Abstract – The recent advances in ac waveform synthesis using quantized voltage pulses have enabled new methods for the measurement of absolute temperature. At the same time, the increased linearity and sampling rates of digital electronics have established frequency-domain processing as the preferred technique in Johnson noise thermometry (JNT). The spectral-ratio quantum-based JNT developed at NIST has achieved measurement uncertainties as low as 12 \( \mu \text{K}/\text{K} \) at 273 K and 40 \( \mu \text{K}/\text{K} \) for temperatures in the range 500 K to 700 K, with even lower uncertainties achievable in the near term. We describe the theory and experimental realization of the quantized voltage noise source (QVNS) and spectral JNT from the metrology standpoint as intrinsic temperature standards. The role of JNT in special point-of-use thermometry applications and future research directions are also discussed.

Introduction

Thermometric fixed points as used in thermometry are well known examples of intrinsic temperature standards.[1] While some of these have temperatures that are assigned on the International Temperature Scale of 1990 (ITS-90) [2], these assignments are based on thermodynamic data from primary thermometers. These primary thermometers, such as acoustic gas thermometers, radiation thermometers and noise thermometers, are themselves also intrinsic standards, but are more complex to implement and usually less reproducible than the best fixed points [3]. Recent advances in AC Josephson standards [4] have allowed the development of a new class of intrinsic temperature standards that are based on quantized voltage noise and Johnson noise and are able to measure absolute temperatures independent of any fixed-point reference [5]. These new standards have been applied to temperature scale research and to the determination of the Boltzmann constant. As a purely electronic standard, other possible applications could be explored to address some long-standing issues in thermometry that exist due to the limitations of material artifacts.

Background

The fluctuations of voltage and current that exist in any conductor at finite temperatures have long been understood as a fundamental noise characteristic proportional to temperature \( T \) or “Johnson noise”. The spectral density of voltage fluctuations \( S_R \) of Johnson noise across a resistance \( R \) has the well-known low-frequency limit given by Nyquist’s formula [6],

\[
S_R = 4k_B T R ,
\]

(1)

where \( k_B = 1.3806504(24) \times 10^{-23} \) J/K is the Boltzmann constant [7]. Johnson noise is an intrinsic property of any conductor in thermal equilibrium. The noise is random and frequency independent (i.e. ‘white’) to second order in \( f \tau_c \) where \( \tau_c \) is typically \( 10^{-13} \) s [8].
Noise waveforms are digitally synthesized with pulse-biased Josephson junction arrays that are designed and fabricated at NIST for JNT applications [9]. The systems that produce these noise waveforms are referred to as quantized voltage noise sources (QVNS). The power spectral density \( S_Q \) of a QVNS array of \( N_J \) junctions operating at a sampling frequency \( f_s \) and a repetition rate \( f_1 \) is given by,

\[
S_Q = \frac{Q^2 N_J^2 f_s^2}{K^2 f_1},
\]

where \( K_J = 483 \, 597.891 \, \text{GHz} \cdot \text{V}^{-1} \) is the Josephson constant and \( Q \) is a dimensionless parameter calculated from the digital synthesis [10]. This noise is non-random (i.e., deterministic) [11], approximately Gaussian in the fluctuating (i.e. time-dependent) voltage amplitudes, and programmed to produce a constant average spectral density up to a maximum frequency \( f_{\text{max}} \). In the frequency domain the synthesized noise spectrum consists of a harmonic series of discrete tones of constant and known amplitude starting at the fundamental tone \( f_1 \) with a tone spacing of \( f_1 \) or \( 2f_1 \) and terminating at \( f_{\text{max}} \).

Referring to Fig. 1, a precision comparison of Johnson noise to quantized voltage noise is possible by use of a dual channel amplifier system with specially designed junction-field-effect transistor (JFET) pre-amplifiers [12]. These systems, typically referred to as ‘voltage noise correlators’ [13], are used to determine the ratio of the two averaged cross-correlated spectral densities, \( \langle S_R \rangle / \langle S_Q \rangle \). Normally the amplitude of the synthesized tones is chosen such that the spectral power ratio \( \langle S_R \rangle / \langle S_Q \rangle \cong 1.000 \) in order to minimize the effects of non-linearities in the amplifiers. When this and certain other operating constraints are satisfied [9], the system output can be used to measure the absolute noise temperature of the resistor according to,

\[
T = \frac{\langle S_R \rangle}{\langle S_Q \rangle} \cdot \frac{Q^2 N_J^2 f_s^2}{4k_B K^2 f_1 R f_1}.
\]

This is the basis of spectral Johnson Noise Thermometry (JNT) in the absolute mode [5] using a QVNS reference. This temperature measurement, as described by Eqn. 3, represents both a primary thermometer and an intrinsic standard of temperature. The practical realization of Equation 3, however, requires significant effort to: (1) produce sufficiently linear (i.e. low distortion) amplification; (2) reject of both common-mode and differential mode electromagnetic interference (EMI); and (3) match both transmission line and source impedances [14]. To date, the development of the QVNS-based systems at NIST has yielded absolute-mode uncertainties of 40 \( \mu \text{K} \cdot \text{K}^{-1} \) in the range 500 K to 700 K [8] and 12 \( \mu \text{K} \cdot \text{K}^{-1} \) at 273 K [15].

The output of the noise correlator is not perfectly flat in frequency over the typical measurement bandwidth of ~600 kHz. This is due to a variety of finite time constants, which exist in any system and are primarily associated with the input-coupling and source impedances [16]. For power ratio spectra below ~600 kHz, these time constants lead to a quadratic frequency dependence. For systems using low-loss insulator materials, this can easily be accounted for by fitting the ratio spectrum to a simple model \( \langle S_R \rangle / \langle S_Q \rangle = a_0 + a_2 f^2 \). In practice, the line and source impedances are closely matched, which generally produces only small remnant quadratic coefficients. For systems where the measured temperature exceeds ~500 K, some dielectric losses in the hot sections of insulators are inevitable and the model should accommodate an additional linear term or \( \langle S_R \rangle / \langle S_Q \rangle = a_0 + a_1 f + a_2 f^2 \) [17].

Prior to the advent of Josephson AC waveform synthesis, the only practical way to perform spectral JNT was using Johnson noise from another resistor \( R_0 \) at a known reference temperature \( T_0 \) as the noise reference (replace ‘QVNS’ with Johnson noise from \( R_0T_0 \) in Fig. 1). This method had some drawbacks due to the power matching requirement of \( RT = R_0T_0 \), which leads to inherently unmatched time constants, relatively large values of \( a_2 \), and often higher-order frequency corrections [18]. The achievable uncertainty under these conditions is degraded compared to what can otherwise be achieved in a well-matched
QVNS-based system, but relative uncertainties of ~100 μK·K⁻¹ are possible. This so called R-based JNT produces only a relative temperature and the extent to which this class of noise thermometer qualifies as an intrinsic standard is subject to interpretation depending on the details of the reference configuration.

There exists a third class of electronic noise-based instruments based on tunnel junctions operating at radio frequencies [19], and these are usually referred to as shot noise thermometers (SNTs). While these instruments qualify as intrinsic standards and also measure absolute temperature, their primary utility is at cryogenic temperatures and uncertainties near 300 K appear limited to ~ 1%. Finally, another JNT instrument has recently emerged [20] using superposition of audio harmonic reference tones that are traceable to electrical artifact standards. These instruments also measure absolute temperature but are not, strictly speaking, intrinsic standards.

Figure 1. Block diagram of a typical spectral JNT system. The temperature is sensed by the resistor \( R \) and the Johnson noise is compared to the QVNS reference noise by alternately switching the source inputs into a dual-channel amplifier and digitizer system. Those digitized time series are then Fourier transformed and multiplied to form the cross-correlated noise power spectrum. The transmission lines (‘X-line’) represent impedances of cable and PC boards, which determine the dominant time constants for a system.

Overall a JNT system offers the advantage of a purely electronic-based primary thermometer that can serve as an intrinsic standard at any arbitrary temperature. Furthermore, the noise temperatures are relatively immune to the chemical purity issues that are always critical to any thermo-physical temperature standard. This conveys advantages in flexibility for its use as a standard in a laboratory and certain industrial environments. The main trade-off occurs in the complexity that is required for the instrumentation, which we outline below.

**Hardware Requirements**

Practically all of the JNT electronics has been custom built from readily available electronic components. The most critical part of the design is the pre-amplifier, the performance of which determines the errors in the correlated noise power [12]. The simultaneous requirements of high common-mode rejection, low voltage noise and low current noise necessitate the use of a differential JFET design. The design currently being used at NIST is a JFET input bipolar-cascode type preamp with digitizer and various buffer amplifiers that drive passive filter stages, which suppress aliasing effects (see Fig. 2). Copies of these amplifiers have now been provided to several laboratories around the world [23].

The digitizers suitable for JNT systems must exhibit a high degree of isolation, linearity, and sampling speed. Analog to digital converters (ADCs) as used in some telecommunication equipment can often be adapted for this purpose. The ADC used in the most recent NIST designs can sample with 16 bit resolution up to 6 MHz with 106 dB spurious-free dynamic range. The digitizer isolation is critically dependent on the output coupling. For this reason, JNT digitizers should have the output optically decoupled. In the NIST JNT systems, we use 50 Mbit s⁻¹ fiber-optic cable links to transmit the data from each channel to a custom bus-interface card installed on the back-plane of a dual-xeon motherboard.
This configuration allows for direct-memory addressing of the data for further processing in a compiled C-language executable program.

Figure 2. The block layout for the NIST JNT electronics configured with two buffered passive filter blocks and three gain blocks.

For a QVNS-based spectral JNT system, the principal QVNS sub-components are a bit-stream generator, the Josephson array, and the immersion cryoprobe for mounting and cooling the array to 4.2 K (see Fig. 3). The bit-stream generator must be capable of producing programmable unipolar pulses that are clocked at 5 GHz or faster and also have a re-circulating memory of capacity $M_s$ to store the binary code that defines the QVNS waveform. Equipment that fulfills these requirements is found in the telecommunications industry and known as pattern generators, which are typically used in bit-error-rate testing systems. Older pattern generators tend to have smaller memories, on the order of 8 Mbit, while newer types can be found with 64 Mbit or larger, which allows longer codes with smaller spacing $f_1$. The Josephson array chip is a highly specialized integrated circuit made from niobium-normal metal-niobium junctions patterned on a 1 cm$^2$ Si substrate. At present only NIST has designed and fabricated QVNS circuits that are appropriate for JNT [10].

Figure 3. Block diagram for the QVNS. The Delta-Sigma modulator takes a time-series waveform and produces an equivalent bit-stream representation, which is then loaded into the pattern generator memory. The code $D$ and its binary complement $\bar{D}$ are generated with a clock rate of $f_s=10$ GHz. The bit stream outputs are high-pass filtered (HPF) and fed into microwave transmission lines that terminate on the Josephson array within the liquid helium bath. The outputs across the entire array circuit are two equivalent shielded pairs.
Spectral JNT is both data intensive and computationally intensive in nature. As an example from the NIST systems, for a maximum bandwidth of approximately 1 MHz with 1 Hz resolution, two 1x10^6 point FFTs must be computed together with three 1x10^6 point x 1x10^6 point array multiplications to form the two autocorrelation and one cross-correlation spectra. Ideally this must be done in much less time than the 1 s sample time required to acquire the data in order to avoid any appreciable deadtime. Fortunately, C-language FFT libraries [24] have been publically available for some time and can easily accomplish this on 32-bit Pentium-class dual-processor systems. In addition, storage requirements for this amount of raw spectra are approximately 33 megabytes every 100 s. For a 0.6 MHz usable bandwidth, an averaging time of ~120 hours is required for a QVNS-based JNT system to achieve a statistical uncertainty of ~7 μK·K⁻¹. Hence, one such measurement requires more than 140 gigabytes of hard drive space. Fortunately, such requirements are not exceptional given current commercial data storage technology.

The thermal probe (‘R thermal’ in Fig. 1) senses the unknown temperature and for any JNT system is subject to a few design constraints. These include: adequate shielding from EMI sources; adequate immersion characteristics to achieve proper equilibration; the use of materials that do not react or degrade with time and temperature exposure; and the use of high-quality, low-loss insulators. The first constraint is the only one really unique to JNT, while the others are common to all forms of Pt resistance thermometry. The use of high-quality, low-OH⁻ content fused-quartz insulators and noble metal alloys will best address the last two criteria. Although not a critical constraint, it is also useful for the alloy of the sensing resistor to exhibit a low-temperature coefficient of resistance (e.g. <~ 100 μΩ·Ω⁻¹·K⁻¹) so that impedance matching is maintained over as wide a temperature range as possible.

Applications as a Laboratory Standard

JNT has always been a primary method well suited to static systems where the temperature is held constant or drifting very slowly. In metrology, this is always the case for fixed-point realizations [25] or temperature-scale evaluations [26]. As an intrinsic standard, a QVNS-based system can be programmed to generate the spectral power density, which matches that of any given RT product. In principle, any artifact thermometer (e.g. Standard Pt Resistance Thermometer (SPRT), etc.) that can be installed in a comparison system can then be calibrated against a set of arbitrary temperatures as measured by the JNT electronics. In practice, the ITS-90 itself is compared to the noise temperatures either by comparison to SPRTs or directly to fixed point cells for the purposes of evaluating the scale. In either case the thermal comparison system or the fixed-point system must have its own temperature control electronics and that circuitry must have relatively good isolation and stability.

Implementation of a JNT system as a laboratory temperature standard can be done in the range 500 K to 900 K with uncertainties ~40 μK·K⁻¹ or better using commercially available comparison furnaces. However, some customization of the furnace controls is necessary when used in conjunction with a JNT system. In particular, the power supplies must be purely linear (i.e. continuously transformed) and not based on switching power supplies. Most switching supplies produce significant EMI, which is difficult to decouple from the DC power necessary to drive the heating elements of the furnace. Likewise, good isolation of the heaters themselves greatly reduces the likelihood of other EMI being coupled in through DC ground loops.

Starting from a desired spectral power or RT product, the calculation of Q involves an inverse Fourier transform from the desired random-phase harmonic tone series to a repeating time series. From that input, a Δ–Σ modulator algorithm is applied to derive the correct binary pattern for loading into the generator’s memory. This process takes approximately 30 minutes on a moderately fast processor. However, this is based on an uncompiled scripting language and could be significantly shortened by porting to a compiled C-language version. In any case, this time is comparable to or less than the time necessary to ramp a furnace to a new temperature and re-stabilize its control so it does not represent any real limitation.

The main role of JNT as a laboratory standard is as a research tool for studying the small defects that exist in artifact-based temperature scales such as the ITS-90 or in establishing the foundation of a
possible future temperature scale. NIST has performed both R-based and QVNS-based JNT as comparison measurements against calibrated SPRTs [18][17]. These measurements were in fact adequate to place new limits on the possible deviations of the ITS-90 at the zinc freezing point \( T_{90} = 692.93 \) K [8]. These experiments allowed our new JNT techniques to be indirectly compared with other primary thermometers, which are based on entirely different physics and different technology [27]. This situation arises because acoustic gas-based thermometry data exists in the range between 273 K and 550 K [28] and radiation thermometry data exists in the range above 690 K [29], all of which are relative to the ITS-90. The currently available JNT data that overlaps these two ranges are in agreement with those other primary thermometers to within the combined experimental uncertainties.

An even more demanding application exists for a planned reformulation of the International System (SI) of units, including the definition of the kelvin itself [30]. In this application it is necessary to measure the absolute value of \( k_B T \) near the water triple point \( (T_{SI} = 273.16 \) K). This temperature currently defines the SI kelvin and consequently is where the lowest uncertainty can be achieved. Since \( T \) can be realized with a lower uncertainty than that of \( k_B \), the measurement becomes a determination of the Boltzmann constant via Eqn. (3). To date the lowest uncertainty determinations of \( k_B \) have been performed using gas-based acoustic resonance methods [31] and all but two of the most competitive new experiments [32] will continue to utilize this approach. The one notable exception is the NIST JNT method, which has recently achieved a new result for \( k \) at \( 12 \times 10^{-6} \) relative uncertainty [15]. Since QVNS JNT is a purely electronic intrinsic standard based on entirely different physics from the gas-based methods, given a sufficient further reduction in uncertainty, it could enable an independent cross-check for the pool of data being collected [7] and analyzed for the future reformulation of the SI.

Applications as in-situ derived Standards

A primary thermometer can also be useful for certain demanding in-situ process measurement applications. These include embedded thermometer installations, which for reasons of personnel safety or remote location cannot be serviced in any conventional sense. The ability to maintain traceability to a primary thermometer at any arbitrary temperature is a key advantage in harsh radiation or high-temperature environments [33]. The co-location of JNT probes and artifact sensors in process environments allows for in-situ validation of the artifacts to correct for the calibration changes in the artifact thermometers that occur due to chemical contamination, oxidation, or radiation damage. This becomes important for quality control or safety in the event of an undetected failure or a very slow degradation over time, which may be difficult to detect. Various JNT systems have been built for such applications in the past by several different groups worldwide [34]. In all these cases, however, the systems were not strictly speaking used as intrinsic standards, since the gain calibration was usually derived from an artifact thermometer.

The JNT systems that have been built for in-situ process monitoring applications have historically been simpler R-based designs, which required a reference temperature for calibration and had uncertainty requirements of only 0.1% or greater. The best examples are found for applications in high-flux ionizing or neutron radiation environments in nuclear reactor facilities. Studies of monitoring the so-called in-core temperatures of a reactor were carried out at Oak Ridge National Lab (ORNL) more than twenty years ago [35]. Other JNT studies have been carried out in Germany in experimental reactors [36], which served to advance the instrumentation technology to what was then state-of-the-art in the early 1990s [37] with uncertainties of \( \sim 0.01 \% \). Newer JNT instrument designs for nuclear applications have also recently emerged from ORNL [38], where the traceability is derived from electrical artifact standards with a target uncertainty of 0.1%. Newer JNT designs have also been developed at Forschungszentrum Jülich [39] and recently tested [40] for possible nuclear applications at temperatures near 300 °C.

A completely different application has recently been found for JNT in micro-thermometry where the difficulties are in finding suitable in-situ calibration methods for micron-scale fabricated thermoelectric materials [41]. At the micron-scale the properties of materials deviate from that found in bulk samples as surface effects and large deposition stress become important. JNT also allows on-chip investigation of
thermal effects in small device structures with high spatial resolution [42]. Again, these methods yield relative temperatures only and are functionally equivalent to derived standards.

**Future Research**

Further research is needed to reduce the uncertainty to the \( \sim 5 \, \mu K \cdot K^{-1} \) level for QVNS-based JNT in laboratory applications at NIST. This level of uncertainty would be competitive with gas-based primary thermometers for temperatures \( \sim 300 \, K \). For temperatures in the range \( \sim 700 \, K \) to \( 930 \, K \), the relative-mode QVNS JNT results at uncertainties of \( \sim 30 \, \mu K \cdot K^{-1} \) are already competitive with the best absolute radiation thermometers and we expect that lower uncertainties will soon be demonstrated even in the absolute mode measurements.

The current estimated deviations for \( T-T_{90} \) provided by the Consultative Committee for Thermometry (CCT) Working Group 4 (WG4) [43] is dominated by absolute radiation thermometry results above 690 K. These estimates are represented by an `analytical smooth interpolation function' for the range 273 K to 1360 K which is essentially monotonic above 400 K. This function represents the degree to which the ITS-90 SPRT reference function (extending to 1235 K) may be in error. But there is nothing in principle that precludes the existence of a more complex form for the error. More data of the lowest (i.e. \( \sim 10 \, \mu K \cdot K^{-1} \)) uncertainty is needed in the range from 500 K to 930 K to establish whether or not a purely monotonic function is adequate to represent the error in this range (see Figure 4).

![Figure 4](image-url)

**Figure 4.** The deviations of the ITS-90 from thermodynamic temperature in the range 300 K to 950 K as determined by primary thermometers based on: constant volume gas; acoustic gas; plank radiation; and noise thermometry methods. (Error bars are \( k=1 \) standard uncertainties)

The only primary thermometry methods capable of resolving this question in the near term are acoustic gas thermometry and QVNS-JNT. For acoustic gas thermometry, new acoustic-duct transduction techniques are required in order to extend the current upper practical temperature limit of \( \sim 550 \, K \). The development of these techniques is underway at NIST [44] but the achievable uncertainties and practical upper temperature limits have not yet been demonstrated. At the same time, the NIST QVNS-JNT
The program is undergoing an electronics upgrade, which is expected to improve the achievable uncertainties by a factor of 2 to 3 in the range 500 K to 930 K.

For possible future in-situ applications, the achievable uncertainties are already more than adequate and the principle focus should be on simplification and automation. A key functionality for any in-situ process JNT is the ability to automate the system to operate without human intervention in the case where the process temperature is changing and must be tracked accurately. The systems discussed up to this point operate as 'open-loop' fixed-temperature instruments and manual intervention is needed to change the intended operation to a new temperature. This manual operation consists of either swapping out a reference resistor in the case of R-based JNT or, in the case of QVNS-based JNT, programming a new code and loading it into the pattern generator. These operations are necessary in order to keep the noise correlator inputs approximately balanced by maintaining the condition \( \frac{S_R}{S_S} \approx 1.000 \).

In theory, a ‘closed-loop’ or auto-balancing mode of operation is also possible for tracking changing temperatures. In R-based JNT, this would involve feeding back the correlator power ratio output and adjusting the value of \( T_0 \) in proportion to changes in \( T \). If such a system is impedance matched such that \( R = R_0 \), then the control circuit set point for the block or thermal control zone is continuously updated to keep the noise power ratio near unity by driving \( T_0 = T \). Since the reference resistor \( R_0 \) exists in a controlled environment away from the process zone, \( T_0 \) can be measured with SPRTs with a low uncertainty. This closed-loop mode can be thought of as a ‘thermal noise mirror’, for R-based JNT systems (see Fig. 5).

Closed-loop operation is also possible for QVNS-based JNT systems. In this case however, considerably more processing power must be introduced to handle the computations necessary for periodic updates of the binary code as the synthesized tone amplitudes change. Referring again to Fig. 5, the element identified as the \( \Delta-\Sigma \) processor would be a dedicated program, probably running on a dedicated multi-core processor to update the binary codes and load these into the pattern generator. An additional benefit to implementing such a system would be the ability to periodically load new random phase distributions for the synthesized harmonic tone series from a preexisting library of codes. This type of randomization reduces the susceptibility of QVNS spectra to distortion effects as the distortion products average out over time [21]. This type of system has not yet been implemented, but in principle the technology necessary to do it can be constructed from existing commercial hardware, with software integration being the main missing component.

There would of course be a practical limit to the rate of change of temperature that can be tracked using any JNT system, but this would depend on the uncertainty target. For the statistical performance of the JNT systems at NIST, a drift rate of \( \sim 170 \mu K \cdot K^{-1} \cdot hr^{-1} \) could be tracked with an uncertainty of approximately 100 \( \mu K \cdot K^{-1} \), while a drift rate 100 times larger than that could only be tracked with an uncertainty of 1000 \( \mu K \cdot K^{-1} \) (i.e. 0.1 %). A more demanding requirement would probably be created by the typical time available to perform the necessary \( \Delta-\Sigma \) algorithm calculations, which ideally should be less than the 100 s input switching interval in order to avoid any deadtime in the data acquisition. Again, the principle challenge would be in optimizing the computational routines to run efficiently on available commercial computer hardware.
In summary, the laboratory applications for JNT are primarily being driven by requirements for temperature scale research and the future redefinition of the SI unit of temperature. In both cases, reducing the achievable uncertainty to be comparable to the best gas-based techniques remains the principle challenge. We see the QVNS-based JNT as best positioned to achieve those uncertainty targets and to continue to gain wider recognition as an intrinsic temperature standard. For possible future in-situ JNT applications, the principle challenge is probably in simplifying the technology from the operational standpoint and adding automation features, while relaxing the uncertainty performance.

References


