Adaptive measurements in nonorthogonal state discrimination

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Abstract. Adaptive measurements represent important resources in quantum information science and quantum technologies. They take advantage of the knowledge of partial measurements of the system to optimize subsequent measurements and perform tasks that are difficult or not possible with nonadaptive measurements. In this note, we briefly describe some of the latest examples of schemes using adaptive measurements for an applications relevant to quantum information science and quantum communications, specifically, for nonorthogonal state discrimination.

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INTRODUCTION

In general, the measurement of a given property of a physical system, which is one of the most important and ubiquitous problems facing modern science, consists of the interaction with this system that can provide information about such property. These measurements can be used to read out encoded messages or information [1, 2], prepare different states of a system from which this subsystem is correlated [3], and measure a given property of second system when the first system is used as a probe or test [4, 5].

Adaptive measurement schemes perform and learn from partial measurements of the system and prepare optimized subsequent measurements [6].

Adaptive measurement schemes for the discrimination of nonorthogonal coherent states [7] have been known for several years [8, 9, 10, 11] and continue to be an active area of research. While demonstration and implementation of these strategies were not realized at that time due to the technological requirements and the complexity of the proposed receiver structures, recent advances in single-photon detection technologies [12] and high-bandwidth systems, have lead to extensive efforts to implement such schemes [13, 14, 15, 16]. These implementations have demonstrated the feasibility of constructing such quantum receivers that can overcome ideal conventional receivers working at the standard quantum limit (SQL). Such receivers could be used to optimize current and future quantum communications [17, 18, 19] and quantum information protocols [20, 3, 21]. Here we briefly describe some of the adaptive measurement schemes for nonorthogonal state discrimination and recent examples of studies and demonstrations of these schemes for minimum error discrimination and unambiguous state discrimination.
STATE DISCRIMINATION BASED ON ADAPTIVE MEASUREMENTS

Nonorthogonal state discrimination is a central problem in quantum and classical communications. Quantum mechanics prevents the perfect discrimination of nonorthogonal states [7]. The inability of distinguishing these states with total certainty is essential in quantum key distribution [17, 18, 19] allowing absolute secure communications. In the other hand, the nonorthogonality of coherent states, produce unavoidable errors in classical communications [22]. For these applications optimized discrimination strategies can improve both classical and quantum communication protocols. Adaptive measurements for nonorthogonal state discrimination [8, 9, 10, 11] can perform better than any ideal scheme for the conventional receivers that ideally perform at the SQL [2]. In these schemes the state to be discriminated is split in time or space and the receiver performs a partial measurement of the state, gains information from the result of this measurement about the state, and uses this information to perform an optimized measurement in subsequent adaptive measurements. Here we focus on two discrimination strategies for nonorthogonal coherent states; one for minimum error discrimination (MED) with error probabilities below the SQL and the other that allows for unambiguous state discrimination (USD) with maximized conclusive results.

Minimum Error Discrimination

The adaptive measurement schemes discussed here for MED share the property that all are optical implementations using coherent states, where the possible input states belong to a discrete set of coherent states with different phases or intensities. Coherent states are nonorthogonal to each other and its nonorthogonality becomes more important at low intensities. All coherent states have a vacuum component, and because of this shared component, these states cannot, even in principle, be discriminated with total certainty. While conventional receivers can in principle discriminate these states with error probabilities reaching the SQL, quantum mechanics allows for a lower limit which is described by the Helstrom bound [7]. Quantum receivers can take advantage of the quantum properties of light to discriminate among nonorthogonal states with error probabilities below this SQL and can approach the Helstrom bound.

The Dolinar receiver [8], shown in Fig. 1, is the canonical example of a quantum state discrimination receiver based on adaptive measurements. Its strategy is based on simple linear optics, photon counting, and adaptive measurements in the form of fast feedback. The Dolinar receiver outperforms any conventional receiver for binary state discrimination and, remarkably, can reach the ultimate Helstrom limit. There is not any other known optimal strategy capable of reaching the ultimate quantum limit without resorting to auxiliary entangled state projections.

In the Dolinar scheme, the state enters the receiver in a given spatial and temporal mode, and the receiver performs an optical displacement of the input state with an optimized phase and intensity. The existence of photons in the displacement state is detected with a single-photon detector (SPD). Depending on the detection result (in
FIGURE 1. Schematic of optimal receiver for minimum error discrimination of two nonorthogonal states. This receiver is based on adaptive measurements consisting of optimized displacements and single-photon counting with fast feedback. For a perfect single-photon detector with 100% detection efficiency and no dark counts, and an infinite bandwidth feedback, this detector reaches the Helstrom limit for the discrimination of two nonorthogonal states.

In this case only a binary detection result is required to distinguish between zero photon counts and one or more photons), the hypothesis of the input state is updated and a new optimal displacement is applied. At the end of the measurement sequence, depending on the parity of the number of measurements yielding a nonzero photon result, the receiver makes its decision of the identity of the input state. With perfect 100% detection efficiency and infinite feedback bandwidth, this receiver is an optimal quantum receiver reaching the Helstrom limit for the discrimination of any two nonorthogonal states [8].

The Dolinar receiver was demonstrated after three decades of its invention [23]. The implementation of this scheme performed the discrimination of two nonorthogonal states, one coherent state with a given intensity and the other corresponding to the vacuum state. Although not below the SQL for binary detection, this implementation observed error rates below the shot noise limit, which is the corresponding limit for this on-off modulation case [23].

Multi-state quantum receiver schemes generalize the binary hypothesis testing of binary receivers to allow a higher number of possible hypotheses for the input state. There are several discrimination strategies for multiple nonorthogonal states below the SQL with and without adaptive measurements, however finding an optimal strategy is still an open problem. The use of multiple nonorthogonal states in quantum communications could increase the mutual information and the secret key shared by two parties [24], and represents an attractive element in future classical communications where bandwidth and capacity are crucial considerations [22]. The discrimination strategies for multiple nonorthogonal states with the highest performance are based on adaptive measurements. These suboptimal discrimination strategies can discriminate nonorthogonal states with lower error probabilities than any ideal conventional standard-quantum limited receiver [13, 10, 14, 15, 16].
The few experimental studies of multiple nonorthogonal state discrimination have used adaptive measurements in a feedback or in a feed forward manner. The multiple-state receiver described in Ref. [15] uses a hybrid strategy for the discrimination of four nonorthogonal states in the quadrature phase-shift keying (4PSK) format. This strategy splits the energy of the incoming field in two paths; a homodyne detector measures one portion of this field and discriminates among two sets of two possible phases of the incoming state; and this information is fed forward to an optimized binary receiver where the two possible remaining states are discriminated. The experimental implementation of this strategy, using postprocessing to mimic the feed forward process and correcting for detection efficiency of the receiver, showed that it can in principle discriminate these states below the SQL, which in this case corresponds to the heterodyne limit. Another adaptive measurement strategy that is capable of discriminating among general M-ary PSK states was studied in [14]. That strategy uses adaptive displacements of the incoming field to the vacuum state and photon counting in multiple projection and measurement stages. It uses a maximum a posteriori probability criteria to find the most probable state of the input field in every measurement from the information gained in previous adaptive measurements, and tests this most probable state in subsequent adaptive measurements. The study in [14] applied postprocessing in a static experimental setup to mimic the dynamics of the adaptive measurement, and the experimental results demonstrated that discrimination of QPSK states below the SQL is possible with moderate detection efficiencies and a few adaptive measurements. As a result, this scheme is an attractive alternative for realistic implementations of quantum receivers for multiple nonorthogonal state discrimination below the SQL. Very recently, a receiver for 4 state pulse-position (4PPM) modulation fields was demonstrated in [16] using feedback, photon counting, and an auxiliary local oscillator field. The discrimination strategy used in that experiment requires an auxiliary field providing a phase reference, together with requiring PPM signals to be coherent with this field, that make it possible to discriminate this 4 alphabet states better than what is possible with direct detection. However, in this experiment the receiver observed error probabilities below that of direct detection, but only after correcting for the detection efficiency of their device. Therefore, unconditional discrimination of multiple nonorthogonal states was not demonstrated.

Unambiguous State Discrimination

An alternative strategy to MED consists of performing measurements with a higher number of outcomes than the number of possible input states of the system under measurement to allow for sometimes obtaining inconclusive results. These schemes make it possible to sometimes obtain measurement outcomes that unambiguously identify the state of the input field, that is with zero probability of error [25, 26, 27]. USD of coherent states is of fundamental interest and it can have application in quantum communication and quantum computation [20, 3, 21, 28]. Optimal USD consists of maximizing the

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1 In a general PPM scheme the signal is not required to be coherent since conventional PPM is not a coherent communication scheme.
probability of conclusive results, since ideally there are no errors, and the maximum probability of conclusive results for USD of linearly-independent equally-likely symmetric states is known [29]. As to how to implement measurements that achieve this maximum possible probability, it is only for binary states that such strategies are known [30]. While for discriminating among more than two states there is no such known optimal strategy, a few suboptimal schemes have been proposed [31, 32], with one being optimal only in the limit of small photon numbers. That strategy is based on optical displacements, photon counting and dynamic updating of the hypothesis of the state of the input field.

Similarly, on the experimental side, work on USD for coherent states is extremely limited [33], and only binary discrimination has been demonstrated [34]. The implementation of USD discrimination strategies for multiple state and the effects of degradation of USD performance under realistic conditions is still to be demonstrated.

**CONCLUSION**

Adaptive measurements are a well known tool for improving the measurement and control of different physical systems. Nonorthogonal multi state discrimination strategies based on adaptive measurements together with the current technological advances will soon allow us to demonstrate quantum receivers discriminating multiple nonorthogonal coherent states for MED and USD. These implementations will allow us to evaluate their performance in realistic scenarios in lossy and noisy environments, and these quantum receivers can in principle be incorporated to advantage in existing classical and future quantum networks.

**REFERENCES**

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