an aging bridge. As shown in Fig. 5, four source uncertainties were identified for a simple case where load and constraint uncertainties were ignored. Three of the four uncertainties came from databases of (1) failure events, (2) nondestructive evaluation (NDE) reports, and (3) materials property test data. The fourth is an uncertainty assessment of the validity of the assumed crack-growth model. The result uncertainty, which can be either the failure probability, the time-to-failure, or the cycles-to-failure, is then estimated from a knowledge of the four source uncertainties.

In Table 2, Fong, et al. [65] gave a more complete formulation by including two additional databases involving (4) loadings and constraints, and (5) physical-chemical composition and dimensional variability. A conceptual representation of the relationship between the result uncertainty of the remaining life estimates and a collection of six source uncertainties is given in Fig. 6.

To illustrate the usefulness of the fracture-mechanics-based failure probability uncertainty equation concept as introduced in Fig. 6 [65], we show its application to structural health monitoring in two practical cases, namely,

(Case 1): An innovative inspection interval design based on “direct” NDE measurements of crack initiation and growth, as explained in Figures 7 [67] and 8 [65].

(Case 2): Estimation of the cumulative average leak probability of a critical piping component in a nuclear power plant as shown in Fig. 9 [69].

We will discuss below the significance of the two applications of the failure probability uncertainty equation concept:

(Case 1) Inspection Interval Design (Figs. 7 and 8)

In an “ideal” plant with an “ideal” critical component undergoing \(N\) cycles of operating loads, we identify a critical area and monitor it with equipment to detect if a crack has initiated with a length of \(a_i\).

Our equipment is calibrated to have a “known” detectable crack length, \(a_d\). By default, we assign the initial crack length, \(a_i\), to be the detectable crack length, i.e., \(a_i = a_d\).

From laboratory test data and a fracture mechanics model for that component, we calculate a critical crack length, \(a_c\), that will cause a catastrophic failure of the component.

In a fatigue life model, we need to know both the initial crack length, \(a_i\), and the final crack length \(a_f\). Again by default, we assign the final crack length, \(a_f\), to be the critical crack length \((a_f = a_c)\).

For a given pair of initial and final crack lengths, a given load range, \(DS\), and three known material property constants, \(C\), \(F\), and \(m\), Dowling [67, pp. 519-524] and Fong, et al. [65, Section IV] have shown that the number of cycles to failure, \(N_f\), is given by:

\[
N_f = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C (F \Delta S \sqrt{\pi})^m (1 - m/2)}
\]

In Fig. 7, we show a plot of the crack length, \(a\), vs. \(N\), the number of cycles for a component that is allowed to operate until failure \((N = N_f)\). Let \(N_p\) be the number of cycles from the initial to the second inspection. Again as shown in Fig. 7, we assume in an ideal situation that the crack has grown from a length of \(a_d\) to something larger, and a repair is made after the second inspection to return the component to its initial undamaged state. In principle, we have prolonged the
useful life of the component by $N_0$ cycles. This scenario of inspection followed by repair can be repeated as many times as the operator wishes, and the component is expected to be as "new" as before the first inspection with a theoretical cycle-to-failure equal to $N_f$. This is the essence of a deterministic approach to inspection interval design.

Obviously, the deterministic approach is incorrect and misleading because of at least four sources of uncertainty. The NDE measurement of initial crack length, $a_i$, and the laboratory measurement of critical crack length, $a_c$, are the first source of uncertainty. The three material property constants, $C$, $F$, and $m$, of equation (3) constitute the second.

Figure 5. A conceptual representation of the information flow involving several databases and a damage model (after Fig. 5 of Ref. [65]). Foremost among the databases considered in Ref. [65] are: The Failure Event Database-1 (Uncertainty-1, or, $e_1$), the Flaw Detection, Location & Sizing or NDE Database-2 (Uncertainty-2, or, $e_2$), the Material Property Database-3 (Uncertainty-3, or, $e_3$), the Deterministic or Probabilistic Damage Model (Uncertainty-M, or, $e_M$), and the Remaining Life Estimates (Uncertainty-4, $e_4$), where the e's are symbolic representations of the error measures of the five types of uncertainty inherent in a damage model. For simplicity, loading and constraints databases were not considered. Photo at the upper left corner is from the 100-year-old Jonathan Hulton Bridge, built in 1909, of Pittsburgh, PA, courtesy of reference [66]. Photo at the lower left corner was taken by the first author (Fong) during a site visit to the bridge in 2006.
III. UNCERTAINTY IN MECHANICS-BASED FAILURE PROBABILITY ESTIMATION (CONT'D)

Table 2. A List of Uncertainty-Contributing Factors in 5 Categories of Databases (after Table 1 of Ref. [65]).

\[ e_4 = f(e_M, e_1, e_2, e_3, e_4, e_5) \]

<table>
<thead>
<tr>
<th>Failure mechanisms</th>
<th>( F_i ), ( i = 1, 2, \ldots, n_F )</th>
<th>DB-1 (( e_i ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>NDE flaw geometry</td>
<td>( N_j ), ( j = 1, 2, \ldots, n_N )</td>
<td>DB-2 (( e_2 ))</td>
</tr>
<tr>
<td>Material property</td>
<td>( M_k ), ( k = 1, 2, \ldots, n_M )</td>
<td>DB-3 (( e_3 ))</td>
</tr>
<tr>
<td>Loadings and constraints</td>
<td>( L_u ), ( u = 1, 2, \ldots, n_L )</td>
<td>DB-4 (( e_4 )) (not treated here)</td>
</tr>
<tr>
<td>Physical-chemical composition (incl. dimensions)</td>
<td>( P_v ), ( v = 1, 2, \ldots, n_P )</td>
<td>DB-5 (( e_5 )) (not treated here)</td>
</tr>
</tbody>
</table>

III. UNCERTAINTY IN MECHANICS-BASED FAILURE PROBABILITY ESTIMATION (CONT'D)

Figure 6. A conceptual representation (after Fong, et al. [65]) of the relationship between the Uncertainty of the Remaining Life Estimates (Uncertainty-4, $e_4$) on the left of the eq. and a collection of six Uncertainties on the right of the eq. as listed below:

1. Uncertainty due to the crack growth damage model (Uncertainty-M, or, $e_M$).
2. Uncertainty due to estimated failure modes as reported in the Failure Event Database-1 (Uncertainty-1, or, $e_1$).
3. Uncertainty due to NDE measurements as reported in the Flaw Detection, Location & Sizing Database-2 (Uncertainty-2, or, $e_2$).
4. Uncertainty due to Material Property as reported in the Material Property Database-3 (Uncertainty-3, or, $e_3$).
5. Uncertainty due to estimated loadings and constraints as reported in a Loading-Constraints Database-$\lambda$ (Uncertainty-$\lambda$, or, $e_\lambda$).
6. Uncertainty due to estimated geometry & chemical composition as reported in the Geometry-Chemical Composition Database-$\pi$ (Uncertainty-$\pi$, or, $e_\pi$).
III. UNCERTAINTY IN MECHANICS-BASED
FAILURE PROBABILITY ESTIMATION (CONT'D)

Figure 7. An application of the crack-length-based approach to fatigue using a
deterministic formulation (after Fig. 11.2, p. 491 of the book by Dowling [67]). In this
case, several plots of crack length $a$ vs. cycle number $N$, appear where two
types of crack lengths are defined:

- $a_d =$ the minimum crack size that can be "reliably" detected by NDE, and
- $a_c =$ the critical crack length that causes a structure or component to fail and is related to
  material properties such as $K_{ic}$.

Three cycle number parameters and a safety factor on life are also defined:

- $N_{if} =$ no. of remaining life cycle after initial detection without further inspection,
- $N_p =$ no. of cycles from the initial to the second inspection,
- $\hat{N} =$ no. of remaining life cycles expected in service after initial inspection with the
detection of $a_d$, and $x_N = \frac{N_{if}}{\hat{N}}$, the safety factor on life.
Figure 8. An application of the crack-length-based approach to fatigue using a stochastic formulation (after Fig. 49 of Ref. [65]). In this case, several plots of crack length $a$ vs. cycle number $N$, appear where two types of crack lengths are defined: (Note: $m$ is the exponent in the crack growth model.)

- $a_d =$ the minimum crack size that can be "reliably" detected by NDE, and
- $a_c =$ the critical crack length that causes a structure or component to fail and is related to material properties such as $K_{IC}$.

Three cycle number parameters and a safety factor on life are also defined:

- $N_I =$ no. of remaining life cycle after initial detection without further inspection,
- $N_p =$ no. of cycles from the initial to the second inspection,
- $\hat{N}$ = no. of remaining life cycles expected in service after initial inspection with the detection of $a_d$, and $X_N = N_{if} / \hat{N}$, the safety factor on life.

$Q_{i,m} =$ Initial crack growth rate.  
$Q_{3N,m} =$ crack growth rate after $3N_p$.

$q_{ad}, q_{ac}, q_{Nif},$ and $q_{Np}$ are uncertainty bounds of $a_d, a_c, N_{if}$ and $N_p$, respectively.

Note: Scale for $N_{if}$ is reduced by design to fit all in one figure.
III. UNCERTAINTY IN MECHANICS-BASED FAILURE PROBABILITY ESTIMATION (CONT’D)

The load range $\Delta S$, also in equation (3), is the third source of uncertainty. Finally, after several intervals of inspection, the component has aged and the initial damage model of crack growth is no longer valid. The rationale for using equation (3) to predict the cycle-to-failure, $N_{if}$, from two crack length measurements, one initial and the other critical, is gone.

In 2009, Fong, et al. [65] developed a stochastic formulation of the same inspection interval design by introducing the concept of a “direct measurement” of crack growth rate. As shown in Fig. 8, the theory developed in [65] permits the estimation of the parameters, $a_d$, $a_c$, $N_{if}$, and $N_p$, all with uncertainty bounds if “direct measurements” of the crack growth rates are available in a continuous health monitoring system. This ends our discussion of an application of the failure probability uncertainty equation concept.

(Case 2) Cumulative Average Leak Probability Estimation

In 2007, Simonen, Gosselin, Lydell, Rudland, and Wilkowski [68] identified ten uncertainty sources as to why two benchmarked PFM models, PRO-LOCA and PRAISE, predicted significantly higher failure probabilities of cracking than those derived from field data by a factor ranging from 30 to 10,000. In 2009, Fong, deWit, Marcal, Filiben, Heckert, and Gosselin [69] developed an uncertainty plug-in that allows a user to address those ten uncertainties and calculate a failure probability with uncertainty bounds. An example of this application of the uncertainty equation concept is given in Figure 9.

Figure 9. A plot (after Fig. 34 of Ref. [69]) of the cumulative average leak probability (CALP) vs. time based on a 3-factor, full factorial, 8-run-center-point design run (DEX-2) with a 40% change in N1, 40% change in M1, and 20% change in M2, as compared with a plot of the same CALP versus time, based on the output file of a PC-PRAISE (v. 4.42) run (see Ref. [64]) using 4 distributional parameters (in red) and 20+ constant parameters. The estimated mean and confidence half-interval of the DEX-2 run differ sharply from those of the PC-PRAISE run.
IV. UNCERTAINTY IN FAILURE CONSEQUENCE ESTIMATION

In the last section, we addressed the uncertainty of the failure probability, \( p \), of the simple risk equation, \( r = p \times c \). Here in Section IV, we will address the uncertainty of the consequence, \( c \), of a failure event.

To illustrate why this second uncertainty factor in a risk equation is just as important as the first, we introduce in Figures 10 and 11 two summary plots from a recent study by Cohn, Fong, and Besuner [63], where the results of a million Monte-Carlo simulations were plotted to show the frequency vs. outage cost severity of all welds piping and seam welds piping, respectively.

To estimate the probability and consequence of a pipe failure for a typical 400 MW, 40-year-old fossil unit, Cohn, Fong, and Besuner [63] used the North American Electric Reliability Corp (NERC)-developed Generating Availability Data System (GADS) data for the period of 1982 through 1997 on main steam and hot reheat piping incidents with forced outages of at least 350 hours. These incidents were described as High Impact Low Probability (HILP) events. Three key elements of the analysis methodology are described below:

1. A generalized gamma distribution model that includes both lognormal and Weibull models as special cases was used to fit the failure statistics with five random variables, of which three were identified as critical: (i) time-to-failure of welds (years), (ii) forced outage duration (hours), (iii) forced outage replacement cost (US$ per MW-hour).
2. A power replacement cost distribution diagram as a function of frequency of simulated events was constructed from experience.
3. A composite distribution to model the discount rate as a random variable was constructed by assuming a median annual discount rate of 9% and a lognormal distribution with a coefficient of variation of 0.15.

![Figure 10. Frequency (i.e., Probability of Exceeding Cost) and Severity (Consequence in Cost) of Piping High Impact Low Probability (HILP) Outages Exceeding $100,000 for All Welds (after Fig. 5 of Ref. [63]).](image-url)
IV. UNCERTAINTY IN FAILURE CONSEQUENCE ESTIMATION (CONT'D)

The survival analysis plus Monte Carlo methodology were implemented by Cohn, Fong, and Besuner [63] using a computer software named Stata [70-75].

A plot of the frequency or probability of exceeding cost vs. the severity or consequence in cost of piping HILP outages exceeding $100,000 for all welds is shown in Fig. 10, where the best estimates of the HILP cost chance is given with 95% confidence bounds. This allows us to estimate a standard deviation of the best consequence estimate, $c$, for any given probability of exceeding cost. For example, as shown in Figure 10, we choose a probability of 0.001 to estimate the present valued cost of the worst-in-1000 units HILP piping event to be equal to 14.0 million dollars ($= \bar{c}$) with a 95% confidence interval of (9.0, 19.0). This translates into an approximate standard deviation, $sd(\bar{c}) = 2.5$, and a coefficient of variation, $c.v.(\bar{c}) = 0.18$.

![Figure 11. Frequency (i.e., Probability of Exceeding Cost) and Severity (Consequence in Cost) of Piping High Impact Low Probability (HILP) Outages Exceeding $100,000 for Seam Welds (after Fig. 9 of Ref. [63]).](image)

It is of interest to the plant operators and maintenance engineers to evaluate if weld type consequences are significantly different. Using the same raw data and the modeling assumptions, Cohn, Fong, and Besuner [63] did calculations for all seam welds, and plotted a similar set of results in Fig. 11. The difference in the consequence estimates is quite significant. For example, the best estimate of the present valued cost of the worst-in-1000 units HILP piping event dropped to 4.7 million dollars ($= \bar{c}$) with a 95% confidence interval of (1.4, 8.0). This translates into an approximate standard deviation, $sd(\bar{c}) = 1.65$ million dollars.

What did we learn from this example? First of all, we are convinced that any estimate of the consequence, $c$, of a failure event needs to include a measure of uncertainty such as an estimated standard deviation or a 95% confidence bound. Secondly, as shown in Eq. (1), we need to couple the consequence estimate with the failure probability estimate in order to obtain a risk estimate. Unfortunately, this was not done in [63].
IV. UNCERTAINTY IN FAILURE CONSEQUENCE ESTIMATION (CONT’D)

To supplement the limited consequence estimation methodology used by Cohn, Fong, and Besuner [63], we refer our readers to a companion paper by Chapman, Fong, Butry, Thomas, Filliben, and Heckert [76], in which they apply a consequence estimation methodology based on an ASTM guide for developing a cost-effective risk mitigation plan for new and existing construction facilities [77].

In that ASTM E 2506 Guide-based methodology [77], a three-step protocol is specified, namely, (1) perform risk assessment, (2) specify combinations of risk mitigation strategies for evaluation, and (3) perform economic evaluation.

The first two steps produce the necessary data and information for use as input to the third step, for which a computer code named “Cost-Effectiveness Tool (CET) for Capital Asset Protection” has been developed by Chapman and Rushing [78]. Using an example of a simple-minded fictional failure event in a power plant scenario involving the rupture of a critical piping component, Chapman, et al. [76] present a case study of a consequence estimation technique comparable to that presented by Cohn, et al [63] and re-stated in this paper.

V. A RISK UNCERTAINTY FORMULA

In Section III, we addressed the uncertainty of the failure probability, \( p \), of the simple risk equation, \( r = p \cdot c \).

In Section IV, we addressed the uncertainty of the consequence, \( c \), and introduced the notation for its mean estimate, \(\bar{c} \), its standard deviation, \( sd(\bar{c}) \), its variance, \( var(\bar{c}) \), and its coefficient of variation, \(c.v.(\bar{c})\).

For the failure probability, \( p \), we also introduce the notation for its mean estimate, \(\bar{p} \), its standard deviation, \( sd(\bar{p}) \), its variance, \( var(\bar{p}) \), and its coefficient of variation, \(c.v.(\bar{p})\). A similar set of notation can also be introduced for risk, \( r \), namely, the mean risk estimate, \(\bar{r} \), its standard deviation, \( sd(\bar{r}) \), its variance, \( var(\bar{r}) \), and its coefficient of variation, \(c.v.(\bar{r})\). The simple risk equation becomes \(\bar{r} = \bar{p} \cdot \bar{c} \).

Based on a variance formula for the product of two quantities given by Ku [79], we obtain the variance of risk as a function of the two variances, \( var(\bar{p}) \) and \( var(\bar{c}) \), as well as the three mean estimates, \(\bar{p} \), \(\bar{c} \), and \(\bar{r} \), as follows:

\[
\text{var}(\bar{r}) = (\text{var}(\bar{r}))^2 \left( \frac{\text{var}(\bar{p})}{(\bar{p})^2} + \frac{\text{var}(\bar{c})}{(\bar{c})^2} \right) \quad (4)
\]

Since \( (\bar{r})^2 = (\bar{p})^2 \cdot (\bar{c})^2 \), Eq (4) becomes as follows:

\[
\text{var}(\bar{r}) = \left( (\bar{c})^2 \cdot \text{var}(\bar{p}) + (\bar{p})^2 \cdot \text{var}(\bar{c}) \right) \quad (5)
\]

Therefore,

\[
\text{sd}(\bar{r}) = \sqrt{\text{sd}(\bar{c})^2 \cdot \text{sd}(\bar{p})^2 + (\text{sd}(\bar{p}) \cdot \text{sd}(\bar{c}))^2} \quad (6)
\]

Since \( c.v.(\bar{r}) = \text{sd}(\bar{r}) / \bar{r} \), \( c.v.(\bar{p}) = \text{sd}(\bar{p}) / \bar{p} \), and \( c.v.(\bar{c}) = \text{sd}(\bar{c}) / \bar{c} \), we write Eq. (6) as follows:

\[
c.v.(\bar{r}) = \sqrt{(c.v.(\bar{p}))^2 + (c.v.(\bar{c}))^2} \quad (7)
\]

Let us illustrate the utility of Equations (6) and (7) with a numerical example based on the consequence estimation given by Cohn, Fong, and Besuner [63] for the case of all welds as documented in the previous Section IV.

As shown in Figure 10, we choose to consider a worst-in-1000 units HILP piping event with \(\bar{c} = 14.0\) million dollars, \(\text{sd}(\bar{c}) = 2.5\) million dollars, and \(c.v.(\bar{c}) = 0.18\).

For the all welds case, Cohn, et al. [63] also reported a \(\bar{p}\) value equal to 0.0566, but did not report a value for \(\text{sd}(\bar{p})\), because it was never calculated.

For illustrative purposes here, let us assume that \(c.v.(\bar{p}) = 0.10\). Then \(\text{sd}(\bar{p}) = 0.10 \cdot 0.0566 = 0.00566\).

From Equations (1) and (6), we now obtain an estimate of risk with uncertainty, \(\bar{r}\) and \(\text{sd}(\bar{r})\), as follows:

\[
\bar{r} = \bar{p} \cdot \bar{c} = 0.0566 \cdot 14.0 = 0.79\text{ million$}, \quad \text{and}
\]

\[
\text{sd}(\bar{r}) = \sqrt{(14.0)^2 \cdot (0.00566)^2 + (0.0566)^2 \cdot (2.5)^2}
\]

\[
= \sqrt{(0.079)^2 + (0.14)^2} = 0.16\text{ million$}.
\]
IV. UNCERTAINTY IN FAILURE CONSEQUENCE ESTIMATION (CONT'D)

Figure 12. Potential Evolution to Nuclear Systems Code (after Fig. 45.3 of Ref. [10]).
Figure 13. A schematic diagram showing that risk research involves a total of seven disciplines, with five responsible for failure probability research (fracture mechanics, materials science, computer soft- and hard-ware, statistical science, and computational science), and two for failure consequence research (applied economics and social science). The latter two are shown with a question mark, because they are generally not included in a traditional engineering curriculum.
VI. DISCUSSION AND SIGNIFICANCE OF RESULTS

If a crude measure of uncertainty of risk, $r$, is the estimated standard deviation of risk, $sd(r)$, then the formula given in Equation (6) provides a recipe for calculating the uncertainty of risk, $sd(r)$, from a knowledge of 4 quantities, namely, the estimated mean probability of failure, $p$, the estimated standard deviation of failure probability, $sd(p)$, the estimated mean consequence of that failure event, $c$, and the estimated standard deviation of the consequence, $sd(c)$.

Let us recall the numerical example given in the last section, where we had a knowledge of the four quantities as follows:

- $p = 0.0566$ (based on failure data).
- $sd(p) = 0.00566$, or, $c.v.(p) = 0.10$ (both assumed).
- $c = 14.0$ million $\$$(from simulations).
- $s.d.(c) = 2.5$ million $\$, or, $c.v.(c) = 0.18$.

From the simple-minded risk equation (1), and its instantiation by mean estimates, $r = p \cdot c$, we obtain:

- $r = 0.79$ million $\$.

From the risk uncertainty equation (6), we obtain:

- $sd(r) = 0.16$ million $\$, or,

from the risk coefficient of variation equation (7), we obtain:

$$c.v.(r) = \sqrt{(0.10)^2 + (0.18)^2} = 0.21.$$  \hspace{1cm} (8)

Equation (8) reminds us of the classical Pythagoras Theorem, where the sum of the squares of the two sides of a right triangle equals the square of the hypotenuse. Equations (7) and (8) for the relationship among the coefficients of variation of the risk, the failure probability, and the failure consequence has the elegant geometric representation as shown in Figure 14 below:

Assuming a normal distribution for the risk, $r$, the availability of an estimate of the standard deviation of risk, $sd(r)$, allows us to report an estimate of risk, $r$, with an approximate 95 % confidence limits as follows:

$$r = 790,000 \pm 320,000, \hspace{1cm} (8)$$

where the quantity following the sign, “$\pm$”, is the 2-sigma value corresponding to an approximate two-sided 95 % confidence limits of the risk, namely, ($470,000$, $1,110,000$). Such a measure would place the risk of a worst-in-1000 units HILP piping event in a risk matrix (see Fig. 3, Section II) as being “UNLIKELY” in the vertical axis (probability) and “SERIOUS” in the horizontal axis (consequence), if we choose to consider the horizontal axis as a logarithmic scale with base 10 and the origin being $100$.

The significance of the risk uncertainty formula, Equation (6) or (7), is two-fold: (1) It places a responsibility on the engineers to estimate not only the mean failure probability of an event and its mean consequence, but also their standard deviations as measures of their uncertainties, and (2) the formula provides an easy-to-remember rule-of-thumb type of a quick estimator of the risk uncertainty, $sd(r)$, in terms of four quantities, $p, sd(p), c, sd(c)$, provided $p$ and $c$ are not correlated and their coefficients of variation, $c.v.(p)$, and $c.v.(c)$, are small ($< 0.1$).
VII. CONCLUDING REMARKS

Before we conclude, we wish to quote a series of cogent remarks made by Bernsen, Simonen, Balkley, West, and Hill III in Section 45.8.3 of their 2009 paper [10]:

“. . . Current ASME Nuclear Code and Standards rely primarily on deterministic and mechanistic approaches to design of components, including piping systems.

“. . . Work is in progress to develop an ASME Nuclear Systems-Based Code, which would include a planned evolution that integrates the various nuclear codes and standards and adopts a risk-informed approach across a facility life cycle, encompassing design, construction, operation, maintenance, and closure.

“. . . Figure 45.3 (reproduced as Figure 12 in this paper) offers a conceptual development.”

[Emphasis in bold given by authors of present paper.]

In addition, Bernsen, et al. [10] noted that

“. . . the USNRC is considering development of a risk-informed performance-based and technology-neutral alternative licensing process for new reactor designs [49].

“. . . The design processes used today are predominantly deterministic and not risk informed.

“. . . It is envisioned that a Systems-Based Code design process would be based on risk-informed probabilistic methodologies that cover a facility’s life-cycle from the start of conceptual design through decontamination and decommissioning [60].

[Emphasis in bold given by authors of present paper.]

Thus we conclude that the result reported in this paper provides a quantitative tool to the combined ASME-NRC-Industry effort in promoting the use of the metric of risk not only for in-service inspection (ISI) and in-service testing (IST), but also throughout the facility’s life cycle from design, licensing, to construction, operation, maintenance, and planned replacement without catastrophic failure.

Furthermore, the analysis and examples provided in this paper on a rational estimation of risk with uncertainty bounds based on hard data and expert judgment in several disciplines, point to a need for educators to strengthen the curriculum for future engineers. As shown in Figure 13, by and large, present-day curriculum for a mechanical or nuclear engineer is quite adequate when it comes to the estimation of failure probability based on fracture mechanics, materials science, computer engineering, statistical science, and computational science. What could strengthen the curriculum are courses in applied economics (for consequence evaluation), social science (for human factors in NDE and plant operations), and a stronger dosage of engineering statistics including the powerful method of design of experiments [80, 81].

Finally, we note that the variance formula given by Ku [79] and used in this paper to derive the risk uncertainty formula has also been applied by Fong and his colleagues [82-85] in estimating uncertainty in fatigue life prediction or failure probability due to a number of source uncertainties such as:

1. assumed failure modes as documented in failure event databases,
2. flaw detection, location, and sizing measurements from NDE data,
3. material property testing data,
4. assumed loads and support conditions vs. actual ones,
5. assumed aging characteristics of base materials and weldments vs. actual ones, and
6. assumed damage model vs. alternative model calibrated for a specific structure.

Such advances in estimating failure probability uncertainty, when combined with economics-based intelligence (EI) tools (see, e.g., Refs. [76, 77, 78]) for estimating consequence uncertainty, are essential in closing the loop for estimating the uncertainty of risk in a multiple-scenario risk-informed maintenance decision-making process.

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