Narrow-Band Photon Pairs Generated from Spontaneous Parametric Down Conversion in a Bragg-Grating Enhanced Waveguide

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ABSTRACT

We propose a new method to narrow the linewidth of photon pairs generated from spontaneous parametric down conversion (SPDC). The single structure device incorporates an internal Bragg grating onto a nonlinear optical waveguide. We study theoretically the spectral characteristics of SPDC under two Bragg grating structures. We show that using the Bragg grating with a midway π-phase shifter is a promising way to implement narrow-line (~GHz to sub-GHz) entangled photon sources.

Keywords: Quantum optics, quantum communication, nonlinear optics, parametric processes, Bragg grating

1. INTRODUCTION

Quantum information science and technology open up a fascinating future for the research, development and application in information technologies. Quantum communication is one of its most important and practical applications. Entangled photon pair sources are necessary for quantum communication systems, such as quantum key distribution [1], entanglement swapping [2], quantum teleportation [3], and quantum repeater [4]. In an entanglement-based quantum communication system, a photon source with broad linewidth limits the fiber transmission distance due to the dispersion effect in the fiber. In the case of time-bin entanglement, the impurity of the entanglement is greatly degraded, which becomes a limitation for high data rates. Thus an entangled photon source with narrow linewidth is crucial for high-speed long-distance quantum communication systems [5].

For a quantum communication system, when the distance is longer than ~100 km, it is necessary to use quantum repeaters, which are formed mainly by an entangled photon source for teleporting long distance and a quantum memory for storing of quantum information. With regard to quantum memory, an atomic ensembles or rare earth ions in solids are used to store quantum information carried by single photons. In this approach, non-degenerate entangled photon pairs are needed to fulfill the purpose: one wavelength is in the communication band for long distance transmission and the other matches the transition line of the quantum memory. The linewidth of the photon sources should be comparable to the transition linewidth of the memory, which was a few MHz in early studies [4]. Recently, this requirement is relaxed to the GHz range by using atomic frequency combs [6] or Raman type approach [7].

The current mainstream method to generate entangled photons is through a nonlinear optical process called spontaneous parametric down conversion (SPDC) [8]. For easy system integration with a low power pump laser, the nonlinear optical crystal is made into a waveguide, and the crystal’s polarity is spatially periodically poled, resulting in quasi-phase matching. However, the entangled photons generated by single-pass SPDC from such a nonlinear optical waveguide have relatively broad linewidth, typically several hundred GHz (or a few nm at communication wavelengths). This kind of entangled photon source is not suitable for the above mentioned applications which need a narrow bandwidth.

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Therefore, reduction of the bandwidth of entangled photons generated by SPDC is important for quantum communication systems. A straight forward way of narrowing the SPDC bandwidth is through passive filtering [9, 10]. This approach suffers the disadvantage of additional loss of the desired signal and idler photons. Below-threshold optical parametric oscillator is another approach [11-17]. Although spectral width at specific signal or idler wavelengths can be quite narrow, because of the closeness of the resonator longitudinal modes, multiple signal and idler lines exist, and it has a bulky and complex configuration.

In this paper, we propose a new method to narrow the linewidth of entangled photons generated by SPDC. A quasi-phase-matched nonlinear optical crystal waveguide is incorporated with an internal distributed Bragg grating. SPDC with a distributed Bragg reflector (DBR) has been studied previously where it was used as one of the reflectors in an external micro-cavity configuration [18] or to increase the pump acceptance bandwidth for higher efficiency [19]. Our scheme is to use the single nonlinear crystal waveguide with distributed Bragg grating as the entangled photon source with a reduced spectral linewidth. This approach, when implemented, has only a single longitudinal mode, and its configuration will be much simple, compact, and robust. We theoretically study the spectral characteristics of SPDC incorporated with internal Bragg grating under finite quasi-phase-matching. Two grating structures are considered. We propose to use a Bragg grating with a midway $\pi$-phase shifter, for the first time to our knowledge, and we show that it is a promising way to implement narrow-line (~GHz to sub-GHz) entangled photon sources.

2. MODEL

The schematic structure of the proposed SPDC with internal distributed Bragg grating is shown in Fig. 1. A Bragg grating is written onto a periodically poled nonlinear optical crystal waveguide. We consider two Bragg grating structures. One has a continuous grating modulation. Another has a half grating period of flat spacer at the middle of the full Bragg grating, which is commonly used in semiconductor distributed feed-backed lasers. Note that fabrication of a distributed Bragg grating on a periodically poled nonlinear optical crystal waveguide is technically practical, as similar technology (fabrication of a continuous Bragg grating on a periodically poled LiNbO$_3$) has been demonstrated for distributed feedback optical parametric oscillation [20]. Fabrication of a Bragg grating with a midway phase shifter should be implementable using a proper grating mask.

![Figure 1. Periodically poled nonlinear optical crystal waveguide with Bragg grating. (a) Continuous grating; (b) grating with midway $\pi$-phase shifter.](image)

Accurate calculation of SPDC with distributed Bragg grating needs to be done by quantum theory [18]. We initially proceed with the calculation by the classical method, i.e., we treat the signal, idler and pump as classical waves, with equivalent noise input fields for the signal and idler. For SPDC, the pump field is virtually non-depleted and we treat it as constant. Thus one considers the coupling evolution of the signal and the idler waves. Including the forward and backward traveling waves, the signal and idler fields take the form

\[
E_s(z,t) = \text{Re}[\hat{E}_s(z) \exp(j \omega_s t)]
\]

\[
E_i(z,t) = \text{Re}[\hat{E}_i(z) \exp(j \omega_i t)]
\]

\[
\hat{E}_s(z) = A_s(z) \exp(-j \beta_s z) + B_s \exp(j \beta_s z)
\]
\[ \bar{E}_i(z) = A_i(z) \exp(-j \beta_i z) + B_i \exp(j \beta_i z) \tag{4} \]

where the \( A_s \) and \( A_i \) are the amplitudes for the forward waves and \( B_s \) and \( B_i \) are the amplitudes for the backward waves. The subscript \( s \) is for signal and \( i \) for idler. The wave equations for the signal and idler fields are

\[
\left[ \frac{d^2}{dz^2} + \omega^2_{\mu}\mu \right] \bar{E}_s = -\omega_s^2 \mu \left( \tilde{P}_{G,s} + \tilde{P}_{NL,s} \right) \tag{5} 
\]

\[
\left[ \frac{d^2}{dz^2} + \omega_i^2 \mu \right] \bar{E}_i = -\omega_i^2 \mu \left( \tilde{P}_{G,i} + \tilde{P}_{NL,i} \right) \tag{6} \]

For simplicity we assume the dielectric constant to vary sinusoidally in \( z \)-direction. For the continuous grating as in Fig. 1(a), we write the modulation of susceptibility as

\[ \Delta \varepsilon(z) = \varepsilon_g \cos \beta_G z \tag{7} \]

We assume a non-degenerate optical parametric down conversion, with signal and idler at distinct wavelengths. For example, with a pump wavelength of 532 nm, signal and idler are at 895 nm and 1310 nm, respectively. Such an entangled photon source is useful in quantum communication systems [21]. In this case the Bragg grating couples strongly only the signal wave, i.e. the Bragg condition is satisfied only for the signal \( 2 \beta_s = \beta_G \). Therefore we neglect the backward pump and idler waves. The Bragg grating perturbation polarization is given by

\[ \tilde{P}_{G,s}(z) = \Delta \varepsilon(z) \left[ A_s(z) \exp(-j \beta_s z) + B_s(z) \exp(j \beta_s z) \right] \tag{8} \]

and \( \tilde{P}_{G,i} \approx 0 \). The nonlinear polarizations for the signal and idler waves are given by, respectively

\[ \tilde{P}_{NL,s}(z) = 2d(z)A_p^* A_s \exp[-j(\beta_p - \beta_i)z] \tag{9} \]

\[ \tilde{P}_{NL,i}(z) = 2d(z)A_p^* A_i \exp[-j(\beta_p - \beta_i)z] \tag{10} \]

Also for simplicity, let the periodically poled nonlinearity be written as

\[ d(z) = d_Q \cos \beta_G z \tag{11} \]

The quasi-phase matching condition is \( \beta_p - \beta_s - \beta_i \approx \beta_Q \). Under the slowly-varying-envelope approximation and keeping the nearly phase-matched and grating-resonant terms, the signal-grating-coupled parametric down conversion process is governed by the following coupled differentially equations:

\[ \frac{dA_s}{dz} = -j \kappa_s^* B_s \exp(j \Delta \beta_G z) - j \kappa_s A_i^* \exp(-j \Delta \beta_G z) \tag{12} \]

\[ \frac{dA_i^*}{dz} = j \kappa_i^* A_i \exp(-j \Delta \beta_G z) \tag{13} \]

\[ \frac{dB_s}{dz} = j \kappa_s A_i \exp(-j \beta_G z) \tag{14} \]

where

\[ \kappa_g = \frac{\mu \varepsilon_s \omega_s c}{4 n_s} \tag{15} \]
\[
\kappa_d = \frac{\mu_0 \omega_0^2 \int dE^E_x(0)}{\sqrt{\omega_0 n_L n}} \tag{16}
\]
\[
\Delta \beta_G = 2 \beta_s - \beta_G \tag{17}
\]
\[
\Delta \beta_Q = \beta_p - \beta_s - \beta_l - \beta_Q \tag{18}
\]

In the above equations, \( A_s \) is the forward signal field, \( A_i \) is the forward idler field, and \( B_s \) is the backward signal field. The field amplitudes are normalized, but keeping the same symbols, such that the modular square of the amplitude gives the photon flux density in a particular wave. \( \Delta \beta_G \) is a measure of frequency detuning away from the Bragg grating line center, and \( \Delta \beta_Q \) is the phase mismatch away from the signal/idler line-center. When the Bragg grating line center and the quasi-phase-matched signal line center line up, \( \Delta \beta_G \) and \( \Delta \beta_Q \) are related by the difference of group indexes of the signal and idler waves

\[
\Delta \beta_Q = [(n_s - n_l) / 2 n_s] \Delta \beta_G \tag{19}
\]

When dispersion is neglected, the group indexes can be approximated by the corresponding refractive indexes. For the second Bragg grating structure, it is equivalent to have a \( \pi \)-phase shift at the midpoint of the full sinusoidal Bragg grating

\[
\Delta \varepsilon(z) = \begin{cases} 
\varepsilon_g \cos \beta_G z & 0 \leq z < L / 2 \\
-\varepsilon_g \cos \beta_G z & L / 2 \leq z \leq L 
\end{cases} \tag{20}
\]

corresponding to \( \kappa_g \) changing a sign in the second half region.

Note that distributed feedback optical parametric amplification (OPA) and oscillation with a continuous Bragg grating have been studied by classical theory, in which phase matching among pump, signal and idler waves is assumed and for OPA only the signal wave has input [22]. However, the classical calculation in [22] cannot be used directly to describe the SPDC process. With definite input signal and idler fields, Eqs. (12)-(14) describe a classical coherent process. The straight-forwardly calculated spectral characteristic of PDC is sensitive to the phases of the input signal and idler and is in general asymmetrical about the phase-matching center wavelength. SPDC is a quantum mechanical process with both signal and idler photons generated from vacuum. There is no first-order correlation between the signal and idler photons generated from SPDC, and the spectral characteristic is symmetrical (in theory) about the phase-matching center wavelength. In order to use the classical theory, as an approximate approach, to calculate the SPDC spectral characteristic, one needs to treat the equivalent input fields of signal and idler as noise. We take equivalent input fields for the forward signal and idler waves with random phases at \( z = 0 \) and zero input field for the backward signal wave at \( z = L \). The physical SPDC spectral characteristic is obtained from the ensemble-average over random phases of input signal and idler.

3. RESULT

As a baseline, Fig. 2 shows the transmission and reflection characteristics of the two Bragg grating structures for the forward and backward signal waves, with grating coupling parameter \( \kappa_g L = 3 \), a reasonable grating coupling strength [22]. The unit of frequency detuning is \( \Delta \beta_G L / 2 \). For the continuous Bragg grating, around zero detuning, the signal wave is mostly reflected with a broadband. In contrast, for the Bragg grating structure with a midway \( \pi \)-phase shift, the transmitted signal wave has a sharp peak at zero detuning. The stop-band of the Bragg grating (full width \( \Delta \beta_G L / 2 \approx 3 \pi \)) corresponds to \( \sim 20 \text{ GHz} \) for a 1-cm LiNbO\(_3\) waveguide.

Fig. 3 shows the spectral characteristics of spontaneous optical parametric down conversion with two types of Bragg grating in resonance with the signal wave. As a comparison, the broadband spectral response of parametric down
conversion is also shown. For illustration, we take phase-mismatch with a nominal value $\Delta \beta = 0.1 \Delta \beta_c$ and pump strength $\kappa_d L = 1.1$. The calculated nominal SPDC linewidth (full width $\Delta \beta G = 2 \approx 8\pi$) is about 55 GHz (~0.3 nm at wavelength of 1.3 µm) for a 1-cm nonlinear optical waveguide and refractive index of 2.2 (LiNbO$_3$). The refractive index difference of commonly used nonlinear optical crystals in type-I configuration is about one-order of magnitude smaller, corresponding to an SPDC linewidth of a few nm for a 1-cm length of nonlinear optical waveguide [21]. For a type-II configuration, the refractive index difference is usually larger than for a type-I configuration, and the SPDC linewidth (with a corresponding larger $\Delta \beta$) is narrower (less than 1 nm).

Figure 2. Transmission and reflection characteristics of two Bragg grating structures. (a) Continuous grating; (b) grating with $\pi$-phase shifter.

Figure 3. Signal and idler photon flux densities with and without a Bragg grating. (a) Continuous grating; (b) grating with $\pi$-phase shifter.

With an internal continuous Bragg grating ($\kappa_s L = 3, \kappa_d L = 1.1$), within the stop-band of the Bragg grating, most of the signal flux is in the backward direction, while the idler’s flux is also reduced. However, the signal and idler photons are generally still broadband. In contrast, with the Bragg grating structure that has a midway $\pi$-phase shift ($\kappa_s L = 3, \kappa_d L = 0.7$), the forward signal and idler waves peak sharply, with comparable high fluxes, at zero detuning. Please note that the spectra of both the signal and idler photons are narrowed even when the Bragg grating is designed only for the signal wavelength. The SPDC linewidth is about 15 times narrower than the stop-band of the Bragg grating and is less than 1.5 GHz for a 1-cm LiNbO$_3$ waveguide. Note that the reduced SPDC linewidth is due to the distributed Bragg grating and should be about the same for type-I or type-II nonlinear interactions. With a longer crystal length, the spectral narrowing will be more prominent with linewidth territory down into sub-GHz.
The narrow-band peak of PDC is reminiscent of spectral resonant characteristics. In the context of the quantum nature of SPDC, the resonant characteristics should be understood as a result of the imposition of a constraint on the nonlinear system by the midway $\pi$-phase-shifted Bragg grating and consequently forcing the spontaneous parametric down conversion into only a narrow band. As mentioned above, the grating structure with a midway $\pi$-phase shift is equivalent to a grating structure that has a midway flat spacer, which can be viewed physically as a micro-cavity of $\lambda_s/4$ length and bounded by two Bragg reflectors. The internal-grating-enhanced SPDC contrasts with the external passive filtering in several ways. First, the internal-grating/SPDC is a single structure device, does not suffer loss in principle and results in high spectral density flux for both signal and idler. For passive filtering, beside suffering additional loss, the spectral density of signal or idler does not increase. Furthermore, implementation of narrow-band passive filtering for both signal and idler would need two filters, and matching and maintaining their accurate center wavelengths would be difficult.

4. SUMMARY

We propose a new approach to narrow the linewidth of entangled photons, by incorporating a Bragg grating structure with a midway $\pi$-phase shifter onto a nonlinear optical crystal waveguide and theoretically study the linewidths of signal and idler photons from the SPDC process. From our theoretical calculation, the approach is promising to generate very narrow linewidth (~GHz to sub-GHz) signal and idler photons via the spontaneous parametric down conversion. This type of entangled photon source may be practical and very useful for quantum information and communication applications.

REFERENCES