A New Convexity Measurement for 3D Meshes

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Abstract

This paper presents a novel convexity measurement for 3D meshes. The new convexity measure is calculated by minimizing the ratio of the summed area of valid regions in a mesh’s six views, which are projected on faces of the bounding box whose edges are parallel to the coordinate axes, to the sum of three orthogonal projected areas of the mesh. The complete definition, theoretical analysis, and a computing algorithm of our convexity measure are explicitly described. This paper also proposes a new 3D shape descriptor CD (i.e., Convexity Distribution) based on the distribution of above-mentioned ratios, which are computed by randomly rotating the mesh around its center, to better describe the object’s convexity-related properties compared to existing convexity measurements. Our experiments not only show that the proposed convexity measure corresponds well with human intuition, but also demonstrate the effectiveness of the new convexity measure and the new shape descriptor by significantly improving the performance of other methods in the application of 3D shape retrieval.

1. Introduction

Shape measurement is a fundamental problem in several research communities including computer vision, pattern recognition, and computer graphics. It is often preferable if shapes can be quantified by some global shape measures with direct intuitive meanings, such as, chirality [17], compactness [32], convexity [34], ellipticity [18], rectangularity [21], rectilinearity [33], symmetry [9], triangularity [22], and so on. While a large amount of 2D shape measures have been proposed in the last few years, there has been much less work on the measurement of 3D shapes. This paper investigates the convexity for 3D polygon meshes.

Convexity is a basic and popular shape descriptor with many applications. As defined in [34] [19], an object is said to be convex if it contains all points on the line segment between any two points that belong to the object. The concept of convexity of 3D shapes is demonstrated by some simple examples shown in Figure 1. Intuitively, the sphere and the cube are considered to be convex, while the other two are concave and the rightmost one should have a smaller value of convexity than its nearest neighbor. We would like to define a shape measure describing the extent to which a 3D object is convex. Trivially, the following two definitions can be obtained by directly generalizing 2D convexity measures [34] [19] into 3D domain.

Definition 1. For a given 3D closed mesh $M$, where the region bounded by the surface $M$ is $M_+$, we define its convexity measure $C_1(M)$ as
\begin{equation}
C_1(M) = P(\alpha X_1 + (1-\alpha)X_2 \in M_+), \forall \alpha \in [0, 1],
\end{equation}
where $P(E)$ denotes the probability of event $E$, $\{\alpha X_1 + (1-\alpha)X_2; 0 \leq \alpha \leq 1\}$ is the line segment between any two points $X_1$ and $X_2$ randomly chosen on the surface $M$.

Definition 2. For a given 3D closed mesh $M$, where the convex hull of $M$ is $CH(M)$, we define its convexity measure $C_2(M)$ as
\begin{equation}
C_2(M) = \frac{Volume(M)}{Volume(CH(M))}.
\end{equation}

Although the above two definitions both satisfy necessary requirements (see [34]) for the convexity measurement, they also have their own limitations. In general, the computational cost of the statistics-based measure $C_1(M)$ is quite expensive. While the volume-based measure $C_2(M)$ is easy to calculate but it cannot always detect small defects of volume, even if the defects have a huge impact on the surface area of 3D models. That is indicated by two examples
shown in Figure 2(a)(b). As we can see, these two models (first row) have similar values of surface area and volume but quite different convex hulls (second row). When the volume of the deep indentation in Figure 2(a) tends to 0, the object’s convexity calculated by \( C_2(M) \) is arbitrary close to 1 although its depth remains unchanged. In other words, the measurement \( C_2(M) \) is not able to detect deep but small-volume indentations into shapes. However, in some applications, we would like to have a 3D convexity measurement that is sensitive to all kinds of defects of surface. To the best of our knowledge, up to now, no boundary-based (i.e., area-based) convexity measure has been proposed for 3D shapes, and the boundary-based 2D convexity measure cannot be directly generalized into 3D. Recall that, for a given 2D polygon, its boundary-based convexity measure is defined as the ratio of the Euclidean perimeter of the boundary of its convex hull to that of the original 2D shape [34]. However, the surface area of a 3D mesh can be either smaller or larger than the surface area of its convex hull (see Figure 2(c)(d)), thereby neither \( \frac{\text{Area}(M)}{\text{Area}(\text{CH}(M))} \) nor \( \frac{\text{Area}(\text{CH}(M))}{\text{Area}(M)} \) is suitable to measure convexity for 3D shapes.

Furthermore, existing methods all calculate a single value to measure the convexity property of a given shape, and thus there exist lots of shapes that are quite different but still have the same convexity value obtained using those traditional methods. To address the problems mentioned above, this paper propose an area-based convexity measure and a new convexity-related shape descriptor for 3D meshes. Given a 3D model, the new convexity measure (see Figures 1 and 2 for some computed results) is calculated by minimizing the ratio of the summed area of valid regions in six orthogonal views to the sum of three orthogonal projected areas, and the new shape descriptor \( CD \) (i.e., Convexity Distribution) is constructed based on the distribution of above-mentioned ratios computed by randomly rotating the mesh around its center. Since our convexity measure and shape descriptor both represent 3D models in quite different manners compared to other existing shape descriptors, and thus provide new and independent information, they are well suited to be incorporated with other shape descriptors to generate discriminative composite signatures. Our experiments validate the effectiveness of the proposed convexity measure and convexity distribution in the application of 3D shape retrieval.

2. Related Work

Convexity is one of the best-known global shape descriptors that have direct intuitive meanings. Up to now, a number of definitions of the convexity measure have been reported for 2D shapes. Among them, the mostly used method that appears in textbooks [26] defines the convexity of a 2D polygon as the ratio of the shape’s area to the area of its convex hull. Similarly, the ratio between the Euclidean perimeter of a polygon’s convex hull and the Euclidean perimeter of the polygon is also a natural solution to measure the convexity of 2D shapes [34]. Alternatively, according to the definition of convex shapes [34] [19], a convexity measure can directly be defined as the probability that, for randomly chosen points \( A \) and \( B \) in a 2D shape, all points on the line segment \([AB]\) belong to the shape [34]. There are also other 2D convexity measures whose definitions are not so intuitive. For example, Stern [27] developed an area-based convexity measure to incorporate the entire topology of the polygon. Boxer [1] estimated the convexity of a given 2D polygon based on the distances between the vertices of the polygon and its convex hull. Recently, Žunić and Rosin [34] presented a boundary-based method to measure convexity by using the minimum ratio of the Euclidean perimeter of the bounding rectangle of a polygon to the polygon’s \( L_1 \) perimeter. Rahtu et al. [19] proposed a convexity measure based on the idea of randomly choosing pairs of points from a 2D object and then computing the probability that a point located on the specified position of corresponding line segments belongs to the shape. One merit of this method is that it can be implemented efficiently using the Fast Fourier Transform. Rosin and Mumford [23] introduced a symmetric convexity measure that is based on the maximally overlapping convex polygon of a 2D shape.

However, there has been considerably less work for the shape measurement of 3D shapes. Regarding convexity, besides the two intuitive definitions presented in Section 1, Rahtu et al. [19] generalized their probability-based method to measure the convexity of N-dimensional data, while Fink and Wood [5] developed a restricted-orientation convexity which was defined in terms of the intersection of a geometric object with lines parallel to the elements of a fixed orientation set. In recent years, more and more researchers have become interested in 3D shape measurements. For instance, Bribiesca [2] proposed a compactness measure which corresponds to the sum of the contact surface areas of the face-connected voxels for 3D shapes. Kazhdan et al. [8] pre-
presented a 3D object’s reflective symmetry descriptor as a 2D function associating a measurement of reflective symmetry to every plane through the model’s centroid. Lian et al. [12] proposed a rectilinearity measure for 3D polygon meshes, which is defined as the maximum ratio of the surface area to the sum of three orthogonal projected areas of the mesh. Motivated by the work presented in [34] and [12], this paper develops a new convexity measure for 3D meshes.

One potential application of shape measurements is 3D shape retrieval. Until recently, a large number of methods have been developed for the retrieval of 3D shapes, such as, D2 [16], SHD [8], LFD [3], and so on. Probably because of the complexity of non-rigid 3D shape analysis, previous efforts have mainly been devoted to rigid 3D shape retrieval (see [30] for more details). Thereby, how to effectively and efficiently calculate the dissimilarity between non-rigid models is still considered to be a challenging problem in content-based 3D object retrieval. Here, we briefly review some representative work published recently for the retrieval of non-rigid 3D shapes. Wang et al. [31] proposed to compare non-rigid 3D models based on a local feature named Intrinsic Spin Images (ISIs), which is designed by generalizing the traditional spin images [7] from 3D space to N-dimensional intrinsic shape space. Tam and Lau [29] used topological and geometric features simultaneously to represent a theoretical framework to directly compare non-rigid 3D models based on the Bag-of-features approach.

3. A New Convexity Measure for 3D Meshes

In this section, we first give some notations used in this paper and then present the definition of our new convexity measure with complete proofs. Throughout this paper, we assume that all considered shapes are 3D closed trian-

\[ P_{\text{face}}(M, \alpha, \beta, \gamma) = P_{\text{face}x}(M, \alpha, \beta, \gamma) + P_{\text{face}y}(M, \alpha, \beta, \gamma) + P_{\text{face}z}(M, \alpha, \beta, \gamma), \]  

(3)

while the sum of areas of the valid regions in six views projected on faces of the bounding box whose edges are parallel to the coordinate axes is defined as

\[ P_{\text{view}}(M, \alpha, \beta, \gamma) = 2 \cdot (P_{\text{view}x}(M, \alpha, \beta, \gamma) + P_{\text{view}y}(M, \alpha, \beta, \gamma) + P_{\text{view}z}(M, \alpha, \beta, \gamma)). \]  

(4)

**Theorem 1.** 1) The inequality

\[ P_{\text{face}}(M, \alpha, \beta, \gamma) \geq P_{\text{view}}(M, \alpha, \beta, \gamma) \]  

(5)

holds for any 3D closed mesh \( M \).
2) A given 3D mesh $M$ is convex if and only if for any choice of the coordinate frame the sum of three projected areas of $M$ equals the sum of areas of the valid regions in six views projected on faces of the bounding box whose edges are parallel to the coordinate axes, i.e.,

$$P_{\text{face}}(M, \alpha, \beta, \gamma) = P_{\text{view}}(M, \alpha, \beta, \gamma), \quad \forall \alpha, \beta, \gamma \in [0, 2\pi].$$

Proof. On the one hand, if a given 3D mesh $M$ is convex, then the projections of the faces of $M$ onto the $YOZ$, $ZOX$, and $XOY$ planes exactly cover the valid regions of six views projected on faces of the bounding box whose edges are parallel to the coordinate axes (see Figure 4(a)). Furthermore, such projections are independent of the choice of the coordinate system, i.e., $P_{\text{face}}(M, \alpha, \beta, \gamma) = P_{\text{view}}(M, \alpha, \beta, \gamma)$, for all $\alpha, \beta, \gamma \in [0, 2\pi]$, when $M$ is convex.

On the other hand, if $M$ is not convex, then there exist points $A$ and $B$ on the mesh $M$ such that the line segment $[AB]$ does not completely belong to $M$ and its interior (see Figure 4(b)). Let a coordinate axis, say $z$, be parallel to the line through points $A$ and $B$, then the projections of the faces of $M$ onto the plane $XOY$ must overlap (see Figure 4(c)). That is, $P_{\text{face}}(M, \alpha, \beta, \gamma) > P_{\text{view}}(M, \alpha, \beta, \gamma)$ for some or all $\alpha, \beta, \gamma$. Moreover, the fact that $M$ is a closed mesh means any point on the valid region of projected views has at least two corresponding points on the surface of $M$. In other words, $P_{\text{face}}(M, \alpha, \beta, \gamma)$ could never be smaller than $P_{\text{view}}(M, \alpha, \beta, \gamma)$. This complete the proof for both statement 1) and 2).

Theorem 1 suggests that the ratio $P_{\text{view}}(M, \alpha, \beta, \gamma) / P_{\text{face}}(M, \alpha, \beta, \gamma)$ can be used as a convexity measure for the 3D mesh $M$. But this ratio depends strongly on the choice of the coordinate system and, in some cases, it can be equal to 1 for concave meshes (see Figure 4(b)), which is not acceptable for a convexity measure. We address the problem by calculating $\min_{\alpha, \beta, \gamma \in [0, 2\pi]} P_{\text{view}}(M, \alpha, \beta, \gamma) / P_{\text{face}}(M, \alpha, \beta, \gamma)$ and define the new convexity measure of 3D meshes as

$$C(M) = \min_{\alpha, \beta, \gamma \in [0, 2\pi]} \frac{P_{\text{view}}(M, \alpha, \beta, \gamma)}{P_{\text{face}}(M, \alpha, \beta, \gamma)}. \quad (7)$$

The following theorem summarizes properties of the proposed convexity measure:

**Theorem 2.** For any 3D mesh $M$, we have:

1. $C(M)$ is well defined and $C(M) \in (0, 1]$;
2. $C(M) = 1$ if and only if $M$ is convex;
3. $\inf_{M \in \Pi} (C(M)) = 0$, where $\Pi$ denotes the set of all meshes;
4. $C(M)$ is invariant under similarity transformations.

Proof. Items 1, 2, 4 can be directly derived from Theorem 1 and Definition 3. In order to prove Item 3, we introduce a 3D mesh $M_n$ (see Figure 5). Since the length of diagonal of the bounding box of $M_n$ is $\sqrt{3n}$, which is the maximum length of line segments between any two points on $M_n$, we have

$$P_{\text{view}}(M, \alpha, \beta, \gamma) < 6 \cdot (\sqrt{3n} \cdot \sqrt{3n}) = 18n^2, \quad (8)$$
for any $\alpha, \beta, \gamma \in [0, 2\pi]$. On the other hand,

$$P\text{face}(M, \alpha, \beta, \gamma) \geq 6n^2 - 2 \cdot n \cdot \frac{1}{2} \cdot (n - 1) + 2 \cdot (n - 1) \cdot (n - 1) \cdot n = 2n^3 + n^2 + 3n,$$

(9)

and

$$\lim_{n \to \infty} \frac{18n^2}{2n^3 + n^2 + 3n} = 0,$$

(10)

which means, for any $\varepsilon > 0$, there exists a $n$ such that

$$0 < C(M_n) = \min_{\alpha, \beta, \gamma \in [0, 2\pi]} P\text{view}(M_n, \alpha, \beta, \gamma) < \frac{18n^2}{2n^3 + n^2 + 3n} < \varepsilon,$$

(11)

or equivalently, for some $M_n$, their convexity $C(M_n)$ can be arbitrary close to $0$. That completes the proof.

4. Computation of Convexity

In this section, we describe how to calculate the proposed convexity measure $C(M)$ for 3D meshes. We first present the computations of $P\text{face}(M, \alpha, \beta, \gamma)$ and $P\text{view}(M, \alpha, \beta, \gamma)$, and then we solve the nonlinear minimization problem to obtain an approximate value of the convexity measure. Note that, before the calculation of convexity, all models should be normalized with respect to the coordinate frame so that their mass centers coincide with the origin and they are bounded by the unit sphere.

4.1. Computation of $P\text{face}(M, \alpha, \beta, \gamma)$

Assume that the 3D mesh $M$ consists of $N$ triangles $(T_1, T_2, \ldots, T_N)$. The coordinates of these triangles’ vertices are denoted by $(x_{i0}, y_{i0}, z_{i0})$, $(x_{i1}, y_{i1}, z_{i1})$, $(x_{i2}, y_{i2}, z_{i2})$, $i = 1, \ldots, N$. After successively rotating the coordinate system around its $x, y, z$ axes with angles $\alpha, \beta, \gamma$, we obtain their new coordinates, which are denoted as $(x'_{i0}, y'_{i0}, z'_{i0})$, $(x'_{i1}, y'_{i1}, z'_{i1})$, $(x'_{i2}, y'_{i2}, z'_{i2})$, $i = 1, \ldots, N$, by using the following formulae

$$(x'_{ij}, y'_{ij}, z'_{ij})^T = R_x(\gamma) \cdot R_y(\beta) \cdot R_z(\alpha) \cdot (x_{ij}, y_{ij}, z_{ij})^T,$$

(12)

where $R_x(\alpha)$, $R_y(\beta)$, and $R_z(\gamma)$ stand for the matrices which result in the rotations of the coordinate frame around its $x$, $y$, and $z$ axes, respectively. Then, the sum of the three projected areas is computed by

$$P\text{face}(M, \alpha, \beta, \gamma) = \sum_{i=1}^{N} (S'_{ix} + S'_{iy} + S'_{iz}),$$

(13)

where $S'_{ix}$, $S'_{iy}$, and $S'_{iz}$ are the projected areas of the triangle $T_i$ on the planes $YOZ$, $ZOX$, and $XOY$, respectively (see Figure 3).

4.2. Computation of $P\text{view}(M, \alpha, \beta, \gamma)$

As mentioned in Section 3, $P\text{view}(M, \alpha, \beta, \gamma)$ denotes the sum of areas of the valid regions in six views projected on faces of the bounding box whose edges are parallel to the coordinate axes. Theoretically, it is possible to calculate the exact area of the valid region shown in a 2D projection of a 3D polygon mesh. However, as shown in Figure 6(a), the boundary of a projected 2D polygon could be very complicated for some 3D meshes due to large amounts of overlapping and intersections. As a matter of fact, the complexity of the exact area calculation for this kind of 2D shapes is often unacceptable in practice. To solve the problem, we apply an image-based method to efficiently calculate the approximate value of $P\text{view}(M, \alpha, \beta, \gamma)$. First, three silhouette views are captured in the directions of the coordinate axes (see Figure 6(b) for an example whose valid region is comprised of a set of black pixels). Then, we calculate the number of black pixels in three views (see Figure 6(c)), which are denoted as $Np_x$, $Np_y$, and $Np_z$, respectively. Note that the exact value of area of the projected image for each view is 4, since after normalization the mesh $M$ is bounded by the unit sphere. Finally, let the total number of pixels in the view be denoted as $Np$, we have

$$P\text{view}(M, \alpha, \beta, \gamma) = 2 \cdot \left(4 \cdot \frac{Np_x}{Np} + 4 \cdot \frac{Np_y}{Np} + 4 \cdot \frac{Np_z}{Np}\right).$$

(14)

Here, OpenGL is utilized to capture depth-buffer views from 3D objects and the resolution of images is experimentally chosen as $400 \times 400$, which implies $Np = 160000$.

4.3. Computation of $C(M)$

Owing to the complexity of $P\text{view}(M, \alpha, \beta, \gamma)$ and $P\text{face}(M, \alpha, \beta, \gamma)$, we find that it is very difficult and computationally expensive to compute the exact value of our convexity measure $C(M)$ for 3D meshes. On the other hand, according to the definition of $C(M)$, we observe that the computation of convexity is basically a nonlinear optimization problem which has been well studied and can be efficiently resolved by intelligent computing approaches.
In this paper, we choose the Genetic Algorithm (GA), which is an optimization technique based on natural evolution [6], to calculate the new convexity measure in the following two steps.

1. **Initialization:** Define and create a group with $N_g$ individuals. Each individual contains a value of fitness $P_{view}(M, \alpha, \beta, \gamma)$ and three chromosomes $\alpha, \beta,$ and $\gamma,$ which are represented by binary codes.

2. **Implementation:** Iterate the genetic algorithm procedure, which consists of encoding, evaluation, crossover, mutation and decoding, for $N_{gen}$ generations. The greatest value of fitness of all individuals in the group is obtained to calculate the approximate value of $C(M)$.

Coefficients of the Genetic Algorithm adopted here are selected as follows: the number of individuals $N_g = 50$; the number of evolution generations $N_{gen} = 200$; the length of each chromosome’s binary codes $L_c = 20$; the probability of crossover $p_c = 0.800$ and mutation $p_m = 0.005$.

5. **Convexity Distribution**

According to the definition and theorems mentioned in Section 3, we know that the ratio

$$R(M, \alpha, \beta, \gamma) = \frac{P_{view}(M, \alpha, \beta, \gamma)}{P_{face}(M, \alpha, \beta, \gamma)}$$  \hspace{1cm} (15)

relates closely to the convexity of a 3D mesh $M$. Since $R(M, \alpha, \beta, \gamma)$ changes when the rotation angles $\alpha, \beta, \gamma$ vary, a set of such ratios can be obtained by randomly (or uniformly) rotating the mesh around its center. We then construct a new shape descriptor CD (i.e., Convexity Distribution) based on the distribution of the above-mentioned ratios, which employs a histogram instead of a single value to better describe the convexity-related properties of 3D shapes compared to other existing convexity measurements.

In this paper, rotation angles are chosen as random floating numbers between 0 to $2\pi$, the number of rotations we make for a mesh is selected as $N_{rot} = 10000$, and the shape descriptor is built by quantizing these $N_{rot}$ ratios $R(M, \alpha, \beta, \gamma)$ whose values range from 0 to 1 into a histogram with $N_{his} = 1024$ bins.

6. **Results**

To demonstrate the effectiveness of our convexity measurement, it is first applied to several specifically-designed 3D models which are obtained from a cube by cutting different numbers of thin cuboids. Figure 7 shows these models in order of their convexity values. As we can see, the more gaps an object has the smaller its convexity will be, which coincides with our intuitive notion. Moreover, we also compute convexity values for other kinds of 3D meshes. Several examples are displayed in Figure 8, from which we observe that the proposed convexity measure is sensitive to the change of area and it generally corresponds well with human perception for the convexity of 3D objects.

Next, we apply our convexity measure and shape descriptor CD in the retrieval of non-rigid 3D shapes. Experiments are carried out on the widely-used McGill Articulated 3D Shape Benchmark [25], which consists of 10 categories containing 255 watertight meshes, and retrieval performance is evaluated by the Precision-recall plot as well as four quantitative measures (NN, 1-Tier, 2-Tier, DCG) [24].

Table 1 shows results for methods that utilize our approaches (i.e., CD and $C(M)$) and other two convexity measures (i.e., $C_1(M)$ and $C_2(M)$) to represent a 3D object and employ the $L_1$ norm to calculate the dissimilarity between two signatures. Note that all these convexity-related shape descriptors are computed on the canonical forms of the 3D meshes. As we can see from Figure 9, models in the same class may appear in quite different poses but can still have very similar canonical forms. Thereby, after the calculation of canonical forms, all feature extraction methods, even those specifically designed for rigid 3D shapes, can be employed to extract isometry-invariant shape descriptors from non-rigid objects. Here, we use a method recently presented in [10] to construct feature-preserved canonical forms for 3D meshes. As we can see from Table 1, representing a 3D object by a convexity measure that contains only a single value results in poor discrimination. While, because our shape descriptor CD provides more information to de-

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Table 1. Retrieval performance of our methods (CD and $C(M)$) and other convexity measures evaluated on the McGill database.

<table>
<thead>
<tr>
<th></th>
<th>NN</th>
<th>1-Tier</th>
<th>2-Tier</th>
<th>DCG</th>
</tr>
</thead>
<tbody>
<tr>
<td>CD</td>
<td>57.3%</td>
<td>41.3%</td>
<td>67.1%</td>
<td>72.9%</td>
</tr>
<tr>
<td>$C(M)$</td>
<td>25.9%</td>
<td>26.3%</td>
<td>45.9%</td>
<td>60.4%</td>
</tr>
<tr>
<td>$C_1(M)$</td>
<td>27.8%</td>
<td>29.6%</td>
<td>51.9%</td>
<td>62.4%</td>
</tr>
<tr>
<td>$C_2(M)$</td>
<td>37.3%</td>
<td>26.0%</td>
<td>43.7%</td>
<td>59.8%</td>
</tr>
</tbody>
</table>

Figure 9. Non-rigid models (a) and their feature-preserved 3D canonical forms (b).
scribe 3D shapes, it obtains much better performance compared to other convexity measures. Finally, we follow the method presented in [12] to generate 8 composite shape descriptors via the linear combination of our convexity-related shape descriptors (i.e., CD and C(M)) with the following isometry-invariant signatures: the shape distribution of Geodesic distance (G2) [13], Heat Kernel Signatures (HK-S) [28], Laplace-Beltrami Spectrum (LBS) [20], and Bag-of-feature SIFT (BF-SIFT) [15]. Compared to the performance of original signatures, reasonable improvements in terms of retrieval accuracy have been achieved (see Figure 10), especially after combining with the new shape descriptor CD. This is mainly due to the fact that our convexity measure and our shape descriptor CD provide new and effective information, which is complementary with other existing signatures, to represent 3D shapes.

7. Conclusion

In this paper, a new convexity measure is developed for 3D meshes. The proposed convexity measure is defined based on the area of faces on the surface. Thereby, compared to the most commonly-utilized method (i.e., the ratio of the volume of a 3D object to the volume of its convex hull), it is more sensitive to deep indentations into objects, especially when the volume of these indentations are negligible. This is validated by our computed results which also demonstrate that the new convexity measure corresponds well with human intuition. The paper also proposes
a convexity-related 3D shape descriptor based on the distribution of ratios $R(M, \alpha, \beta, \gamma)$ (Equation 15), which are computed by randomly rotating the mesh around its center, to better describe the object’s convexity-related properties compared to existing methods. Finally, experiments were carried out on a widely-used benchmark to show the effectiveness of our convexity measure and the new shape descriptor in the application of non-rigid 3D shape retrieval.

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