Modeling of an air-backed diaphragm in dynamic pressure sensors: Effects of the air cavity

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ABSTRACT

As the key structure of most dynamic pressure sensors, a diaphragm backed by an air cavity plays a critical role in the determination of sensor performance metrics. In this paper, we investigate the influence of air cavity length on the sensitivity and bandwidth. A continuum mechanics model neglecting the air viscous effect is first developed to capture the structural–acoustic coupling between a clamped circular diaphragm and a cylindrical backing air cavity. To facilitate sensor design, close-form approximations are obtained to calculate the static sensitivity and the fundamental natural frequency of the air-backed diaphragm. Parametric studies based on this analytical model show that the air cavity can change both the effective mass and the effective stiffness of the diaphragm. One new finding is that the natural frequency of the air-backed diaphragm behaves differently in three different cavity length ranges. In particular, due to the mass effect of the air cavity being dominant, it is shown for the first time that the natural frequency decreases when the cavity length decreases below a critical value in the short cavity range. Furthermore, a finite element method (FEM) model is developed to validate the continuum mechanics model and to study the damping effect of the air cavity. These results provide important design guidelines for dynamic pressure sensors with air-backed diaphragms.

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1. Introduction

Dynamic pressure sensors have been widely used in a variety of consumer, commercial, and military applications including telecommunication [1], speech recognition [2], hearing aids [3], and sound source localization [4]. In terms of transduction methods, these sensors can be based on piezoelectric [5,6], piezoresistive [7], optical [8–10], and capacitive principles [11–13]. For almost all of these sensors, the first stage of transduction involves the deflection of a flexible diaphragm in response to a net differential pressure across its thickness [14,15]. On the backside of the diaphragm, there exists a cavity that is most often filled with air. In general, the air cavity has the following effects that are important to the performance of a pressure sensor: (i) it provides damping to the diaphragm motion due to the viscosity of air (i.e., resistance to the air flow in the cavity), (ii) it increases the effective stiffness of the diaphragm due to the air spring effect, and (iii) it increases the effective mass of the diaphragm due to air particles moving together with the diaphragm.

Because the air cavity plays a critical role in determination of sensor performance, it is imperative to study the mechanics of an air-backed diaphragm, which is the key structure for most dynamic pressure sensors. This is particularly important as...
Nomenclature

\( a \) radius of the diaphragm
\( A_m \) coefficient to normalize \( U_d(r) \)
\( B_d \) vector used to superpose the modal coordinates of the diaphragm displacement
\( B_n \) coefficients used to normalize \( U_d(r) \)
\( c_0 \) speed of sound in the air
\( c_d \) speed of longitudinal wave in the diaphragm
\( D \) flexural rigidity of the diaphragm
\( E_d \) Young’s modulus of the diaphragm
\( f_{c1} \) fundamental natural frequency of the air backed diaphragm
\( h_d \) thickness of the diaphragm
\( H_{a,mn} \) transfer function relating the displacement \( \bar{w}_a \) to the reaction pressure \( \bar{p}_R \)
\( H_{c,mn} \) transfer function of the coupled system as defined in Eq. (23)
\( H_c \) transfer function matrix of the coupled system as defined in Eq. (32)
\( H_{d,m} \) transfer function of the diaphragm for the \( m \)th mode
\( I_{0, I'_0} \) modified Bessel function of the first kind and its derivative
\( I_M \) identity matrix of order \( M \)
\( J_0, J'_0 \) Bessel function of the first kind and its derivative
\( K_a \) stiffness matrix of the air cavity as defined in Eq. (24)
\( K_c \) stiffness matrix of the coupled system as defined in Eq. (33)
\( K_d \) stiffness matrix of the diaphragm as defined in Eq. (28)
\( l \) length of the air cavity
\( \langle l(a)_{cr,ss} \rangle \) critical cavity length when the air cavity and the diaphragm have equal stiffness
\( \langle l(a)_{cr,long} \rangle \) Critical cavity length separating the long and medium cavity length regions
\( \langle l(a)_{cr,short} \rangle \) critical cavity length separating the medium and short cavity length regions
\( m, n \) order of mode shape
\( M_{a,n} \) equivalent mass of the air cavity as defined in Eq. (27)
\( \tilde{M}_{a,n} \) equivalent mass of the air cavity as defined in Eq. (20)
\( M_a \) mass matrix of the air cavity as defined in Eq. (25)
\( M_c \) mass matrix of the coupled system as defined in Eq. (34)
\( M_d \) mass matrix of the diaphragm as defined in Eq. (29)
\( N_0 \) in-plane force of the diaphragm
\( N_d \) vector in expanding the applied pressure in terms of the modal coordinates of the diaphragm
\( p_0 \) static pressure of the air
\( p_d \) net pressure applied to the diaphragm
\( \bar{p}_d \) net pressure normalized by Young’s modulus of the diaphragm
\( p_e \) pressure applied to the external surface of the diaphragm
\( p_R \) reaction pressure at the diaphragm–air interface
\( \bar{p}_R \) reaction pressure normalized by the static pressure \( p_0 \)
\( P_{ed,m} \) coefficients in expanding \( p_e \) in terms of the diaphragm’s modes
\( \bar{P}_{ed} \) vector of the normalized pressure applied to the top surface of the diaphragm
\( P_{la,n} \) modal coefficients in expanding \( p_k \) in terms of the air cavity’s modes
\( P_{ld,m} \) coefficients in expanding \( p_k \) in terms of the air cavity’s modes
\( Q \) non-dimensionalized variables as defined in Eq. (49)
\( Q_1 \rightarrow Q_4 \) non-dimensionalized variables as defined in Eq. (47)
\( r \) normalized radial coordinate, \( 0 \leq r \leq 1 \)
\( s_{dyn} \) dynamic sensitivity of pressure sensors
\( t \) time
\( T_{mn} \) transformation coefficients between the modes of the diaphragm and the air cavity matrix whose elements are \( T_{mn} \)
\( U_d(r) \) radial part of the mode shape of the air cavity
\( U_d(r) \) radial part of the diaphragm’s mode shape
\( w_a \) displacement \( w_a \) normalized by the radius \( a \)
\( W_a \) transverse displacement of the diaphragm
\( \tilde{W}_d \) transverse displacement of the diaphragm normalized by the diaphragm radius \( a \)
\( W_{a,n} \) modal coefficients in expanding \( w_a \) in terms of the air cavity’s modes
\( W_{d,m} \) coefficients in expanding \( w_d \) in terms of the diaphragm’s modes
\( \tilde{W}_d \) vector of the normalized displacement of the diaphragm
\( Z(z) \) axial part of the mode shape of the air cavity
\( \alpha_1, \alpha_2 \) variables in the characteristic equation of the diaphragm
\( \beta \) variable in the characteristic equation for the air cavity
\( \chi \) normalized tension parameter of the diaphragm
\( \delta_{mn} \) Kronecker delta, \( \delta_{mn} = 0 \) for \( m \neq n \); \( \delta_{mn} = 1 \) if \( m = n \)
\( \gamma \) adiabatic index of the air
\( \lambda \) sound wavelength in the air
\( \nu \) Poisson’s ratio of the diaphragm
\( \theta \) Azimuthal coordinate
\( \rho_0 \) static density of the air
\( \rho_d \) density of diaphragm
\( \sigma \) a non-dimensionalized variable as defined in Eq. (45)
\( \omega \) radial frequency
\( \omega_d \) natural frequency of the diaphragm
\( \xi \) damping ratio
\( \zeta \) normalized parameter as defined by Eq. (11)
\( \theta \) a non-dimensionalized variable as defined in Eq. (39)
\( \Lambda \) natural frequency parameter of the diaphragm
sensors are miniaturized or new diaphragm materials are used, as design guidelines used in conventional sensors may not be applicable. In this article, our goal is to determine how the air cavity affects the sensitivity and bandwidth of a dynamic pressure sensor. To this end, we conduct a theoretical and numerical investigation into the structural–acoustic interaction in the air-backed diaphragm.

A well-known approach for studying the interaction between a vibrating diaphragm and its backing fluid medium is to employ the dynamical analogy by converting the involved mechanics to an equivalent electric circuit [16,17]. In this approach, only the fundamental mode of the diaphragm is considered and the air cavity is simply modeled as an equivalent elastic spring. However, this simplified approach has several limitations. First, since the full structural–acoustic interaction is not taken into account, the pressure field in the air cavity cannot be predicted. More importantly, when this approach is used to guide the design of pressure sensors, especially those sensors with a short air cavity, such as micro-electro-mechanical systems (MEMS) based pressure sensors [15] or miniature fiber optic pressure sensors [10,18,19], it is often difficult to accurately predict the performance of these sensors.

Depending on the governing equations used to describe the air cavity, there are generally two approaches to construct an analytical mechanics model to fully capture the structural–acoustic coupling. One approach is based on the sound wave equation where the viscous terms are usually neglected [20–30], and the other is utilizing the Reynolds equation for thin viscous fluid films, often referred as squeeze film damping [31–35].

In the first approach, the diaphragm is usually modeled as a thin-plate or a membrane, while the air cavity is described by a wave equation in terms of the pressure or velocity potential. A geometric compatibility condition is assumed between the diaphragm and the air cavity; that is, continuous displacement and velocity at the interface. By using this approach, in a previous effort, Guy studied the response of an air cavity backed plate under external airborne excitations [22]. In subsequent studies, Rajalingham et al. employed a receptor-rejector system model to study the vibration of rectangular and circular membranes backed with an air cavity. It was found that the natural frequency of the coupled system was distinct to those of the isolated membrane, the open-end cavity, and the closed-end cavity [27,28]. More recently, Gorman et al. studied the coupling effect of a circular plate backed with a cylindrical air cavity by using an analytical–numerical method [21]. The natural frequencies and mode shapes obtained from the numerical simulations were verified through experimental studies and finite element analysis of a thin steel disc (radius of 38 mm and thickness of 0.38 mm) backed with an air cavity of different lengths (a short cavity of 81 mm and a long cavity of 255 mm). It was found that a strong acoustical–structural coupling occurs when the acoustic and structural subsystems have close natural frequencies and affined mode shapes. Gorman et al. also investigated the use of kinetic energy of the diaphragm relative to the air cavity to distinguish the predominantly structural modes from the predominantly acoustic modes [36]. It should be noted these models mostly focus on the free vibration problem and the viscous effect is neglected. Therefore, the obtained fundamental natural frequency is an undamped natural frequency.

In the second approach, the squeeze film damping is generally modeled by the linearized Reynolds equation for small displacements [31]. For some small pressure sensors, such as MEMS based pressure sensors [11,15,37], when the air cavity is short, the large damping due to the squeeze film effect can significantly reduce the sensor bandwidth [12], which is typically designed to be the flat region of the response spectrum below the fundamental natural frequency of the air-backed diaphragm. At a small squeeze number, the viscous damping force dominates due to the air flows into and out of the plate region; while for a large squeeze number, the air is trapped between plates, and therefore the elastic damping/spring force dominates [31]. In addition, an isothermal assumption is usually used in the Reynolds equation to simplify the problem. By contrast, the sound wave equation generally assumes an adiabatic process. By assuming an incompressible flow between the diaphragm and the back electrode, Škvor derived analytical expressions for the normalized resistance and inerntance of the air gap [35]. Veijola et al. used frequency–independent equivalent spring and damping parameters in an equivalent circuit model to calculate the response of a silicon accelerometer in both viscous and molecular damping regions [38]. To model flexible plates under the effect of squeeze film damping, Nayfeh and Younis used a perturbation method on the Reynolds equation to express the pressure distribution in the film in terms of the structural modes of the plate, and solved the resulting equation by using a finite element method (FEM) [39]. In another study, Le Van Suu et al. analyzed a rectangular clamped diaphragm under the influence of a thin air gap between

<table>
<thead>
<tr>
<th>$\Theta(\theta)$</th>
<th>Azimuthal part of the mode shape of the diaphragm and the air cavity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Psi(r, \theta, z, t)$</td>
<td>Velocity potential within the air cavity</td>
</tr>
<tr>
<td>$\Omega_a$</td>
<td>Normalized frequency for the air cavity</td>
</tr>
<tr>
<td>$\Omega_{c1}$</td>
<td>Normalized fundamental natural frequency of the air backed diaphragm</td>
</tr>
<tr>
<td>$\Omega_{c1, long}$</td>
<td>Normalized natural frequency in the long cavity region</td>
</tr>
<tr>
<td>$\Omega_{c1, medium}$</td>
<td>Normalized natural frequency in the medium cavity region</td>
</tr>
<tr>
<td>$\Omega_{c1, short}$</td>
<td>Normalized natural frequency in the short cavity region</td>
</tr>
<tr>
<td>$\Omega_d$</td>
<td>Normalized radial frequency of the diaphragm</td>
</tr>
<tr>
<td>$\nu_{an}$</td>
<td>Modal coefficients in expanding $\Psi$ in terms of the air cavity’s modes</td>
</tr>
<tr>
<td>$\Omega_{ns}$</td>
<td>Normalized natural frequency of an close-ended cavity</td>
</tr>
<tr>
<td>$\Omega_{ns}$</td>
<td>Normalized natural frequency of an open-ended cavity</td>
</tr>
</tbody>
</table>
the diaphragm and the backplate [32] and the mode shapes obtained from their analytical model were found to agree well with the experimental results.

Although in the aforementioned efforts, the structural–acoustic coupling of the air-backed diaphragm has been considered, few have studied this problem from the perspective of pressure sensor design, in which practical guidelines are needed to predict the performance characteristics of a pressure sensor. In particular, it is not clear in the literature how the aforementioned three effects (stiffness, mass, and damping) of the air cavity scale differently with the cavity length. In a related effort, Qasi studied the free vibration of a rectangular plate-cavity system by obtaining the mass and stiffness matrices of each subsystem numerically [26]. A simplified equation was provided to calculate the fundamental natural frequency that was shown to increase with decreasing cavity length. However, since the fundamental frequency obtained by using this simplified equation only takes into account the fundamental mode of the plate and the air spring mode of the cavity, it is not valid for a short cavity case, in which the higher order modes of the plate and the air cavity have to be included to provide accurate calculation. In addition, the viscous effect of air was not considered, which can also affect the fundamental natural frequency.

Regardless of the models used to describe the structural–acoustic coupling, a dynamic pressure sensor can be simply described by using an equivalent harmonic oscillator; that is, a one degree-of-freedom (DOF) mass-spring-damper system with equivalent mass, stiffness, and damping. Based on the frequency response to external pressure stimulus, we can characterize the sensor performance in terms of its static sensitivity and bandwidth. The latter is determined by the natural frequency and damping ratio of the equivalent harmonic oscillator. Built upon the previous studies on the diaphragm and air cavity coupling problem, we seek answers to the following fundamental questions from the perspective of dynamic pressure sensor design: (i) How does the air cavity affect the characteristics of the vibrating diaphragm? (ii) Under what conditions can the air cavity effect be neglected (i.e., the diaphragm–cavity system can be treated as a diaphragm in vacuo)? (iii) Can a simple, empirical formula be obtained to predict the sensitivity and bandwidth of a pressure sensor with an air-backed diaphragm? (iv) Is there any difference of the structural–acoustic coupling behavior in different cavity length regions?

To answer these questions, we focus on how the sensitivity and bandwidth are affected by the air cavity length of a large range from orders of magnitudes larger than the diaphragm size to orders of magnitudes smaller. Due to the large cavity length range, some of the assumptions made in the Reynolds equation to describe a squeeze film do not hold. For example, when the film thickness (i.e., the cavity length) is larger than or comparable to the diaphragm size, the pressure cannot be assumed to be uniform in the direction perpendicular to the film, and the air flow cannot be considered planar. Rather than conducting a full-scale Navier–Stokes analysis, a continuum mechanics model of a clamped circular diaphragm backed with a cylindrical air cavity will be presented in Section 2. The diaphragm is modeled as a thin plate with in-plane force, and the air cavity is described by a sound wave equation that neglects the viscous effect of the air. The goal is to gain analytical insights into the fundamental scaling laws of the static sensitivity and the natural frequency with respect to the cavity length. Built upon from our previous work [40], the model in this paper uses a normalized formulation in order to obtain analytical solutions with broad applicability. Two models in matrix forms will be presented: a full model with frequency-dependent coefficients and a simplified model with frequency-independent coefficients under the assumption that the cavity length is much shorter than the sound wavelength. In Sections 3 and 4, the effects of the air cavity on the static sensitivity and fundamental frequency will be investigated, respectively. Close-form expressions for the static sensitivity, fundamental natural frequency, and critical cavity lengths will be provided, which were not attempted in our previous work [40]. To study the damping effect, An FEM model will be presented in Section 5, which is developed by using the thermoacoustic–shell interaction module in COMSOL. This model will be used to show how the damping is related to the cavity length, and to further validate the results obtained from the analytical model. Finally, the concluding remarks will be provided in Section 6.

2. Continuum mechanics model

Without loss of generality, consider a typical pressure sensor configuration shown in Fig. 1, which consists of a clamped circular diaphragm of radius \(a\) and thickness \(h_d\) and an air backed cylindrical cavity of length \(l\). A cylindrical coordinate system \((r, \theta, z)\) is established at the center of the diaphragm. The coordinates are normalized so that \(0 \leq r \leq 1, 0 \leq \theta \leq 2\pi,\) and \(0 \leq z \leq 1\). The clamped circular diaphragm is modeled as a plate with in-plane tension. Depending on a normalized tension parameter, this diaphragm model can capture the behaviors of a pure plate with zero tension, a pure membrane with a large initial tension, and the in-between cases [41]. This model is particularly useful for MEMS pressure sensors where residual stresses in the diaphragm are often inevitable due to the fabrication process. The air cavity is described by using the wave equation in terms of velocity potential. Modal analysis will be employed to derive the transfer function between the external pressure stimulus and the diaphragm response. For most pressure sensors, the diaphragm radius is much smaller than the wavelength. As a result, only the axisymmetric modes will be considered.

\[^{1}\] Certain commercial equipment, instruments, materials, or software are identified in this paper to foster understanding. Such identification does not imply endorsement by NIST, nor does it imply that the items or software identified are necessarily the best available for the purpose.
2.1. Diaphragm model: clamped circular plate with in-plane tension

For a clamped circular plate with in-plane tension, based on the thin plate theory, the normalized transverse displacement of the plate $\tilde{w}_d(r, \theta, t)$, which is related to the displacement $w_d(r, \theta, t)$ by $\tilde{w}_d = w_d/a$, is described by

$$\rho_d h_d a^2 \frac{\partial^2 \tilde{w}_d}{\partial t^2} + D \frac{\partial^4 \tilde{w}_d}{\partial r^4} - N_0 \frac{\partial^2 \tilde{w}_d}{\partial r^2} = E_d \rho_p \tilde{p}_d(r, \theta, t),$$

where $\rho_d$, $D$, $N_0$, $\rho_p$ and $a$ are the density, flexural rigidity, in-plane force, and external pressure, respectively, and subscript $d$ denotes the diaphragm. $D$ is related to the plate Young's modulus $E_d$ and Poisson’s ratio $\nu$ by $D = E_d h_d^3/[12(1-\nu^2)]$. The normalized net pressure term $\tilde{p}_d(r, \theta, t)$ is related to the pressure $p_d(r, \theta, t)$ by $\tilde{p}_d = p_d/E_d$. The intrinsic damping of the diaphragm is neglected as it is usually much smaller than the air damping.

To obtain the transfer function of the diaphragm, we first need to solve the free vibration problem for the mode shapes and natural frequencies. The axisymmetric mode shapes $U_{d,m}(r) \cdot \Theta_d(\theta)$ can be obtained as

$$\Theta_d(\theta) = 1/\sqrt{2\pi}$$

$$U_{d,m}(r) = A_m [I_0(\alpha_{1,m})J_0(\alpha_{2,m}) - J_0(\alpha_{2,m})I_0(\alpha_{1,m})], \quad m = 1, 2, \ldots$$

where $A_m$ are coefficients used to normalize the mode shapes to ensure their orthogonality; that is, $\int_{\theta=0}^{2\pi} U_{d,m}(r) U_{d,n}(r) r d\theta = \delta_{mn}$, where $\delta_{mn}$ is the kronecker delta. $\alpha_{1,m}$ and $\alpha_{2,m}$ can be obtained from the following characteristic equations:

$$\alpha^2 = \alpha^2 + \chi$$

$$a_2I_m(\alpha_1)J_m(\alpha_2) - a_1J_m(\alpha_2)I_m(\alpha_1) = 0,$$

where $\chi = N_0 a^2/D$ is the normalized tension parameter. $I_m$ and $J_m$ are the Bessel function of the first kind and the modified Bessel function of the first kind, respectively, and $I_m'$ and $J_m'$ are the first derivatives of $I_m$ and $J_m$, respectively. Our previous study showed that the diaphragm transitions from a plate behavior ($\chi < 4$) to a membrane behavior ($\chi > 200$) [41]. The natural frequencies of the plate can be calculated from $\alpha_{1,m}$ and $\alpha_{2,m}$ as $\omega_{d,m} = \omega_m c_d/c_a h_d$, where $\omega_m = \alpha_{1,m} c_d/\sqrt{12}$, and $c_a = \sqrt{E_d/\rho_p (1-\nu^2)}$ is the speed of longitudinal wave in the plate. The solutions to the characteristic equations for various values of tension parameter $\chi$ are listed in Table 1. It can be observed that an in-plane tensile stress (positive $\chi$) leads to an increase in the natural frequency while a compressive stress (negative $\chi$) results in smaller natural frequencies.

Assume the net pressure to the diaphragm is described by $\tilde{p}_d = \sum_{m=1}^{\infty} \tilde{p}_d U_{d,m}(r) \Theta_d(\theta) e^{j\omega t}$, and the resulting diaphragm response is $\tilde{w}_d = \sum_{m=1}^{\infty} \tilde{W}_{d,m} U_{d,m}(r) \Theta_d(\theta) e^{j\omega t}$. Substituting into the governing equation (Eq. (1)), we can utilize the orthogonal property of the mode shapes and obtain the transfer function of the diaphragm as

$$H_{d,m} = \tilde{W}_{d,m}/\tilde{P}_d = (1-\nu^2) \left( \frac{a}{h_d} \right)^3 \frac{1}{\Lambda_m - \Omega_d^2},$$

where $\Omega_d = \omega a^2/(c_d h_d)$ is the normalized frequency.

2.2. Air cavity model: cylindrical air chamber with a flexible top

The air cavity is modeled as a cylindrical air chamber with a flexible top, which can be described by using the wave equation in terms of velocity potential $\Psi(r, \theta, z, t)$:

$$\frac{1}{\rho_c} \left( \frac{\partial^2 \Psi}{\partial t^2} + \frac{\partial \Psi}{\partial r} + \frac{1}{r} \frac{\partial \Psi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Psi}{\partial \theta^2} \right) + \frac{1}{c_0^2} \frac{\partial^2 \Psi}{\partial z^2} = 0.$$
where has a zero root labeled as \( \Psi \). This assumption is valid for acoustics, where the frequency range of pressure fields is from 20 Hz to 20 kHz for the audible range, or above 20 kHz for ultrasound. A no-slip boundary condition is assumed at the interface of the plate and the air; that is \([1/\partial(\partial \Psi/\partial z) - \partial \Psi/\partial t]_{z=0} = 0\), where \( w \) is the displacement within the air cavity. The gradient of \( \Psi \) normal to the other walls is equal to zero (i.e., zero displacement on rigid walls).

Through separation of variables, the solution to the wave equation subjected to a harmonic excitation at the top surface is assumed to be \( \Psi(r, \theta, z, t) = U_a(r)\phi(\theta)Z(z)e^{\omega t} \), where \( Z(z) \) is the part of solution that depends solely on \( z \). Substituting \( \Psi(r, \theta, z, t) \) into Eq. (7), the radial part is obtained as the following:

\[
U_{a,n}(r) = B_0 J_0(\beta_n r), \quad n = 0, 1, 2, \ldots,
\]

where \( \beta_n \) can be solved from the following characteristic equation:

\[
j'_0(\beta_n) = 0,
\]

and \( B_n \) are coefficients used to normalize the mode shapes to ensure the orthogonal property, i.e., \( \int_0^1 U_{a,n}(r)U_{a,m}(r)\rho_{\text{air}}(r) dr = \delta_{mn} \). The first four solutions to the characteristic equation are listed in Table 2. Note that Eq. (9) has a zero root labeled as \( \beta_0 = 0 \), which corresponds to the fundamental piston mode of the air cavity.

The \( z \)-dependence part is obtained as

\[
Z_n(z) = \cosh[\varsigma_n(z - 1)],
\]

where

\[
\varsigma_n = (l/a)\sqrt{\beta_n^2 - \Omega_n^2}
\]

and \( \Omega_n = \omega \alpha / c_0 \) is the normalized frequency. The coefficient in Eq. (10) is chosen to ensure \( Z_n(z=1) = 1 \).

In the case of a close-ended air cavity (i.e., rigid top surface), the normalized natural frequencies can be obtained as

\[
\Omega_n^{\text{ce}} = \frac{\omega \alpha}{c_0} = \sqrt{\beta_n^2 + \pi^2 s^2 \left( \frac{a}{l} \right)^2}, \quad s = 0, 1, 2, \ldots
\]

On the other hand, the normalized natural frequencies for an open-ended air cavity (open top surface) are obtained as

\[
\Omega_n^{\text{ce}} = \frac{\omega \alpha}{c_0} = \sqrt{\beta_n^2 + \pi^2 (2s + 1)^2 \frac{a}{l}^2}, \quad s = 0, 1, 2, \ldots
\]
To obtain the relationship between the displacement excitation at the top $w_o(r, \theta, t)$ and the reaction pressure at the interface $p_R(r, \theta, t)$, it is assumed that $w_o(r, \theta, t) = \sum_m W_{a,n} U_{a,n}(r) \theta_0(\theta)e^{i\omega t}$ and $p_R(r, \theta, t) = \sum_m p_{R_m} U_{a,n}(r) \theta_0(\theta)e^{i\omega t}$, and the velocity potential solution to the wave equation is $\Psi(r, \theta, t) = \sum_m \Psi_{a,n} U_{a,n}(r) \theta_0(\theta)e^{i\omega t}$. Here, the subscript $a$ denotes that the expansion is in terms of the mode shapes of the air cavity. From the no-slip boundary condition, it can be obtained that

$$
\left. \left( j_0 W_{a,n} - \Psi_{a,n} \frac{1}{T} \frac{\partial \zeta_n}{\partial z} \right) \right|_{z=0} = 0.
$$

(14)

The pressure is related to the velocity potential by

$$
p_R(r, \theta, t) = (-p_0 \partial\Psi/\partial t)|_{z=0}.
$$

(15)

Combining Eqs. (14) and (15), the relationship between the modal coordinates $(\tilde{W}_m = w_m/a)$ and those of the reaction pressure $(\tilde{p}_R = p_R/p_0)$ can be established as $\tilde{P}_{R_m} = H_{a,n} \tilde{W}_{a,n}$. Here, $H_{a,n}$ is the transfer function, which can be written as

$$
H_{a,n}(\omega) = -\frac{j a}{a^2coth((l/a)\Omega_i)} \zeta_n
$$

(16)

To study the effects of the air cavity on the characteristics of the diaphragm, it is important to understand the roles of different air cavity modes. For the first mode of the air cavity (i.e., $n=0$ and $\beta_0=0$), the transfer function (Eq. (16)) reduces to

$$
H_{a,0} = -\frac{j a}{a^2coth(l/a)\omega} = \gamma \frac{\Omega_0}{\tan(l/a)\Omega_0}
$$

(17)

This air cavity mode is equivalent to a mechanical spring with a frequency-dependent spring constant $H_{a,0}$, which increases with decreasing cavity length $l$. In the case when $\Omega_0 l/a = \omega l/c_0 \ll 1$ (i.e., $l \ll 1$), Eq. (17) can be approximated by its upper bound

$$
H_{a,0} \approx \frac{\gamma}{\Omega_0}
$$

(18)

For the higher order modes of the air cavity ($n \geq 1$), let us assume $\Omega_n \ll \Omega_0$ or $a \gg l$. Therefore, it can be obtained that $\zeta_n \approx a \mu l/a$, and the transfer function of the air cavity reduces to

$$
H_{a,n} = -\tilde{M}_{a,n} \Omega_n^2
$$

(19)

where

$$
\tilde{M}_{a,n} = \gamma coth(\mu l/a)/\mu
$$

(20)

Because the sign of $H_{a,n}$ is negative and the reaction pressure is applied to the bottom surface of the diaphragm, the effect of these higher order air cavity modes is equivalent to increasing the mass of the diaphragm. From Eq. (20) it can be seen that this mass effect becomes more dominate for smaller $l/a$.

2.3. Coupled system model: air backed diaphragm

Here, the transfer functions obtained for the diaphragm and the air cavity will be used to obtain the transfer function of the coupled system (i.e., the air-backed diaphragm). Assume in response to the pressure applied to the top surface of the plate $p_d(r, \theta, t) = \sum_m P_{d,m} U_{d,m}(r) \theta_0(\theta)e^{i\omega t}$, the resulting reaction pressure at the interface is $p_R(r, \theta, t) = \sum_m p_{R_m} U_{a,n}(r) \theta_0(\theta)e^{i\omega t}$, and the transverse displacement of the diaphragm is $w_d(r, \theta, t) = \sum_m W_{d,m} U_{d,m}(r) \theta_0(\theta)e^{i\omega t}$. Here, the subscript $d$ in $P_{d,m}$, $p_{R_m}$, and $W_{d,m}$ denotes that these expansions are in terms of the diaphragm mode shapes. After the transfer function of the air cavity (Eq. (16)) is rewritten in terms of the mode shapes of the diaphragm, it can be obtained that

$$
\frac{p_R}{E_d} \tilde{P}_{R_m} = \sum_k \sum_l T_{km} H_{a,k} T_{kl} \tilde{W}_{d,n},
$$

(21)

where

$$
T_{km} = \int_0^1 U_{a,k}(r) U_{d,m}(r) r \, dr, \quad T_{kl} = \int_0^1 U_{a,k}(r) U_{d,l}(r) r \, dr.
$$

(22)

Combining Eq. (21) with the transfer function of the diaphragm (Eq. (6)), the following relationship can be obtained for the coupled system $\tilde{P}_{ed,m} = \Sigma_n H_{c,mn} \tilde{W}_{d,n}$, where the transfer function of the coupled system can be obtained as

$$
H_{c,mn} = \left[ \frac{p_0}{E_d} \sum_k (T_{km} H_{a,k} T_{kn}) + H_{d,m}^{-1} \delta_{mn} \right].
$$

(23)

Here, the subscript $c$ denotes the coupled system. The natural frequencies and mode shapes can be obtained by finding the roots when the determinant of the matrix $H_c$ is zero. In the absence of the air cavity (i.e., the chamber underneath the diaphragm is in vacuo), $H_{a,k}=0$, and thus, Eq. (23) reduces to the form for the diaphragm in vacuo described in Eq. (6).
2.4. Coupled system in matrix form

If only the first $M$ modes of the diaphragm and the first $N$ modes of the air cavity are considered, the normalized pressure vector $\tilde{P}_{ed}$ and the normalized displacement vector $\tilde{W}_d$ (both are $M \times 1$ vectors) can be written as $\tilde{P}_{ed} = [\tilde{P}_{ed,1} \quad \tilde{P}_{ed,2} \quad \ldots \quad \tilde{P}_{ed,M}]^T$ and $\tilde{W}_d = [\tilde{W}_{d,1} \quad \tilde{W}_{d,2} \quad \ldots \quad \tilde{W}_{d,M}]^T$, respectively.

Define the stiffness and mass matrices ($M \times M$) of the air cavity as

$$
K_a = \frac{p_0}{E_d} \begin{bmatrix}
K_{a,0} & 0 & \cdots & 0 \\
0 & K_{a,1} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & K_{a,N}
\end{bmatrix}_{N \times N},
$$

(24)

$$
M_a = \frac{p_0}{E_d} \begin{bmatrix}
0 & M_{a,1} & \cdots & M_{a,N} \\
M_{a,1} & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
M_{a,N} & 0 & \cdots & 0
\end{bmatrix}_{N \times N},
$$

(25)

where

$$
K_{a,0} = \frac{\rho_d}{\tan (l/2a)} \
M_{a,n} = \frac{\rho_n}{\coth (\beta_n l/a)} \left( \frac{c_n}{c_0} \right)^2 \left( \frac{l}{a} \right)^2.
$$

(26)

(27)

Define the stiffness and mass matrices ($M \times M$) of the diaphragm as

$$
K_d = \frac{1}{1 - \nu^2} \left( \frac{h_d}{a} \right)^3 \begin{bmatrix}
\Lambda_1^2 & 0 & \cdots & 0 \\
0 & \Lambda_2^2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \Lambda_M^2
\end{bmatrix},
$$

(28)

$$
M_d = \frac{1}{1 - \nu^2} \left( \frac{h_d}{a} \right)^3 I_M.
$$

(29)

The transformation matrix ($N \times M$) takes the following form:

$$
T = \begin{bmatrix}
T_{01} & T_{02} & \cdots & T_{0M} \\
T_{11} & T_{12} & \cdots & T_{1M} \\
\vdots & \vdots & \ddots & \vdots \\
T_{(N-1)1} & T_{(N-1)2} & \cdots & T_{(N-1)M}
\end{bmatrix}.
$$

(30)

Neglecting the damping terms, the relationship between $\tilde{P}_{ed}$ and $\tilde{W}_d$ can be written as the following compact matrix form:

$$
\tilde{P}_{ed} = \mathbf{H}_c \tilde{W}_d.
$$

(31)

Here, $\mathbf{H}_c$ is the transfer function in matrix form, which can be written as

$$
\mathbf{H}_c = \mathbf{K}_c - \mathbf{M}_c \Omega_d^2,
$$

(32)

where

$$
\mathbf{K}_c = T^T \mathbf{K}_d T + \mathbf{K}_d,
$$

(33)

and

$$
\mathbf{M}_c = T^T \mathbf{M}_d T + \mathbf{M}_d.
$$

(34)

Note that $\mathbf{K}_c$ (Eq. (24)) is frequency dependent; hence $\mathbf{H}_c$ is also frequency dependent. This model is referred to as the full model in the following analysis. If the cavity length is much smaller than the sound wavelength ($l \ll \lambda$), $\mathbf{K}_c$ becomes frequency-independent. For example, for an acoustic pressure sensor, the upper frequency limit of the audible range is 20 kHz. To ensure the validity of the simplified model, the cavity length should satisfy $l \ll 17$ mm. This frequency independent model is referred to as the simplified model hereafter.

Since the diaphragm radius is assumed to be much smaller than the sound wavelength, the sound field impinging on the diaphragm can be considered as a uniformly distributed pressure $p_e \exp(i\omega t)$. As a result, the pressure input vector $\tilde{P}_{ed}$ can be calculated as $\tilde{P}_{ed} = (p_e/E_d) \mathbf{N}_d$, where $\mathbf{N}_d$ is a $M \times 1$ vector and its $m$th component is $N_{d,m} = \sqrt{2\pi} \int_0^l U_{d,m}(r) r \, dr$. 
After obtaining the displacement vector based on Eq. (31), it needs to be converted to a physical variable that can be measured by using a chosen sensing mechanism. For example, in the case that the physical variable is the center displacement, it can be obtained as $w_c = B_d^T W_d$, where $B_d$ is $M \times 1$ vector and its $m$th component is $B_{d,m} = U_{d,m}(r = 0)\theta_0 = U_{d,m}(r = 0) / \sqrt{2\pi}$. If the pressure sensing mechanism is based on the strain measurements near the clamped edge (e.g., piezoelectric and piezoresistive pressure sensors), the strain to be measured can be determined from the modal response and the strains in the respective mode shapes.

### 3. Air cavity effect on static sensitivity

A representative pressure sensor with an air backed silicon diaphragm will be used here for the parametric study on the effects of the air cavity in the following sections. Detailed parameters of this representative sensor are listed in Table 3. The results obtained for this specific sensor can be generalized to study sensors with different parameters.

In Fig. 2, the static sensitivities as a function of cavity length are obtained for the diaphragm in vacuo, the air cavity alone, and the air backed diaphragm. Here, the static sensitivity is defined as the diaphragm center displacement per unit pressure at zero frequency. For a sensor that is designed to have a constant sensitivity frequency response below the fundamental natural frequency, the static sensitivity is the sensitivity of the sensor. The simplified model is used to obtain the static sensitivity of the air-backed diaphragm. Recall that the simplified model is derived from the full model based on the assumption of the cavity length being much smaller than the sound wavelength. Since the static sensitivity is defined at zero frequency (i.e., the sound wavelength is infinity), there will be no difference in the static sensitivity obtained with the simplified model compared with that obtained with the full model. For the ease of discussion, the static sensitivities for the air cavity, the diaphragm in vacuo, and the air-backed diaphragm will hereafter be referred to as $SS_a$, $SS_d$, and $SS_c$, respectively. As shown in Fig. 3, $SS_c$ decreases as the air cavity becomes shorter. For a short air cavity (e.g., $l/a < 5$ for this specific sensor), $SS_c$ is limited by the air cavity; while for a long air cavity (e.g., $l/a > 500$ for this specific sensor), $SS_c$ is close to that of the diaphragm in vacuo and the air cavity effect can be ignored.

### Table 3

Parameters of a representative pressure sensor.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diaphragm (silicon)</td>
<td></td>
</tr>
<tr>
<td>Young’s modulus $E_d$</td>
<td>169 GPa</td>
</tr>
<tr>
<td>Poisson’s ratio $\nu_d$</td>
<td>0.25</td>
</tr>
<tr>
<td>Density $\rho_d$</td>
<td>$2.3 \times 10^3$ kg/m$^3$</td>
</tr>
<tr>
<td>In-plane tension $N_0$</td>
<td>0 N/m</td>
</tr>
<tr>
<td>Radius $a$</td>
<td>500 $\mu$m</td>
</tr>
<tr>
<td>Thickness $h$</td>
<td>0.5 $\mu$m</td>
</tr>
<tr>
<td>Air cavity</td>
<td></td>
</tr>
<tr>
<td>Pressure $p_0$</td>
<td>$1.01 \times 10^5$ Pa</td>
</tr>
<tr>
<td>Sound speed $c_0$</td>
<td>343 m/s</td>
</tr>
</tbody>
</table>

Fig. 2. Static sensitivity as a function of the normalized cavity length. Parameters listed in Table 3 are used in the simulation.
In Table 4, the static sensitivities obtained with different number of diaphragm modes and air cavity modes are compared. It can be seen that even with one diaphragm mode and one air cavity mode \((N=1, M=1)\) the static sensitivities can be well predicted, which are only slightly higher than those obtained with additional diaphragm and air cavity modes. At the intersection of the lines for the diaphragm in vacuo and air cavity models in Fig. 2, it can be obtained that \(SS_a=SS_d\). The cavity length corresponding to this intersection can be defined as the critical cavity length \((l/a)_{cr,SS}\) for the static sensitivity. It is important to calculate this critical cavity length, since it indicates the transition of the static sensitivity from the air cavity dominated region to the pure diaphragm dominated region. For any pressure sensor with an air backed diaphragm, when the cavity length is much smaller than \((l/a)_{cr,SS}\), the sensitivity of the sensor is determined by the air cavity stiffness. On the other hand, when the cavity length is much larger than \((l/a)_{cr,SS}\), it is determined by the static sensitivity of the diaphragm in vacuo.

By using the simplified model with one diaphragm mode and one cavity mode \((N=1, M=1)\), the critical cavity length can be determined. In this case, the stiffness matrix \(K_c\) (Eq. (33)) reduces to a scalar form as

\[
K_c = \frac{1}{1-\nu^2} \left( \frac{h_d}{a} \right)^3 A_1^2 + \nu_0 \frac{p_0}{E_d} \frac{a}{T}.
\]  

(35)

where the first and second terms represent the contributions from the diaphragm and the air cavity, respectively. The resulting static sensitivity of the air backed diaphragm in terms of center displacement per unit static pressure \(p_e\) can be estimated as

\[
SS_c = \frac{p_e N_d B_d}{E_d} \frac{K_c}{1-\nu^2}.
\]  

(36)

![Fig. 3. Static sensitivity of air-backed diaphragm versus normalized cavity length for various Young’s modulus of the diaphragm: (a) absolute static sensitivity and (b) change of static sensitivity (ratio) from the condition of \(E_d\) in Table 3. All the other parameters used in the simulations are the same as those provided in Table 3.](image)

<table>
<thead>
<tr>
<th>Cavity length (l/a)</th>
<th>Static sensitivity (m/Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(M=1, N=1)</td>
</tr>
<tr>
<td>0.1</td>
<td>(1.129 \times 10^{-9})</td>
</tr>
<tr>
<td>1</td>
<td>(1.108 \times 10^{-8})</td>
</tr>
<tr>
<td>10</td>
<td>(9.363 \times 10^{-8})</td>
</tr>
<tr>
<td>100</td>
<td>(3.671 \times 10^{-7})</td>
</tr>
<tr>
<td>1000</td>
<td>(5.186 \times 10^{-7})</td>
</tr>
<tr>
<td>10,000</td>
<td>(5.410 \times 10^{-7})</td>
</tr>
</tbody>
</table>

Note: \(M\) and \(N\) refer to the numbers of modes for the diaphragm and air cavity, respectively. The numbers provided in the parentheses are the relative changes of static sensitivity as additional modes are used in the calculation. The static sensitivity of the diaphragm in vacuo is \(5.202 \times 10^{-7}\) m/Pa. Parameters in Table 3 are used in the simulation.
Based on Eq. (35), the critical normalized cavity length can be determined as

$$\left( \frac{l}{a} \right)_{cr,SS} = \frac{\gamma(1-\nu^2)T_{01}}{\lambda_1^2 \left( \frac{a}{h_d} \right)^3 \rho_0 \frac{E_d}{E_d}}$$

(37)

If the diaphragm is a pure plate (without in-plane tension), then $\lambda_1 = 2.95$ according to Table 1, and $T_{01} = 0.73$. For the specific sensor parameters that are used to obtain Fig. 2, the critical cavity length can be obtained to be $(l/a)_{cr,SS} = 48.05$ (or $l_{cr,SS} = 24.03$ mm).

Since the results shown in Fig. 2 are obtained for a specific sensor, to generalize these results, parametric studies are conducted to investigate how the behavior of $SS_c$ as a function of cavity length changes with respect to the changes of the diaphragm parameters including Young’s modulus $E_d$ (Fig. 3), thickness $h_d$ (Fig. 4), radius $a$ (Fig. 5), the static air pressure in the cavity $p_0$ (Fig. 6), and the normalized tension parameter $\chi$ (Fig. 7).

When $l/a$ is much larger than $(l/a)_{cr,SS}$ (e.g., $l/a > 10(l/a)_{cr,SS}$), $SS_c$ is mainly determined by the material properties and dimensions of the diaphragm. In this case, $SS_c$ becomes the same as $SS_d$, which is proportional to $a^3E_d h_d^{-3}$. Fig. 3 shows the change in static sensitivity with respect to cavity length as $E_d$ varies from 1/4 to 4 times the value assumed in Table 1. It can be seen from this figure that if $E_d$ is decreased four-fold in the long cavity regime, $SS_c$ will increase by the same ratio in the long cavity region. Fig. 4 shows the effect of diaphragm thickness. If the thickness $h_d$ is doubled in the long cavity regime, $SS_c$ will be reduced by a factor of 8. In Fig. 5, the static sensitivity is obtained for various radii. It can be seen that if the radius $a$ is doubled, $SS_c$ will increase by a factor of 16 in the long cavity regime. Furthermore, in Fig. 6, the static sensitivity as a function of normalized cavity length is compared for different radii.
of $l/a$ is obtained with varying the static air pressure. Note that in the case of varying the static air pressure, it is assumed the air pressure in the cavity is equal to the varied static pressure. As the temperature is assumed to be constant in room temperature, the density changes by the same proportion as the air pressure. As can be seen from Fig. 6, in the long cavity region, changing the static air pressure has no influence on $SS_c$.

On the other hand, when $l/a$ is much smaller than $(l/a)_{cr, SS}$ (e.g., $l/a < 0.1(l/a)_{cr, SS}$), $SS_c$ is almost the same as $SS_a$, which is proportional to $l/p_0$, but independent of the diaphragm dimensions and properties (i.e., $a$, $h_d$, and $E_d$), as can be seen from Figs. 3–6. This means that if the air cavity length is designed in this region, the sensitivity of the sensor will be solely determined by the air cavity, which cannot be changed by tailoring the diaphragm dimensions and properties. Note that the apparent change of $SS_c$ with respect to different $a$ in Fig. 5 for $l/a < 1$ is due to the change of cavity length since the $x$-axis of the plot is the normalized cavity length $l/a$. In addition, it is also important to note that $(l/a)_{cr, SS}$ decreases as the diaphragm becomes stiffer (i.e., increasing $E_d$ and $h_d$ or decreasing $a$).

In the above parametric studies, the diaphragm is considered as a pure plate with zero in plane tension ($N_0 = 0$). Next, the influence of in-plane tension on the static sensitivity is studied, as shown in Fig. 7. An in-plane tension (i.e., residual stress) is often inevitable in MEMS pressure sensors. Due to the tensile residual stress, the diaphragm will become stiffer, resulting in a smaller sensitivity. For example, with an in-plane tension of $N_0 = 0.27$ N/m (i.e., $\chi = 36$), the static sensitivity of the diaphragm in vacuo decreases by a factor of 3.5. When $N_0$ increases to 0.75 N/m ($\chi = 100$), the static sensitivity further decreases by a factor of 7.9. The residual stress also affects the critical cavity length $(l/a)_{cr, SS}$. Note that two terms ($\Lambda_1$ and $T_{01}$)

![Fig. 6. Static sensitivity of air-backed diaphragm versus normalized cavity length for various static air cavity pressure values: (a) absolute static sensitivity and (b) change of static sensitivity (ratio) from the condition of $p_0$ in Table 3. All the other parameters used in the simulations are the same as those provided in Table 3.](image)

![Fig. 7. Static sensitivity of air-backed diaphragm versus normalized cavity length for various normalized tension parameter ($\chi$) values: (a) absolute static sensitivity and (b) change of static sensitivity (ratio) from the condition of $\chi = 0$ in Table 3. All the other parameters used in the simulations are the same as those provided in Table 3.](image)
in Eq. (37) are affected by the change of $\chi$. Because $T_{01}$ is an indicator of similarity between the fundamental modes of the diaphragm and the air cavity, it can be treated as a constant ($T_{01}$ only changes slightly from 0.73 to 0.77 as $\chi$ increases from 0 to 100). On the other hand, $A_1$ increases as $\chi$ increases. Therefore, $(l/a)_{cr,SS}$ decreases with increasing in-plane tension of the diaphragm, which can be observed from Fig. 7(a).

The above results can provide important guidelines for the design of miniature pressure sensors such as MEMS pressure sensors or fiber optic pressure sensors, in which the length of the cavity is usually comparable or even smaller than the size of the diaphragm (i.e., $l/a < 1$). This means that the sensitivity of these sensors will be dominated by the air spring effect, resulting in a significantly lower sensitivity than that of the diaphragm in vacuo. In another words, the common method of enhancing the sensitivity by using a more flexible diaphragm will not be effective since the sensor sensitivity is dominated by the air cavity sensitivity. This sensitivity limitation is often overlooked in existing sensor designs. In Table 5, several acoustic pressure sensors reported in the literature with sensitivities severely limited by the air cavity are listed. For example, in Ref. [42], an acoustic sensor consisting of a 125 $\mu$m-diameter graphene diaphragm backed with a 70 $\mu$m long air cavity was presented. The measured sensitivity of this sensor is 1.1 nm/Pa, which is much less than the predicted sensitivity of 2.86 nm/Pa by using the pure plate model with even the highest possible Young’s modulus of a single layer graphene. Based on the coupled system model developed in this study, it can be found that the measured sensitivity is largely limited by the sensitivity of the air cavity.

To overcome the sensitivity limitation due to the air cavity, one straightforward approach is to make the cavity length long enough such that the sensitivity of sensor will be dominated by the diaphragm itself. For example, in condenser microphones, there is usually a large-volume back chamber underneath a perforated backplate to overcome the sensitivity limitation due to the small air gap between the diaphragm and the backplate. Another approach is to employ a vacuum or

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Diaphragm material</th>
<th>Diaphragm dimension</th>
<th>Air cavity length ($\mu$m)</th>
<th>Critical cavity length $(l/a)_{cr,SS}$ ($\mu$m)$^a$</th>
<th>Sensor sensitivity (nm/Pa) $^b$</th>
<th>Measured</th>
<th>Predicted without air cavity</th>
<th>Predicted with air cavity $^c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[42]</td>
<td>Graphene</td>
<td>Radius of 125 $\mu$m thickness of 100 nm</td>
<td>70</td>
<td>128.36</td>
<td>1.1</td>
<td>2.85</td>
<td>0.97</td>
<td>2.85</td>
</tr>
<tr>
<td>[18]</td>
<td>Silicon</td>
<td>Radius of 800 $\mu$m thickness of 1.5 $\mu$m</td>
<td>200$^c$</td>
<td>6800</td>
<td>34.6</td>
<td>147</td>
<td>19.2</td>
<td>147</td>
</tr>
</tbody>
</table>

$^a$ The critical cavity length is calculated by using Eq. (37).
$^b$ The static sensitivity is calculated by using the model developed in this paper.
$^c$ The effective cavity length is calculated by dividing the volume of the trapped air ($2.1 \times 10^{-9}$ m$^3$, obtained from Dr. J. Bucaro through personal communications) by the diaphragm area. This rough estimate partly explains the discrepancy between the estimated and measured sensitivities.

![Fig. 8. Fundamental natural frequency of air-backed diaphragm (normalized by that of the same diaphragm in vacuo) versus normalized cavity length. Here, the results obtained with the full model and the simplified model are compared to those obtained for the close-ended air cavity and the diaphragm in vacuo where the air effects are completely eliminated. The parameters listed in Table 3 are used in the simulations.](image-url)
partial vacuum chamber underneath the diaphragm, which can help eliminate or reduce the air spring effect and thus increase the sensitivity of the sensor.

4. Air cavity effect on fundamental natural frequency

Here, in Fig. 8, the fundamental natural frequency (undamped) of the air-backed diaphragm \( f_{c,1} \) is obtained as a function of the normalized cavity length by using the full model, the simplified model \((N=5, M=5)\), and the dynamical analogy lumped model \((N=1, M=1)\). For most diaphragm based pressure sensors, \( f_{c,1} \) can be used to define the bandwidth of the sensor. Therefore, in order to increase the bandwidth of the sensor, a large \( f_{c,1} \) is desirable. As shown in Fig. 8, when the air cavity is long \( (e.g., l/a > 200)\), the fundamental frequency obtained with the full model is close to that of the closed-ended air cavity, which is much lower than that of the diaphragm in vacuo. This result indicates that an acoustic mode is exhibited at this fundamental frequency, which can be explained by the fact that the diaphragm is much stiffer than the long air cavity so that the diaphragm can be regarded as a rigid wall for the air cavity. As the air cavity becomes shorter, the fundamental natural frequency first increases, and then saturates before decreasing at a much shorter air cavity \( (e.g., l/a < 0.1)\). To understand this result, recall that in Section 2, it has been discussed that the air cavity has two functions; one is to increase the equivalent stiffness of the diaphragm and the other is to increase the equivalent mass of the diaphragm. As the length of the air cavity starts to decrease, the fundamental natural frequency increases due to the air stiffness effect that is dominating for long air cavities \( (e.g., l/a > 10)\). As the air cavity becomes shorter \( (e.g., l/a < 0.1)\), the mass effect becomes dominating, and thus the fundamental natural frequency decreases. This result can provide important guidelines for pressure sensor design. For example, according to this result, it can be found that when the air cavity length is properly designed \( (e.g., 0.003 < l/a < 20)\), the resulting air-diaphragm coupled system will have a larger fundamental frequency than that of the diaphragm in vacuo, rendering a larger sensor bandwidth without the necessity of changing the properties or dimensions of the diaphragm itself.

As shown in Fig. 8, the result obtained with the simplified model (which uses five diaphragm modes and five cavity modes) has a good agreement with that obtained with the full model \( (N=5, M=5)\) for a relatively short air cavity \( (e.g., l/a < 3)\). For a longer cavity \( (e.g., l/a > 3)\), the discrepancy between the full model and the simplified model can be attributed to the break-down of the assumption that the cavity length is much shorter than the sound wavelength. Note that based on this assumption, the denominator in Eq. (26) \( (i.e., \tan(\Omega l/a))\) can be approximated as \( \Omega l/a\). Since for a long cavity, it can be obtained that \( \Omega l/a < \tan(\Omega l/a)\), the simplified model overestimates \( f_{c,1} \) by predicting the higher frequency diaphragm structure mode.

It is important to recognize that the bandwidth of a pressure sensor is determined by the fundamental natural frequency of the diaphragm \( (a structural mode)\), not the acoustic mode of the air cavity. In the long cavity regime, the simplified model does predict the structural mode of the diaphragm correctly, despite missing the low frequency acoustic modes of the air cavity. Therefore, in order to facilitate pressure sensor design, the simplified model will be used in this paper to obtain close-form expressions for the fundamental natural frequency. It should also be recognized that the long cavity regime is not relevant to most miniature dynamic pressure sensors, where it is desired to keep the cavity length comparable to or shorter than the diaphragm diameter for practical reasons.

To determine the minimum number of modes needed in the simplified model to accurately predict the fundamental frequency of the air-backed diaphragm, the fundamental natural frequencies are obtained by using different numbers of
modes, as shown in Fig. 9. It can be seen that for a relatively long cavity (e.g., \( l/a > 10 \)), the fundamental frequency can be simply captured by using just one diaphragm mode (\( M = 1 \)) and one air cavity mode (\( N = 1 \)). To further capture the saturation behavior (i.e., the plateau shown in Fig. 9) of the fundamental frequency at a shorter cavity (e.g., \( l/a < 0.2 \)), one more diaphragm mode (i.e., \( M = 2, N = 1 \)) is needed. Furthermore, when an additional air cavity mode that contributes to the mass effect is included (i.e., \( M = 2, N = 2 \)), the fundamental frequency can be well predicted. Further increasing the number of modes will only slightly change the result (i.e., \( M = 5, N = 5 \)).

In order to provide a straightforward way for estimation of the bandwidth of a pressure sensor with an air-backed diaphragm, here, the simplified model is further used to obtain close-form approximations of the fundamental frequency for different cavity lengths. As can be seen from Fig. 10, the variation of fundamental natural frequency as a function of air cavity length can be characterized by using three regions: the long cavity region, the medium cavity region (i.e., the plateau region), and the short cavity region. Based on this observation, close-form approximation of the fundamental natural frequency for these three regions can be obtained along with the two critical cavity lengths that can be used to define these regions.

In the long cavity region, the fundamental frequency is estimated by using the simplified model (\( M = 1, N = 1 \)) as the following:

\[
\Omega_{c1,\text{long}} = \sqrt{\Lambda_1^2 + T_{01}^2 / \theta},
\]

where

\[
\theta = \frac{1}{\gamma(1 - \nu^2)\rho_0 \frac{h_d}{a}^3 \frac{l}{a}}
\]

If the diaphragm is a pure plate (without in-plane force), we find that \( \Lambda_1 = 2.95 \) according to Table 2 and \( T_{01} = 0.73 \). A more accurate approach to estimate the fundamental frequency is to use Eq. (26) and find the root of the following equation [26]:

\[
\Lambda_2^2 + \frac{T_{01}^2}{\theta} \frac{\Omega_c l/a}{\tan(\Omega_c l/a)} - \Omega_d^2 = 0,
\]

where \( \Omega_d = \Omega_c (c_d/c_0) (h_d/a) \). However, this approach cannot provide a close-form expression for the fundamental natural frequency. On the other hand, by using Eq. (38), the critical cavity length for the long cavity region can be determined, which will be useful to analyze the variation of \( \Omega_{c1} \). Moreover, Eq. (38) approximates Eq. (40) well for pressure sensors for which the cavity length is much shorter than the sound wavelength, which is often a valid assumption in miniature pressure sensor designs.

To estimate the fundamental frequency for the medium cavity region, the first two modes of the diaphragm and the air spring mode of the cavity (i.e., \( M = 2, N = 1 \)) are used. In this case, the transfer function of the air-backed diaphragm \( H_c \)
where the fundamental frequency \( \Omega_{c1} \) can be obtained by finding the root of the zero determinant of \( \mathbf{H}_c \); that is,

\[
\Omega_{c1} = \frac{a^2}{2} \sqrt{\frac{A_1^2 + A_2^2 + (T_{01}^2 + T_{02}^2)/\theta}{T_{01}T_{02}/\theta}} - \frac{1}{2} \sqrt{(T_{01}^2 + T_{02}^2)/\theta^2 + 2(A_1^2 - A_2^2)(T_{01}^2 - T_{02}^2)/\theta + (A_1^2 - A_2^2)^2}.
\]

Note that as \( l \) is approaching 0, \( \theta \) goes to 0 and the fundamental frequency approaches the plateau value. The fundamental frequency for the medium cavity region can thus be obtained as

\[
\Omega_{c1,medium} = \sqrt{\frac{A_1^2 + A_2^2}{2} + \frac{A_1^2 - A_2^2}{2}T_{01}^2 - T_{02}^2}{T_{01} + T_{02}^2} \tag{43}
\]

If the diaphragm is a pure plate (without in-plane tension), \( A_1 = 2.95 \) and \( A_2 = 11.48 \) according to Table 1, and thus \( \Omega_{c1,medium} \) is equal to 10.10, which is 3.43 times \( A_1 \). It should be noted that this plateau value is the limit when \( l \) approaches 0 and the mass effect becomes dominating. Therefore, Eq. (43) can only be regarded as an upper bound of \( \Omega_{c1} \).

To estimate the fundamental frequency in the short cavity region, the simplified model with \( M = 2 \) and \( N = 2 \) is used and \( \mathbf{H}_c \) defined by Eq. (32) reduces to

\[
\mathbf{H}_c = \frac{p_0a}{E_dI} \begin{bmatrix} T_{01}^2 + \theta A_1^2 - (\sigma T_{11} + \theta)\Omega^2 & T_{01}T_{02} - \sigma T_{11}T_{12}\Omega^2 \\ T_{01}T_{02} - \sigma T_{11}T_{12}\Omega^2 & T_{02}^2 + \theta A_2^2 - (\sigma T_{12} + \theta)\Omega^2 \end{bmatrix}, \tag{44}
\]

where

\[
\sigma = \frac{1}{p_0^2} \left( \frac{E_d}{I} \right)^2 \left( \frac{l}{a} \right)^2. \tag{45}
\]

Similarly, the fundamental natural frequency can be determined as

\[
\Omega_{c1} = \frac{(A_1^2 + A_2^2)\theta^2 + Q_2\theta + Q_1}{2\theta^2 + 2Q_3\theta} - \frac{\sqrt{[(A_1^2 + A_2^2)\theta^2 + Q_2\theta + Q_1]^2 - 4(\theta^2 + Q_3\theta)(A_1^2A_2^2\theta^4 + Q_4\theta)}}{2\theta^2 + 2Q_3\theta} \tag{46}
\]

where

\[
Q_1 = \sigma(T_{02}T_{11} - T_{01}T_{12})^2, \quad Q_2 = \sigma(T_{11}^2A_1^2 + T_{12}^2A_2^2) + (T_{01}^2 + T_{02}^2) \\
Q_3 = \sigma(T_{11}^2 + T_{12}^2), \quad Q_4 = (T_{01}^2A_2^2 + T_{02}^2A_1^2). \tag{47}
\]

Note that Eq. (46) is the closed-form equation for the fundamental frequency of the coupled system in all three regions. However, as \( l/a \to 0, \theta \to 0, \Omega_{c1} \to 0 \). In this case, \( \Omega_{c1} \) can be approximated by

\[
\Omega_{c1, short} = \sqrt{\frac{Q_4\theta}{Q_1}} = \sqrt{\frac{\rho_d h_d l}{\rho a}} \tag{48}
\]

where

\[
Q = \sqrt{T_{01}^2A_2^2 + T_{02}^2A_1^2} / (T_{02}T_{11} - T_{01}T_{12})p_{\text{d}}. \tag{49}
\]

If the diaphragm is a pure plate (without in-plane force), it can be obtained that \( Q = 51.57 \).

The two critical air cavity lengths, marked in Fig. 10, which can be used to determine the short, medium, and long cavity regions, can be calculated from the intersections of Eqs. (38), (43), and (48) as

\[
\left( \frac{l}{a} \right)_{c, \text{long}} = \gamma (1 - \nu) \left( \frac{a}{h_d} \right)^3 \frac{T_{01}}{K_{c, \text{medium}}^2 - \lambda_1^2}, \tag{50}
\]

\[
\left( \frac{l}{a} \right)_{c, \text{short}} = \frac{\rho a}{\rho d h_d} \left( \frac{\Omega_{c1, \text{medium}}}{Q} \right)^2. \tag{51}
\]
For the representative sensor (see Table 1 for parameters), these critical cavity lengths values are calculated as \( l/a_{cr,\text{long}} = 6.14 \) and \( l/a_{cr,\text{short}} = 0.02 \). Note that the critical cavity length in Eq. (50) is related to the critical length for the static sensitivity in Eq. (37) by

\[
\frac{l}{a_{cr,\text{long}}} = \frac{\Lambda_{1}}{\Omega_{1,c1,\text{medium}}} - \frac{\Lambda_{1}}{\Omega_{1,c1,\text{SS}}}. \tag{52}
\]

For a pure plate without in-plane tension, \( l/a_{cr,\text{long}} = 0.093 \) (\( l/a_{cr,\text{SS}} \)).

Since the results in Figs. 9 and 10 are obtained for a specific case, in which Young’s modulus, density, thickness of the diaphragm, and the static pressure in the cavity are fixed, here, parametric studies are carried out to study the effects of these parameters on the fundamental natural frequency. The two critical cavity lengths defined in Eqs. (50) and (51) will be used to investigate the effects of the air cavity on the fundamental natural frequency in different cavity length regions. It should be noted that the relative stiffness and mass between the diaphragm and the air cavity plays a key role in determination of the fundamental frequency of the coupled system.

First, the fundamental natural frequency as a function of normalized cavity length for varying diaphragm Young’s modulus \( E_d \) is shown in Fig. 11. Note that \( E_d \) determines the stiffness of the diaphragm, and thus, it can be observed that increasing \( E_d \) leads to a higher \( f_{c1} \), no matter which of the three regions the cavity length is in. According to Eqs. (50) and (51), \( l/a_{cr,\text{long}} \) is inversely proportional to \( E_d \), while \( l/a_{cr,\text{short}} \) is independent of \( E_d \). Therefore, for a larger \( E_d \), the critical cavity length \( l/a_{cr,\text{long}} \) that separates the long cavity region and the medium plateau region becomes smaller; while
that separates the medium cavity region and the short cavity region stays constant (see Fig. 11(a)). The combined effect for a larger $E_d$ is to have a smaller plateau region, which will eventually disappear when $E_d$ is larger than a critical value. Furthermore, the scaling of $f_{c1}$ with respect to $E_d$ can be observed from Fig. 11(b). In the long cavity region, where the diaphragm behavior dominates, the change of $f_{c1}$ with respect to $E_d$ is similar to the case of a diaphragm in vacuo and $f_{c1}$ is proportional to $E_d^{1/2}$. In the short cavity region, according to Eq. (48), the normalized natural frequency $\Omega_{c1}$ is independent of $E_d$; that is, $f_{c1}$ is proportional to $E_d^{1/2}$, which exhibits the same scaling factor as that in the long cavity region. In the medium region, as shown in Fig. 11(b), $f_{c1}$ is less sensitive to the change of $E_d$. This result indicates that for a sensor with an air cavity in the medium cavity region, in order to increase the bandwidth of the sensor, increasing the stiffness of the diaphragm material becomes less effective than the case of sensors with a diaphragm in vacuo or backed with a relative shorter or much longer air cavity.

Next, the influence of diaphragm density $\rho_d$ on the fundamental frequency is studied, which affects the effective mass of the diaphragm. As shown in Fig. 12, except for short cavity lengths, increasing $\rho_d$ always leads to a smaller $f_{c1}$. According to Eqs. (50) and (51), $(l/a)_{cr, short}$ is inversely proportional to $\rho_d$, while $(l/a)_{cr, long}$ is independent of $\rho_d$. Therefore, the plateau region gets smaller as $\rho_d$ increases, as shown in Fig. 12(a). Furthermore, the scaling of $f_{c1}$ with respect to $\rho_d$ is investigated. In the long cavity range, the scaling of $f_{c1}$ with respect to $\rho_d$ is similar to that of a diaphragm in vacuo, for which $f_{c1}$ is linearly proportional to $(1/\rho_d)^{1/2}$, as can be seen in Fig. 12(b). On the other hand in the short cavity range, $f_{c1}$ is insensitive to the change of $\rho_d$. This can be explained by the fact that the normalized $\Omega_{c1}$ is linearly proportional to $\rho_d^{1/2}$, according to

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig13}
\caption{Fundamental natural frequency $f_{c1}$ versus normalized cavity length $l/a$ for various diaphragm thicknesses $h_d$: (a) absolute $f_{c1}$ and (b) change of $f_{c1}$ (ratio) from the condition of $h_d$ in Table 3. The parameters listed in Table 3 are used in the simulations.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig14}
\caption{Fundamental natural frequency $f_{c1}$ versus normalized cavity length $l/a$ for various static air cavity pressure $p_0$: (a) absolute $f_{c1}$ and (b) change of $f_{c1}$ (ratio) from the condition of $p_0$ in Table 3. The parameters listed in Table 3 are used in the simulations.}
\end{figure}
Eq. (48), and thus, the $f_{c1}$ will not change with respect to $\rho_d$. It should be noted that for sensors with a diaphragm in vacuo, decreasing $\rho_d$ will help increase the fundamental natural frequency. However, the result here shows that in the short cavity region, to increase the fundamental natural frequency, decreasing the diaphragm density $\rho_d$ becomes less effective.

Furthermore, the effect of the diaphragm thickness $h_d$ on the fundamental frequency is studied, as shown in Fig. 13. Note that both the stiffness and the mass of the diaphragm depend on $h_d$. As such, $f_{c1}$ with respect to the change of $h_d$ is equivalent to the combination effect of changing $E_d$ and $\rho_d$. Based on Eqs. (50) and (51), it can be found that both $|(l/a)_{cr, long}|$ and $|(l/a)_{cr, short}|$ decrease as $h$ increases. Therefore, both the short cavity and long cavity regions shift to shorter lengths, as shown in Fig. 13(a). Since $(l/a)_{cr, long}$ scales with $h_d^{-3}$ while $(l/a)_{cr, short}$ scales with $h_d^{-1}$, the plateau region is smaller for a larger $h_d$. Further, we study how $f_{c1}$ scales with $h_d$. In the long-cavity region, $f_{c1}$ scales linearly with $h_d$, which is the same as that for a diaphragm in vacuo, as the effects of air cavity can be neglected in this region. In the short cavity region, $f_{c1}$ is more sensitive to the change of $h_d$ than that in vacuum. As discussed previously, $f_{c1}$ scales with $E_d^{1/2}$ but is insensitive to $\rho_d$. As the stiffness of the diaphragm is proportional to $h_d^3$, $f_{c1}$ scales with $h_d^{3/2}$ in the short cavity region, as seen in Fig. 13(b). It is interesting to note that based on this result, it can be found that in the region when the cavity length changes from the plateau region to the long cavity region, compared with that of the diaphragm in vacuo, increasing the thickness of the diaphragm will become less effective on the increase of the fundamental natural frequency; in the short cavity region, however, this will become more effective.

The effect of the static pressure in the air cavity $p_0$ on $f_{c1}$ is studied, as shown in Fig. 14. Assume that the temperature is kept at room temperature and the air density $\rho_0$ changes proportionally with $p_0$. In this case, changing the static pressure is the same as changing the air density in the cavity, which will lead to the scaling of the stiffness and mass of the air by the same ratio. Since the ratio of $(l/a)_{cr, long}$ to $(l/a)_{cr, short}$ is proportional to $p_0/\rho_0$, the length of the plateau region will remain the same when $p_0$ changes. As shown in Fig. 14(a), increasing the static pressure $p_0$ shifts the $f_{c1}$ curve to the right. Furthermore, a larger $p_0$ (and $\rho_0$) causes an increase of $f_{c1}$ in the transition region between the long cavity region and the medium region, but a decrease of $f_{c1}$ in the short cavity region. This result confirms that the mass effect of the air cavity dominates the short cavity region, in which the fundamental natural frequency decreases as the air density increases; while due the stiffness effect of the air cavity that dominates the long cavity region, the fundamental frequency increases as the air density increases. It should be noted that due to the rarefraction effects at low pressure, the no-slip condition at the interface and the continuum medium assumptions may break down.

In addition, the effect of tension parameter $\chi$ on $f_{c1}$ is studied, as illustrated in Fig. 15. Note that increasing $\chi$ is equivalent to stiffening the diaphragm. Therefore, it can be observed that increasing $\chi$ leads to a higher $f_{c1}$, no matter which of the three regions the cavity length is in. Furthermore, as $\chi$ increases, the critical cavity length shifts to the shorter region, because the air cavity needs to be shorter to match with the increased diaphragm stiffness.

5. Air cavity effect on damping

In addition to the static sensitivity and the fundamental natural frequency studied in the previous sections, damping ratio is another important parameter that affects the sensor performance, especially the bandwidth. Note the damping due to the air cavity is related to the viscous boundary layer thickness. At 15 kHz, the viscous boundary layer thickness is estimated to be about 18 $\mu$m [43]. When the cavity length is comparable to or smaller than this thickness, the squeeze film effect has to be considered.
in order to determine the sensor bandwidth. However, because the cavity length considered in this study spans orders of
magnitudes above/below the diaphragm size, some of the assumptions made in the commonly used Reynolds equation become
invalid. Here, a three dimensional FEM model is developed by using the thermoacoustic–shell interaction Module in COMSOL 4.4.
By using this model, the damping at various cavity lengths is obtained. Furthermore, the FEM model is also used to validate the
previous analytical results on how the static sensitivity and natural frequency are influenced by the cavity length.

As shown Fig. 16, the FEM model consists of a cylindrical air cavity with the top surface modeled using shell elements.
To account for the pressure/velocity gradient in the boundary layer, six finer boundary layer of meshes are added near the
diaphragm at the top and near the hard wall on the bottom and side. The thickness of the first layer is 1.8 μm, which is about
one tenth of the boundary layer thickness at 15 kHz, and increases progressively by 20 percent for the next five layers. The
model uses the same number of hexagonal elements (7488 in total) for the air cavity of various lengths. The cylindrical
volume can be set as a thermoacoustic domain to include the viscous effect of the air, or as a pressure acoustic domain that is
inviscid. The structural damping of the diaphragm is neglected as it is usually much smaller than the air damping. The peripheral
edge of the diaphragm has a fixed constraint to assume the clamped boundary condition. Eigenfrequency studies are carried out
to obtain the natural frequencies and mode shapes for the inviscid case, and frequency domain studies are used to obtain the
frequency response of the air-backed diaphragm for both the viscous and inviscid cases.

When the air-backed diaphragm is considered as an equivalent mass-spring-damper system, the mechanical sensitivity $s_{\text{dyn}}$ of the sensor with such a diaphragm at any working frequency $f$ in terms of the static sensitivity $S_{\text{sc}}$, natural frequency

![Diaphragm and Air cavity](image)

**Fig. 16.** FEM model using the thermoacoustic–shell interaction model in COMSOL. Boundary layer meshes are added near the edges.

![Frequency response curves](image)

**Fig. 17.** Frequency response curves obtained by using the viscous FEM model: (a) $l/a = 0.07$, (b) $l/a = 0.05$, (c) $l/a = 0.04$, and (d) $l/a = 0.03$. The results are obtained for the representative sensor with parameters listed in Table 3.
and bandwidth as evidenced in the results shown in Fig. 17. In this article, we investigate how the sensitivity and bandwidth of dynamic pressure sensors can be influenced by the length of the air cavity backing a flexible diaphragm. A continuum mechanics model of a circular diaphragm backed by a cylindrical air cavity is presented, which represents the key component in many dynamic pressure sensors. The diaphragm is modeled as a thin plate with in-plane force, and the air cavity is described by a sound wave equation. The viscous effect of the air is neglected in this model because of the following reasons. First, the damping is usually small for long cavity lengths. In this case, the bandwidth is mainly determined by the fundamental natural frequency obtained from the free vibration analysis. As such, this inviscid analytical model is adequate for the perspective of sensor design, there is no benefit to have a cavity length less than 0.04 \( l/a \), due to the reduced sensitivity and bandwidth as evidenced in the results shown in Fig. 17.

Fig. 17 shows the frequency response of the mechanical sensitivity \( s_{dyn} \) obtained by using the FEM model for various cavity lengths. The simulation data is fitted by using Eq. (53) to determine the static sensitivity \( S_S \), natural frequency \( f_{c1} \), and damping ratio \( \zeta \), which are listed in Table 6 along with the static sensitivity and natural frequency obtained with the analytical inviscid model and inviscid FEM model.

First, as can be seen from Table 6, the results obtained with the analytical model are in good agreement with those obtained with the inviscid FEM model, with maximum deviation in the static sensitivity and natural frequency to be less than 0.3 percent. Similar results can be observed from Fig. 18(a)–(b). According to Table 6, when \( l/a \) decreases from 10 to 1, the fundamental frequency \( f_{c1} \) of the air-backed diaphragm increases. As \( l/a \) decreases from 1 to 0.1, \( f_{c1} \) reaches the plateau value. When \( l/a \) is reduced further to 0.01, \( f_{c1} \) starts to decrease. On the other hand, the static sensitivity always decreases with decreasing cavity length. This again confirms that the interplay between the stiffness and mass effects of the air cavity in different cavity length regions.

Second, the scaling of the static sensitivity and the natural frequency obtained from the viscous FEM agrees well with the results from the analytical model for \( l/a > 0.4 \). This again verifies how the stiffness/spring and mass effects of the air cavity play different roles in different cavity length regions; the stiffness effect dominates in the long/medium cavity regions whereas the mass effect dominates in the short cavity region. For a short cavity length (\( l/a < 0.4 \) or \( l < 20 \, \mu m \) in this case), the entire air cavity is trapped in the boundary layer. As a result, the velocity gradient normal to the diaphragm is larger than that obtained with the inviscid model and the mass effect becomes more prominent. For this reason, the inviscid model overestimates the natural frequency in this short cavity region, as shown in Fig. 18(b).

Third, we study how the damping ratio is influenced by the cavity length. As shown in Fig. 18(c), for \( l/a > 1 \), the damping ratio increases with the cavity length, as the viscous boundary layer extends along the side wall. For \( l/a < 1 \), the damping ratio becomes larger for shorter cavity length due to the squeeze film effect. When the air cavity is completely trapped in the boundary layer (\( l/a < 0.04 \)), the damping is approaching the critical damping. This result indicates that for a particular sensor configuration that the equivalent damping should be treated differently in the different cavity length regions. From the perspective of sensor design, there is no benefit to have a cavity length less than 0.04\( a \), due to the reduced sensitivity and bandwidth as evidenced in the results shown in Fig. 17.

### 6. Concluding remarks

In this article, we investigate how the sensitivity and bandwidth of dynamic pressure sensors can be influenced by the length of the air cavity backing a flexible diaphragm. A continuum mechanics model of a circular diaphragm backed by a cylindrical air cavity is presented, which represents the key component in many dynamic pressure sensors. The diaphragm is modeled as a thin plate with in-plane force, and the air cavity is described by a sound wave equation. The viscous effect of the air is neglected in this model because of the following reasons. First, the damping is usually small for long cavity lengths. In this case, the bandwidth is mainly determined by the fundamental natural frequency obtained from the free vibration analysis. As such, this inviscid analytical model is adequate for

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**Table 6**

Comparison of sensor performance metrics obtained with the analytical model and the FEM model.

<table>
<thead>
<tr>
<th>Cavity length ( l/a )</th>
<th>Static sensitivity (m/Pa)</th>
<th>Fundamental natural frequency (kHz)</th>
<th>Damping ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Analytical</td>
<td>FEM inviscid</td>
<td>FEM viscous</td>
</tr>
<tr>
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</tr>
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<td>8.790 x 10^{-8}</td>
<td>8.784 x 10^{-8}</td>
<td>1.015 x 10^{-8}</td>
</tr>
</tbody>
</table>

Note: The parameters in Table 3 are used in the simulations.
characterizing the sensor sensitivity and bandwidth. Second, the damping can be tuned post-fabrication by changing the design of the holy plate underneath the diaphragm [44,45]. This means that the static sensitivity and natural frequency are often the two more important metrics than the damping ratio in the early design stage. Third, the commonly used Reynolds equation is only valid for a short cavity length (i.e., thin air film), and a full-scale Navier–Stokes analysis has to be conducted when the cavity lengths span a large range. To complement the inviscid analytical model, a viscous FEM model was developed to study how the damping changes with respect to the cavity length.

The continuum mechanics model developed in this article generally has frequency-dependent stiffness matrix. When the cavity length is much smaller than the sound wavelength (i.e., \( l \ll \lambda \)), it reduces to a simplified model with a frequency-independent stiffness matrix. Through analytical studies, it has been discovered that the effect of the lowest air cavity mode is to increase the equivalent stiffness of the diaphragm; while the higher order air cavity modes add equivalent masses to the diaphragm. And these two effects scales differently with the cavity length.

![Graphs showing comparison of analytical results with FEM simulations in various cavity length regions: (a) static sensitivity, (b) undamped natural frequency, and (c) damping ratio obtained with the viscous FEM model.](image-url)
Without loss of generality, through a case study of a pressure sensor with an air backed diaphragm, we have investigated and found that the static sensitivity of the air-backed diaphragm has contributions from both the diaphragm and the air cavity. In general, the static sensitivity always decreases as the cavity length becomes shorter. However, depending on the cavity length, the sensitivity can be either dominated by the air cavity sensitivity or that of the diaphragm in vacuo. To determine the dominating region, the critical cavity length \( (l/a)_{cr,SS} \) is defined, at which the diaphragm and the air cavity have equal static stiffness. It is important to note that for a cavity length that is much shorter than the critical cavity length, the sensitivity is determined by the air cavity stiffness regardless of how flexible the diaphragm is. Although this finding is quite intuitive, it is not uncommon for one to readily assume that the air cavity stiffness is much smaller than that of the diaphragm and therefore can be neglected, which may not be true, especially for miniature acoustic sensors with size constraints. When the cavity length is much longer than the critical cavity length, the static sensitivity scales with material and dimensional parameters in the way similar to the diaphragm in vacuo. However, the static sensitivity becomes much less sensitive to the change of these parameters when the cavity length is much smaller than the critical value.

The effects of the air cavity on the undamped fundamental natural frequency \( f_{c1} \) are more complicated. The air cavity has a stiffness effect that dominates the long-cavity region and a mass effect that dominates the short-cavity region. The variation of \( f_{c1} \) with respect to the normalized cavity length \( l/a \) can be characterized into three regions; namely, the long cavity region, the medium cavity region (i.e., the plateau region), and the short cavity region. As the cavity length decreases, \( f_{c1} \) increases in the long-cavity region, but decreases in the short-cavity region. In the medium region, \( f_{c1} \) is limited by a plateau value. The two critical cavity lengths that separate these three regions are defined, and parametric studies are carried out to study how the material and geometrical parameters of the diaphragm affect \( f_{c1} \). Five parameters are investigated: the Young’s modulus \( E_d \), density \( \rho_d \), and thickness \( h_d \) of the diaphragm, the air pressure \( p_0 \) in the cavity, and the tension parameter \( \gamma \) of the diaphragm. The rule of thumb is \( f_{c1} \) becomes larger by increasing the overall stiffness or decreasing the overall mass. However, the scaling law of \( f_{c1} \) is different in different regions. In the long cavity region where the air cavity effect can be neglected, the scaling of \( f_{c1} \) is similar to that of a pure diaphragm in vacuo; that is, \( f_{c1} \) scales with \( E_d \rho_d^{-1/2} h_d \). However, the scaling in the short cavity region is quite different, in which \( f_{c1} \) scales with \( E_d^{1/2} \rho_d h_d^{-1/2} \). Furthermore, as \( p_0 \) decreases, \( f_{c1} \) increases in the short cavity region but decreases in the longer cavity region. Therefore, in the short cavity region, changing of these parameters can be either more effective or less effective in tuning \( f_{c1} \).

Combining the parametric study results on the static sensitivity and the undamped fundamental natural frequency, there is a trade-off between these two metrics in the long cavity region; that is, increasing the bandwidth will sacrifice the sensitivity. On the other hand, reducing the cavity length shorter than the medium region will decrease both the static sensitivity and the fundamental natural frequency. In the short cavity length region, the trade-off between the static sensitivity and fundamental natural frequency can be mitigated. In some instances, where a small static sensitivity is acceptable, such as sensors designed for high pressure measurements, a pressure sensor can be designed to operate in the short cavity region, where the static sensitivity is dominated by the air cavity. In this case, the diaphragm can be designed to be stiffer (e.g., with a higher Young’s modulus or a larger thickness) to increase the sensor bandwidth without sacrificing the static sensitivity.

There are different approaches to address the adverse effects of the air cavity in sensor design. One is to avoid the air cavity effects completely, by sealing the cavity in vacuum. The second approach is to design a sensor with a cavity that is long enough to minimize the air cavity effects. The third approach is to take into account the air cavity effects in the sensor design stage by using the close-form expressions provided in this study and constructively achieve desired sensor performance. For example, with a post-fabricating air cavity structure, the sensitivity and bandwidth of the sensor can be tuned by changing the cavity length in the subsequent bonding and packaging process.

A FEM model is developed to investigate two damping scenarios: one being inviscid, which is used to validate with the analytical model, and the other being viscous, which is used to study the effect of damping. The static sensitivity and natural frequency obtained with the inviscid model agree well with those obtained with the analytical model, and thus validate the static sensitivity and mass effects of the air cavity. It is also found that the viscous FEM model largely conforms to the inviscid analytical model, except when the air cavity length is comparable to or less than the viscous boundary layer thickness (i.e., \( l/a < 0.04 \)). In this case, the mass effect of the air cavity is more pronounced so that the natural frequencies obtained with the viscous model are smaller than those predicted by the inviscid analytical model. Nevertheless, it can be concluded from this study that the analytical inviscid model is adequate to predict the overall trend of static sensitivity and fundamental natural frequency as the cavity length is varied.

The damping ratio with respect to the cavity length \( l \) is obtained by using the viscous FEM model. For \( l/a > 1 \), the damping ratio is found to decrease as the cavity becomes shorter. However, when \( l \) decreases further into the region for \( l/a < 1 \), the damping ratio starts to increase due to the squeeze film damping. Although the inviscid analytical model cannot accurately predict the natural frequency for cavity lengths \( l/a < 0.04 \), from sensor design point of view, there is no benefit to design the cavity length in this region, due to the reduced sensitivity and bandwidth.

It should be noted that the insights gained in this study is not limited to the sensor geometry (circular diaphragm backed by a cylindrical air cavity) or the detection method based on the diaphragm center displacement. For the case of a back cavity with perforated holes, an equivalent cavity length can be calculated. This work can not only further the understanding of the structural–acoustic coupling in a structural-fluid hybrid system, but more importantly, it can provide simple, but effective guidelines for better design of pressure sensors that employ an air backed diaphragm.
Acknowledgments

Supports received from the National Science Foundation (CMMI 0644914) and National Institute of Standards and Technology (70NANB12H211) are gratefully acknowledged.

References


