Radiative damping in waveguide-based ferromagnetic resonance measured via analysis of perpendicular standing spin waves in sputtered permalloy films

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The damping $\alpha$ of the spin-wave resonances in 75, 120, and 200 nm thick permalloy films is measured via vector-network-analyzer ferromagnetic resonance (VNA-FMR) in the out-of-plane geometry. Inductive coupling between the sample and the waveguide leads to an additional radiative damping term. The radiative contribution to the over-all damping is determined by measuring perpendicular standing spin waves (PSSWs) in the permalloy films, and the results are compared to a simple analytical model. The damping of the PSSWs can be fully explained by three contributions to the damping: the intrinsic damping, the eddy-current damping, and the radiative damping. It was not necessary to invoke any additional damping contributions to explain the data. Furthermore, a method to determine the radiative damping in FMR measurements with a single resonance is suggested.

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I. INTRODUCTION

Excited magnetic moments relax towards their equilibrium orientation due to damping. Several physical mechanisms can cause damping. Many mechanisms, such as eddy-current damping [1] in conducting ferromagnets, were already identified in the 1950s. More recently, enhanced damping due to spin pumping [2] from a ferromagnet into an adjacent metallic layer was identified, and remains a topic of ongoing investigation [2–5]. Furthermore, wave-number-dependent contributions to the damping caused by inralayer spin pumping have been theoretically predicted [6,7] and currently are the subject of experimental investigation [8,9]. Another damping process, referred to as radiative damping [10,11], has been known to exist since the 1970s and is purely due to inductive coupling between the sample and a microwave cavity in ferromagnetic resonance (FMR) experiments. More recently, this phenomenon has been further investigated in the context of strong magnon-photon coupling experiments, with possible applications in quantum information processing [12]. In these quantum-coherent experiments, radiative damping was identified as a manifestation of nonresonant magnon-photon coupling, when the FMR is detuned from the cavity resonance, and it was determined that such coupling [13] is indeed a source of extrinsic linewidth in cavity-based FMR studies. When the FMR and cavity resonance are coincident, the inductive coupling results in pumping [2] from a ferromagnet into an adjacent metallic layer that carries the resultant power away in the CPW give rise to two contributions to magnetic damping. Historically, the damping caused by eddy currents in the ferromagnet $\alpha_{\text{eddy}}$ is called eddy-current damping, while the damping caused by the eddy currents in the waveguide is called radiative damping $\alpha_{\text{rad}}$.

Eddy-current damping has been recognized since the 1950s [1,14]. For the lowest order mode in FMR, $\alpha_{\text{eddy}} = \frac{C}{16} \frac{\gamma \mu_0^2 M_s}{\rho} \delta^2$, with resistivity $\rho$, saturation magnetization $M_s$, vacuum permeability $\mu_0$, gyromagnetic ratio $\gamma$, and sample thickness $\delta$ (see the derivation in Appendix D). We introduce a correction factor $C$ to account for details of the eddy-current damping.
spatial profile. As shown in a later section, \( \alpha_{\text{rad}} \) for all higher order PSSW modes investigated in this study is much smaller than that of the lowest order mode.

We now turn to the radiative damping \( \alpha_{\text{rad}} \). We consider the experimental geometry sketched in Fig. 1(a). A ferromagnetic sample with thickness \( \delta \) and length \( L \) is placed on top of the center conductor of a coplanar waveguide with width \( W \). The sample dimension along \( x \) is much larger than \( W \). The sample and CPW are separated by a gap of height \( d \). An external dc magnetic field \( H_0 \) is applied perpendicular to the sample plane, and the spin-wave resonances (SWRs) are driven by microwaves in the CPW at resonance frequency \( \omega \). A fraction of the ac magnetic induction \( B \) due to the dynamic component of the magnetization \( m^y_m(H_0, I; z) \) wraps around the center conductor.

To derive a quantitative expression for \( \alpha_{\text{rad}} \), we start by calculating \( h^y(I; x, z) \), the \( x \) component of the driving field \( h_{\text{mw}} \) that is generated by an excitation current \( I \) in the center conductor. We assume \( h^y(I; x, z) \) is uniform along \( y \), but we allow for variation along \( x \) and \( z \). To estimate \( h^y(I; x, z) \) we use the Karlqvist equation [15],

\[
h^y(I; x, z) = \frac{I}{2\pi W} \left[ \arctan \left( \frac{x + W/2}{z} \right) - \arctan \left( \frac{x - W/2}{z} \right) \right].
\]

This microwave field can excite PSSWs in the sample. Schematic mode profiles for the fundamental mode \( (n = 0) \) and the first three PSSW modes are shown in Fig. 1(b), where we use unpinned boundary conditions at the top surface and pinned boundary conditions at the bottom surface. As shown in Fig. 1(b), the mode profiles describe a \( z \) dependence of the dynamic magnetization components \( m^x \) and \( m^y \). In the perpendicu- lar geometry used here, \( |m^x| = |m^y| \) everywhere, i.e., the precession is circular. In what follows, we will only discuss \( m^y \), the dynamics of which are inductively detected in the measurement. For a PSSW with mode number \( n \), \( m^y_n(x, z) = q_n(z)\chi_n(q_n(z)h^y(I; x, z)) \) where \( \langle \rangle \) denotes spatial averaging in \( x \) and \( z \) directions, as defined in the Appendix, \( \chi_n = \chi_n^{xz} \) is the diagonal component of the magnetic susceptibility of the \( n \)th order mode, and \( -1 \leq q_n(z) \leq 1 \) is the normalized mode profile (eigenmode), an example of which is sketched in Fig. 1(b) for \( n = 3 \). The mode inductance \( L_n \) is given by \( L_n = \chi_n L_0 \), where, as detailed in the Appendix, we define a normalized mode inductance \( \tilde{L}_n \) for the \( n \)th PSSW mode,

\[
\tilde{L}_n = \frac{\mu_0}{T_1} \langle \chi_n \rangle \frac{L_0}{Z_0} \delta.
\]

\( \tilde{L}_n \), as explained in the Appendix, no longer has any dependence on magnetic field or excitation frequency. In the simplest case of a uniform magnetization profile \( q_0(z) = 1 \) (FM mode) and uniform excitation field \( h^y(I; x, z) = h^y(I; 0, 0) = I/(2W) \), the normalized inductance is \( \tilde{L}_n = \mu_0 \delta I/(4W) \).

The \( x \) component of the dynamic magnetization \( m^x_n(x, z) \) produces a net flux \( \Phi_n = \chi_n I \tilde{L}_n \) that threads through CPW center conductor, leading to a power dissipation

\[
P_n = \frac{\omega^2}{2Z_0} \langle \chi_n I \tilde{L}_n \rangle^2,
\]

where \( Z_0 \) is the waveguide impedance (in our case \( Z_0 = 50 \) \( \Omega \)) and \( \omega \) is the angular frequency of the magnetization precession. With Eq. (4), the power dissipation rate \( (P_n/E_n) = P_n/E_n \) can be calculated, where \( E_n \) is the energy of the dynamic component of the magnetization derived in the Appendix. This power flow from the sample to the waveguide leads to the radiative damping contribution

\[
\alpha_{\text{rad}} = \frac{1}{2\omega} \left( \frac{1}{T_1} \right) = \eta \gamma \mu_0 M_s \tilde{L}_n / Z_0,
\]

where \( \eta = \delta/(4 \int_0^d dz |q_n(z)|^2) \) is a dimensionless parameter that accounts for the actual mode profile in the sample; see Appendix Sec. A. In the case of sinusoidal PSSWs, \( \eta = 1/2 \), and for a completely uniform mode profile, i.e., \( q_n(z) = 1 \), \( \eta = 1/4 \). From Eq. (5), it is evident that \( \alpha_{\text{rad}} \) is proportional to \( \tilde{L}_n \) for \( n > 0 \). In the simplest case of uniform driving field \( h^y = I/(2W) \), the radiative contribution is given by

\[
\alpha_0 = \frac{\eta \gamma \mu_0 M_s |\tilde{L}_0|}{Z_0} = \frac{\eta \gamma M_s Z_0}{2Z_0 W} \delta I.
\]
Note that the radiative damping thus depends on the sample and waveguide dimensions, in particular linearly on the sample thickness. Unlike eddy-current damping, $\sigma^\text{rad}$ is independent of the conductivity of the ferromagnet, hence this damping mechanism is also operative in ferromagnetic insulators.

### III. SAMPLES AND METHOD

We deposit Ta(3)/Py(δ)/Si$_3$N$_4$(3), Ta(3)/Py(δ)/Ta(5), Ta(3)/Py(δ), and Py(δ) layers on 100 $\mu$m thick glass substrates by dc magnetron sputtering at a Ar pressure of 0.7 Pa ($\approx 5 \times 10^{-3}$ Torr) in a chamber with a base pressure of less than $5 \times 10^{-6}$ Pa ($\approx 4 \times 10^{-3}$ Torr); where $\delta = 75$ nm, 120 nm and, 200 nm is the permalloy thickness. The Py thickness was calibrated by x-ray reflectivity. We estimate that the damping enhancement due to spin pumping into the Ta layer is two orders of magnitude smaller than the intrinsic damping of the permalloy layer for permalloy samples of thickness $\ll 5$ $\mu$m.

Alternatively if we assume Neumann boundary conditions (completely pinned), only odd modes would be detected. A uniform excitation field and Dirichlet boundary conditions (completely unpinned), only the fundamental mode would be detected.

Two effects can contribute to our ability to detect all the PSSW modes. First, the excitation field profile might not be uniform due to eddy-current shielding [20,21]. Second, the interfacial boundary conditions might be asymmetrical, as alluded to above. According to the criterion in Ref. [21], the threshold sheet resistance for the onset of eddy-current shielding at 20 GHz is 0.065 $\Omega$/sq. We estimate that the sheet resistance for our 200 nm is in excess of 0.345 $\Omega$/sq, so we conclude that the eddy current shielding is relatively weak for our samples.

On the other hand, all modes are in principle detectable if we assume asymmetric interfacial anisotropy. For the sake of simplicity of the analysis, we will assume interfacial anisotropy, $H_n^{\text{ex}}$ is the exchange field (defined below), and $\Delta H_s$ is the linewidth. An example of the resulting fits for the complex $S_{21}$ data is shown in Figs. 2(a) and 2(b).

![FIG. 2. (Color online) Measured $S_{21}$ transmission parameter (black circles) at 20 GHz and the multipeak-susceptibility fit (red line) for the (a) real part and (b) imaginary part obtained with the Ta(3)-Py(200)-Si$_3$N$_4$(3) sample. The first six modes are shown. (c) The exchange field $H_n^{\text{ex}}$ (black squares) and exchange field fit, from Eqs. (9) and (10) (red crosses) for all 13 detected modes plotted as a function of the fitted wave numbers $k_n$.](184417-3)
Here, $k_n$ is the spin-wave wave vector, and $A_{\text{ex}}$ is the exchange energy that is related to the spin-wave stiffness $D$ via $D = \frac{2 A_{\text{ex}} g \mu_B}{2f}$. On the other hand, if we want to include interfacial anisotropy for a single interface in our analysis, we can numerically solve the transcendental equation [22]

$$\left( \frac{1}{2} k_n a + \frac{K_s}{2 A_{\text{ex}} k_n} + 1 \right) \tan(k_n \delta) = \frac{K_s}{2 A_{\text{ex}} k_n}, \quad (10)$$

where $K_s$ is the interfacial anisotropy, and $a = 0.3547 \text{ nm}$ is the lattice constant [23]. We minimize the residue of the fit of Eq. (9) to $H_{\text{ex}}^n$ with the fitting parameters $M_s$, $A_{\text{ex}}$, and $K_s$ from Eq. (10) by use of a Levenberg-Marquardt optimization algorithm. This yields the pairs $(k_n, H_{\text{ex}}^n)$ shown in Fig. 2(c) for all modes.

From the fit, we obtain a saturation magnetization of $\mu_0 M_s = 1.02 \pm 0.01 \text{ T}$, in agreement with that determined by magnetometry. The exchange stiffness constant of $D = 3.22 \pm 0.04 \text{ meV nm}^2$ is close to a value of $D \approx 3.1 \text{ meV nm}^2$ reported by Maeda et al. [24].

The exchange fit also yields a single surface anisotropy $K_s$ that depends on the cap and seed layer configurations. For the Ta(3)-Py(5)-Si$_3$N$_4$(3) sample series, $K_s = (5.1 \pm 0.8) \times 10^{-4} \text{ J/m}^2$, while all the other samples have a higher $K_s$ of $(7 \pm 1) \times 10^{-4} \text{ J/m}^2$. All values for $K_s$ are in the range of other reported interface anisotropies for permalloy layers of these thicknesses [25].

We now turn to the linewidth $\Delta H_n$ and the amplitude $A_n$ for the individual modes. The Gilbert damping parameter $\alpha_n$ is extracted from the slope of the linewidth vs frequency $f$ plot [9] shown in Fig. 3(a) via

$$\Delta H_n = \frac{4 \pi \alpha_n f}{|\gamma| \mu_0} + \Delta H^0_n, \quad (11)$$

where $\Delta H^0_n$ is the inhomogeneous broadening that gives rise to a nonzero linewidth in the limit of zero frequency excitation. The normalized inductance of the modes $\tilde{L}_n$ is extracted in a similar fashion from the dependence of the mode amplitude $A_n$ on the frequency $f$; see Fig. 3(b) and Eq. (A24) in the Appendix:

$$A_n = 2 \pi f \frac{\tilde{L}_n}{Z_0} + A^0_n, \quad (12)$$

where $A^0_n$ is an offset for each mode. $A^0_n$ is a phenomenological fitting parameter, which is not yet fully understood.

We plot $\alpha_n$ and $\tilde{L}_n$ as a function of mode number $n$ in Fig. 3(c). The damping and the normalized inductance are found to be proportional. In order to explore this correlation, we plot $\alpha_n$ vs $\tilde{L}_n$ in Fig. 4(a). Here, the data for $\alpha_n$ vs $\tilde{L}_n$ are linearly correlated for all modes except for $n = 0$, as seen by the linear fit (line) to the data for $n \geq 1$. This is as expected for the radiative damping model, as summarized in Eq. (5). The additional damping of the fundamental mode is interpreted as the result of eddy current damping, as quantified in Eq. (1). In Fig. 4(b), we plot the residual $\Delta \alpha_n$ of the linear fit shown in Fig. 4(a) for all modes. $\Delta \alpha_n$ is negligible for all modes except for $n = 0$. We extract $\Delta \alpha_{n=0}$ for all the samples and plot $\Delta \alpha_{n=0}$ vs $\delta^2$ in Fig. 4(c).

It appears that $\Delta \alpha_{n=0}$ for all the samples scales linearly with $\delta^2$, as expected from Eq. (1) for eddy current damping.

V. EXTRACTION OF THE RADIOACTIVE CONTRIBUTION TO THE DAMPING

By use of Eq. (1) and our fitted value of $C = 0.4$, we subtract the eddy current contribution to the damping of all the $n = 0$ modes to obtain a corrected damping value $\alpha'_n|_{n=0}$, where $\alpha'_n|_{n=0} = \alpha_{n=0} - \alpha_{\text{eddy}}(C = 0.4)$. The corrected data for all the modes are plotted in Fig. 5.

Figures 5(a)–5(c) group all data obtained for a set of samples with identical Py thickness $\delta$. The lines are linear fits to Eq. (5). For each thickness $\delta$, we observe a significant correlation of $\alpha_n$ and $\tilde{L}_n$ for all seed and cap layer configurations, as expected for a radiative damping mechanism.

Furthermore, by use of Eq. (6) for the $n = 0$ mode of the 75 nm thick sample, using a value of $\eta \approx 0.46$ as determined in the Appendix, we estimate $\alpha_{\text{eddy}}^0 \approx 0.00023$
The experimentally determined value is $\alpha^{\text{rad}}_0 \approx 0.00035 \pm 0.0001$. The deviation from the calculated value is possibly due to nonuniformities of both the excitation field and magnetization profile in Eq. (6), that requires the solution of the integral in Eq. (A11). Nevertheless the estimated value for $\alpha^{\text{rad}}_0$ is of the correct order of magnitude.

We determine the intrinsic damping $\alpha^{\text{int}}$ from the $\bar{L}_n = 0$ intercept of the linear fits in Fig. 5. We plot $\alpha^{\text{int}}$ for the three values of $\delta$ in Fig. 6 (right scale). We find that $\alpha^{\text{int}}$ is approximately constant to within $\pm 5\%$ for all samples. In addition the average value over all the film thicknesses is in reasonable agreement with the previously reported value of $\alpha^{\text{int}} = 0.006$ (dotted red line) [27].

The other fitting parameter $\eta$, extracted from the slope of $\alpha_n$ vs $\bar{L}_n$, as an average of all cap and seed layer configurations for one thickness, is also plotted in Fig. 6 (left scale). The fitted $\eta$ displays robustness towards variation of the interface conditions. For antisymmetric boundary conditions, $\eta = 1/2$ is expected, whereas for the uniform mode, $\eta = 1/4$.

We see that the fitted values lie exclusively within these extremes, within error bars. The dependence of $\eta$ on $\delta$ still requires further investigation, but is beyond the scope of this work.

There have been recent reports of a nonzero, wave-number-dependent component of damping for both localized eigen-modes in magnetic nanostructures [9] and PSSWs in thick permalloy films [8]. Such exchange-mediated damping of the form $\alpha^{\text{ex}} := A_{\text{ex}} k^2$ was originally predicted by Baryakhtar based on symmetry alone [7]. Nembach et al., [9], obtained a value of $A_{\text{ex}} = 1.4 \text{ nm}^2$, whereas Li et al. [8], found a much smaller value of 0.09 nm$^2$. To determine whether wave-number-dependent damping is apparent in our data, we examined the residual damping after subtraction of both the intrinsic damping $\alpha^{\text{int}}$ and the radiative damping $\alpha^{\text{rad}}_0$ from all the modes, as well as subtraction of the eddy current damping from the $n = 0$ mode. The residual damping $\alpha^{\text{res}}$ is plotted in Fig. 7(b). Within the scatter of $\Xi \pm 0.001$, $\alpha^{\text{res}}$ does not have any clear dependence on $k$. Thus, we obtain an upper bound of $A_{\text{ex}} \leq 0.045 \text{ nm}^2$ for this particular system, given the sensitivity of our measurements. For comparison, and to ensure that the subtraction of $\alpha^{\text{int}}$, $\alpha^{\text{rad}}_0$, and $\alpha^{\text{eddy}}$ did not hide a potential $k^2$ contribution, the measured damping of the Ta(3)-Py(200)-Si$_3$N$_4$(3) sample up to the $n = 10$ mode is shown in Fig. 7(a). For $n \geq 5$ the measured damping scatters around the determined intrinsic damping $\alpha^{\text{int}}$ for the 200 nm samples and no trend for higher mode numbers (larger $k$ values) is discernible.

Tserkovnyak, et al., calculated the damping coefficient $A_{\text{ex}}$ in terms of a microscopic model for the diffusive transport of dissipative transverse spin current within a ferromagnetic metal [6]. The theory in Ref. [6] framed the exchange-mediated damping in terms of a so-called transverse spin conductivity $\sigma_{\perp}$,

$$A_{\text{ex}} = \left( \frac{\gamma}{M_s} \right) \left( \frac{\hbar}{2e} \right)^2 \sigma_{\perp},$$

(13)
dependence of the residual damping on

The residual damping for all detected modes for all samples is

\[ \alpha \] 

function of Py thickness \( \delta \). The intrinsic damping is close to

\[ \eta \] 

conditions (dashed black line) and the value of 1

In another experiment, we further validate the presence of radiative damping and demonstrate an alternative method to determine \( \alpha_{\text{rad}} \) by varying the distance \( d \) in Eq. (2) between the sample and waveguide. To this end, we insert a \( d = 200 \, \mu \text{m} \) glass spacer between the sample and waveguide. By comparing \( h(0,0) \) to \( h(0,200 \, \mu \text{m}) \) via Eq. (2), we estimate that the insertion of the spacer decreases the microwave magnetic field by about a factor of 6.25. Referring to Eq. (3), the normalized mode inductance \( \tilde{L}_n \) decreases by a factor of \( \approx 40 \). To determine the effect of the reduced inductive coupling on the radiative damping, we used VNA-FMR to measure the first four modes for the Ta(3)-Py(120) sample with and without the spacer. The effect of the spacer can be seen in the raw data, reducing the linewidth of the first two modes measured at 10 GHz in the 120 nm samples by approximately 0.6 mT, well outside error bars. The fitted values of \( \tilde{L}_n \) are shown in

FIG. 6. (Color online) Mode profile parameter \( \eta \) (black squares, left axis) and intrinsic damping \( \alpha_{\text{int}} \) (red circles, right axis) as a function of Py thickness \( \delta \). The mode profile parameter \( \eta \) lies between the value of 1/2 for sinusoidal PSSWs with antisymmetric boundary conditions (dashed black line) and the value of 1/4 for the uniform mode (both dashed black lines). The intrinsic damping is close to \( \alpha_{\text{int}} = 0.006 \) (dotted red line).

\[
\sigma_\perp := \left( \frac{\sigma}{\tau} \right) \left( 1 + \frac{\tau_\perp}{\tau} \right)
\]

(14)

with the exchange splitting \( h_{\text{ex}} \), the conductivity \( \sigma \), the spin scattering time \( \tau \), and transverse spin scattering time \( \tau_\perp \). Given that \( h_{\text{ex}} \approx 1 \, \text{eV} \) for permalloy, the maximum value for \( A_{\text{ex}} \) predicted by the transverse spin current theory is 0.001 nm\(^2\). Insofar as we are not able to observe any such wave-number-dependent damping down to the level of 0.045 nm\(^2\), our results are consistent with the predictions of the microscopic theory.

While the theory in Ref. [6] is specific to the microscopic mechanism of transverse spin accumulation in a metallic ferromagnet, the phenomenology of exchange mediated damping, as described in Ref. [7], is not limited to such a microscopic mechanism. As such, it remains plausible that extrinsic material-specific parameters that have not yet been identified could be responsible for the previously reported values for \( k^2 \) damping. For example the presence of anti-symmetric exchange at interfaces, i.e., the Dzyaloshinskii-Moriya interaction (DMI) could enhance the coupling between magnons and Stoner-excitations insofar as the DMI gives rise to exotic spin textures [28] with nanometer length scales, that are comparable to the wavelength of low-energy Stoner excitations [29]. Thus, the results of Ref. [9] could be a manifestation of interfacial enhancement for \( A_{\text{ex}}^2 \), insofar as the magnetic films used in Ref. [9] are only 10 nm thick.

In another experiment, we further validate the presence of radiative damping and demonstrate an alternative method to determine \( \alpha_{\text{rad}} \) by varying the distance \( d \) in Eq. (2) between the sample and waveguide. To this end, we insert a \( d = 200 \, \mu \text{m} \) glass spacer between the sample and waveguide. By comparing \( h(0,0) \) to \( h(0,200 \, \mu \text{m}) \) via Eq. (2), we estimate that the insertion of the spacer decreases the microwave magnetic field by about a factor of 6.25. Referring to Eq. (3), the normalized mode inductance \( \tilde{L}_n \) decreases by a factor of \( \approx 40 \). To determine the effect of the reduced inductive coupling on the radiative damping, we used VNA-FMR to measure the first four modes for the Ta(3)-Py(120) sample with and without the spacer. The effect of the spacer can be seen in the raw data, reducing the linewidth of the first two modes measured at 10 GHz in the 120 nm samples by approximately 0.6 mT, well outside error bars. The fitted values of \( \tilde{L}_n \) are shown in

FIG. 7. (a) The measured damping for the first 11 PSSW modes of the Ta(3)-Py(200)-Si\(_3\)N\(_4\)(3) sample. The enhanced damping due to inductive coupling to the waveguide and eddy currents in the sample only affects the first five modes at wave vectors \( \leq 7 \times 10^6 \text{ cm}^{-1} \). (b) The residual damping for all detected modes for all samples is plotted against their respective wave vector \( k \). Within the scatter, no dependence of the residual damping on \( k \) is observed.

FIG. 8. (Color online) Measurement of the first four PSSWs of the Ta(3)-Py(120) sample with and without a spacer inserted between sample and CPW. (a) Inductance \( \tilde{L}_n \) determined for the sample directly on the CPW (black squares) and for a 200 µm spacer between sample and CPW (red circles). (b) The resulting damping constants for both measurements (same symbols and colors). The red line is the previously extracted intrinsic damping \( \alpha_{\text{int}} \). (c) The difference between the damping with and without the spacer (black squares) is in good agreement with the radiative damping from Fig. 5(b) (gray line).
Fig. 8(a). Indeed, $\bar{L}_n$ decreases on average for all modes by a factor of $\approx 50$ after inserting the spacer, in good agreement with the predictions of Eqs. (2) and (3). Thus, we will assume that $\alpha_n^{rad}$ is negligible when the spacer is used. The data for the damping $\alpha_n$ of the first four modes, both with and without the spacer, are plotted in Fig. 8(b). Indeed, the damping determined from the measurement with the spacer layer (circles) is consistently lower than that found without the spacer layer (squares). The line in Fig. 8(b) is the previously determined intrinsic damping. Under the assumption that the radiative damping contribution is given by $\alpha_n^{rad} = \alpha_n(d = 0) - \alpha_n(d = 200 \mu m)$, we plot $\alpha_n^{rad}$ vs $L_n(d = 0)$ in Fig. 8(c). The line is the calculated $\alpha_n^{rad}$, where we used Eq. (5) with $\eta = 0.35$ and $\delta = 120 nm$, as determined from the fits in Fig. 5. Good agreement between the calculated and measured values for $\alpha_n^{rad}$ are obtained, which demonstrates the self-consistency of our analysis. Of great importance is that the spacer-layer approach can also be used to determine the radiative contribution to the damping in the absence of PSSWs (single resonance). By measuring $\alpha$ for varying distance $d$ between sample and waveguide and extrapolating $\alpha$ to $d \to \infty$, both the intrinsic value for the damping and the radiative contribution can be determined, under conditions where eddy-current damping is negligible.

VI. SUMMARY

In summary, we identified three contributions to the damping in PSSWs: intrinsic damping $\alpha_n^{int}$, eddy-current damping $\alpha_n^{edd}$, and radiative damping $\alpha_n^{rad}$. The latter exhibits a linear dependence on the normalized sample inductance $\bar{L}_n$ in a waveguide based FMR measurement. We attribute this linear dependence to radiative losses that stem from the inductive coupling between the sample and the waveguide. The radiative damping term is inherent to the measurement process and is thus present in all FMR measurements. The radiative damping constitutes up to 40% of the total damping of the spin-wave modes in our 200 nm thick permalloy films. Furthermore, the radiative damping can be already important for much lower film thicknesses, in materials with small intrinsic damping.

As an example, the radiative damping calculated from Eq. (6) for a 20 nm thick and 1 cm long sample of yttrium-iron-garnet (YIG), measured on a 100 $\mu m$ wide guide is $\alpha_n^{rad} \approx 1.26 \times 10^{-4}$. When compared to the reported value for the damping of $\alpha = 2.3 \times 10^{-4}$ [30], we see that the radiative part of the damping, among others [31], can substantially influence the determination of $\alpha_n^{int}$. As such, careful analysis of $\alpha$ vs inductance is required to isolate the radiative damping contribution.

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APPENDIX A: DERIVATION OF $\alpha_n^{rad}$

In this section, we derive model equations for the normalized mode inductance $\bar{L}_n$, mode amplitude $A_n$ and the radiative damping $\alpha_n^{rad}$. We are restricting our analysis to the case of ideal perpendicular standing spin-wave modes that only vary through the film thickness without any lateral variation. It is assumed that the excitations are in the perpendicular geometry with the magnetization saturated out of the film plane. As such, the response to the $z$-coordinate component of the microwave excitation field above the waveguide can be neglected. In addition, the magnetization precession is always circular. As such, the magnetization dynamics in the $x$ and $y$ coordinates in response to the microwave field generated by the waveguide are degenerate, outside of a phase factor of $\pi/2$. This eliminates the need to explicitly consider the full Polder susceptibility tensor in the calculation of the sample response to the excitation field. The sample dimensions are $l$ along the waveguide direction, $\delta$ in thickness, but infinite in the lateral direction.

We begin by introducing the concepts of a spin-wave mode susceptibility $\chi_n$, and the dimensionless, normalized spin-wave amplitude $q_n(z)$ for the $n$th spin-wave mode, such that the magnetic excitation of amplitude in the $x$ direction $\tilde{m}_n^x(H_0, I; z)$ that results from the application of a microwave magnetic field of amplitude $h^x(I; x,z)$, given in Eq. (2), driven by an ac current $I = V_{ac}/Z_0$ in an applied field $H_0$ is given by

$$\tilde{m}_n^x(H_0, I; z) := \tilde{m}_n^x(z) = q_n(z)\chi_n(A_n)(q_n(z)h^x(I; x,z)), \quad (A1)$$

where the quantity in brackets is simply the overlap integral of the excitation field and the spatial profile of the $n$th spin-wave mode. The magnetic excitation of amplitude in the $y$ direction $\tilde{m}_n^y(z)$ can be written in a similar way. In the trivial case of a uniform excitation field and uniform spin-wave mode, we recover the usual relation between the excitation field and the magnetization dynamics via the Polder susceptibility tensor component, $\chi^{xx}$. However, if the product of the mode profile and excitation field has odd spatial symmetry, dynamics are not excited, as we expect. The overlap integral is nothing more than the spatial average of the mode-excitation-product:

$$\langle q_n(z)h^x(I; x,z) \rangle = \frac{1}{W} \int_{-\infty}^{\infty} dx \int_{-d}^{d} dz q_n(z)h^x(I; x,z). \quad (A2)$$

First, the power transferred to the waveguide via inductive coupling with the spin-wave dynamics is given by

$$P_n = \frac{|\partial_t \Phi_n(H_0, I)|^2}{2Z_0}, \quad (A3)$$

where

$$\partial_t \Phi_n(H_0, I) = \mu_0 \ell \int_{-\infty}^{\infty} dx \int_{-d}^{d} dz [\partial_t \tilde{m}_n^y(z)]h^x(I; x,z). \quad (A4)$$

It is important to recognize at this point that the power dissipation is not constant with time, given that $P_n$ is proportional only to $\partial_t m_n^y$. As such, the damping associated with the re-radiation of the microwave energy back into the waveguide is best characterized with an anisotropic damping tensor, to be elaborated upon more fully later in this appendix. To calculate the energy of the spin-wave mode, we start by
defining a spatially averaged spin-wave excitation density \[32],

\[
\langle \mu_n^2(H_0, I) \rangle = \frac{\int_{-\infty}^{\infty} dx \int_{-d}^{d} dz \left[ \partial_t m_n^z(z) \right] \left[ m_n^z(z) \right]^* - \left[ \partial_t m_n^z(z) \right] \left[ m_n^z(z) \right]^*}{4 \omega \delta W}.
\]  

We can then calculate the magnon density \( N_n \) associated with the \( n \)th spin wave excitation as

\[
N_n = \frac{\langle \mu_n^2(H_0, I) \rangle}{2 \mu_0 M_s}.
\]  

The total energy associated with the spin-wave mode is given by

\[
E_n = \frac{\omega \mu_0 \langle \mu_n^2(H_0, I) \rangle}{\gamma M_s} \delta W.
\]  

The energy dissipation rate \((1/T_1)_n\) for the \( n \)th mode is therefore

\[
\left( \frac{1}{T_1} \right)_n = \frac{P_n}{E_n} = \frac{2 \mu_0 \ell \omega M}{Z_0} \int_{-\infty}^{\infty} dx \int_{-d}^{d} dz \left[ \partial_t \hat{m}_n^x(z) \right] \left[ \hat{h}^x(x, z) \right]^2 + \int_{-\infty}^{\infty} dx \int_{-d}^{d} dz \left[ \partial_t \hat{m}_n^z(z) \right] \left[ \hat{h}^z(x, z) \right]^2 + \int_{-\infty}^{\infty} dx \int_{-d}^{d} dz \partial_t \hat{m}_n^y(z) \left[ \hat{h}^y(x, z) \right]^2.
\]  

where \( \omega_M = \gamma \mu_0 M_s \). We then apply the Fourier transform to move into the frequency domain, where \( \partial_t m_n^z(H_0, I; z) \leftrightarrow i \omega n^z(H_0, I; z) \), such that the energy relaxation rate \((1/T_1)_n\) for magnetization oscillations along the \( x \) axis is

\[
\left( \frac{1}{T_1} \right)_n^x = \frac{\omega \mu_0 \ell \omega M}{Z_0} K_n,
\]  

where

\[
K_n := \frac{\int_{-\infty}^{\infty} dx \int_{-d}^{d} dz \left[ \hat{m}_n^z(z) \right] \left[ \hat{h}^x(x, z) \right]^2}{\epsilon \int_{-\infty}^{\infty} dx \int_{-d}^{d} dz \left[ \partial_t \hat{m}_n^x(z) \right] \left[ \hat{q}_n(z) \right]^2},
\]  

with \( \epsilon = |\hat{m}_n^z|/|\hat{m}_n^x| \).  

Since the energy dissipation rate for the case of radiative damping is anisotropic, it must be generally treated in the damping tensor formalism, where the Gilbert damping torque \( \hat{T} \) is given by

\[
T_k = \epsilon_{ijk} \alpha_{t ij} \hat{n}_t \left( \partial_t \hat{m}_n \right)_j.
\]  

The equation of motion is

\[
\partial_t \hat{m} = -\gamma \mu_0 \hat{m} \times \hat{H} + \hat{T}
\]  

and \( \hat{m} = \vec{M}/M_s \) is the normalized magnetization. For the coordinates in Fig 1, the only nonzero radiative damping tensor components are \( \alpha_{zz} \) and \( \alpha_{yy} \). For the perpendicular FMR geometry, the relationship between the energy relaxation rate and the Gilbert damping components is

\[
\left( \frac{1}{T_1} \right)_n^x = \alpha_{xz} \omega_x,
\]  

and

\[
\left( \frac{1}{T_1} \right)_n^y = \alpha_{yz} \omega_y,
\]  

where \( \omega_x \) and \( \omega_y \) are the respective stiffness frequencies, defined as

\[
\omega_i := \gamma \partial^2 U_m M_s \partial \hat{n}_i
\]  

and \( U_m \) is the magnetic free energy function. The frequency-swept linewidth \( \Delta \omega = \gamma \mu_0 \Delta H \), where \( \Delta H \) is the field-swept linewidth in Eq. (11), is given by

\[
\Delta \omega = \frac{(\frac{1}{T_1})^x + (\frac{1}{T_1})^y}{2} = \alpha_{zz} \omega_x + \alpha_{yy} \omega_y.
\]  

For perpendicular FMR, \( \omega_x = \omega_y = \omega \), and the specific case of anisotropic radiative damping, \( \alpha_{zz} = \alpha^{rad}_n \), \( \alpha_{yy} = 0 \), and we obtain

\[
\alpha^{rad}_n = \frac{1}{2 \omega} \left( \frac{1}{T_1} \right)_n^x = \frac{\mu_0 \ell \omega M}{2 Z_0} K_n
\]  

and \( \Delta \omega^{rad} = \alpha^{rad}_n \omega \). This is in contrast to the case of isotropic damping processes, such as eddy currents and intrinsic damping, where we obtain \( \Delta \omega^{iso} = 2 \alpha^{iso}_n \omega \) instead. Thus, the net damping due to the sum of anisotropic radiative damping, and any other isotropic processes, is given by

\[
\alpha_n = \alpha^{rad}_n + \alpha_n^{iso} + \alpha^{iso}_n \frac{\omega}{2}
\]  

where \( \alpha_n \) is the damping parameter in Eq. (11) for the field-swept linewidth.

We use a vector network analyzer (VNA) to measure the two-port \( S \)-parameter matrix element for the \( n \)th spin-wave mode, \( \Delta S_n^{21} \). The matrix element is defined as the ratio of the voltage induced in the waveguide by the \( n \)th spin-wave mode \( V_n(H_0) \) in an applied magnetic field \( H_0 \), and the excitation voltage \( V_m \).

\[
\Delta S_n^{21} := \frac{V_n(H_0)}{V_m}.
\]
If we model the reactance of the $n$th spin-wave mode as nothing more than a purely inductive element of inductance $L_n$ in series with an impedance matched transmission line, and if we assume the sample inductance is much smaller than the transmission line impedance, we can approximate $\Delta S_n^{\text{21}}$ as

$$\Delta S_n^{\text{21}}(H_0) \cong -\frac{i\omega L_n(H_0)}{Z_0}, \quad \text{(A22)}$$

where $L_n(H_0) = \Phi_n(H_0, l)/l$.

We define a normalized, field-independent mode inductance $\tilde{L}_n$ as

$$\tilde{L}_n := \frac{L_n(H_0)}{\lambda_n(H_0)}, \quad \text{(A23)}$$

and a dimensionless, field-independent mode-amplitude $A_n$,

$$A_n := \frac{i\omega \tilde{L}_n}{Z_0}, \quad \text{(A24)}$$

such that

$$\Delta S_n^{\text{21}}(H_0) = -A_n \lambda_n(H_0). \quad \text{(A25)}$$

Thus, $A_n$ is the dimensionless amplitude parameter that we obtain when fitting data for $\Delta S_n^{\text{21}}(H_0)$. By use of Eqs. (A4), (A24), and (A23), we can rewrite the mode amplitude as

$$A_n := \frac{i\omega \mu d}{W_0} \left( \int_{-\delta}^{\delta} dx \int_{-\delta}^{\delta} dz q_n(z) \delta(x, z) \right)^2. \quad \text{(A26)}$$

Remembering that the normalized mode inductance has a factor identical to the numerator of Eq. (A11), we can rewrite the radiative damping in terms of the normalized mode inductance,

$$\frac{\alpha_n^{\text{rad}}}{\tilde{L}_n} = \frac{i\omega M \eta_n}{Z_0}, \quad \text{(A27)}$$

where

$$\eta_n := \frac{\delta}{4} \int_{-\delta}^{\delta} dz |q_n(z)|^2. \quad \text{(A28)}$$

We emphasize that Eq. (A27) is a very general result, regardless of the details of the excitation field profile. Thus, even if the field profile is highly nonuniform due to the combination of eddy-current and capacitive coupling effects [21,33], there should still be a fixed scaling between the radiative damping and the normalized inductance.

In the case of the uniform mode, $\eta = 1/4$ and Eq. (A27) reduces to

$$\frac{\alpha_n^{\text{rad}}}{\tilde{L}_n} = \frac{i\omega M}{4Z_0}. \quad \text{(A29)}$$

However, for a sinusoidal mode of the form

$$q_n(z) = \cos \left( \frac{(2n + 1)\pi z}{2\delta} \right) \quad \text{(A30)}$$

that is expected in the case of a pinned boundary condition at one interface and an open boundary condition at the other interface, as shown in Fig. 1(b), we obtain $\eta = 1/2$ and

$$\frac{\alpha_n^{\text{rad}}}{\tilde{L}_n} = \frac{i\omega M}{2Z_0}. \quad \text{(A31)}$$

For the case of the wave-number values extracted from the data shown in Fig. 2(c) for a 200 nm Py film, we can determine the value for $\eta_n$ and the degree to which it can vary with mode number. We use the form for the spin-wave profile

$$q_n(z) = \cos(k_n z), \quad \text{(A32)}$$

consistent with our assumption, when extracting $k_n$ from our PSSW data, that an unpinned boundary condition applies to only one of the interfaces, i.e., at $z = 0$. Using these extracted values for the wave number, we obtain values for $\eta_n$ shown in Fig. 9.

We see in Fig. 9 that the variation in $\eta_n$ with varying mode number is less than 10%. Thus, to within first order, we can treat $\eta_n$ as a constant for the purposes of fitting our data, i.e., $\eta_n \equiv \eta$.

**APPENDIX B: DERIVATION OF $\alpha^{\text{eddy}}$**

For the derivation of the eddy-current damping $\alpha^{\text{eddy}}$ uniform magnetization dynamics are assumed. The notation stays the same as for the radiative damping.

Then the total flux passing through the magnetic film is

$$\hat{\partial}_t \Phi = \mu_0 \delta(\hat{\partial}_t \vec{m}), \quad \text{(B1)}$$

where $(\hat{\partial}_t \vec{m})_x = \hat{x} \cdot \hat{\partial}_t \vec{m}$. The electrical power dissipated by the eddy currents is

$$P_{\text{eddy}} = \frac{1}{2} \frac{|\hat{\partial}_t \Phi|^2}{Z_0} \left( \frac{2\mu_0}{\delta \omega W} \right) \quad \text{(B2)}$$

$$= \frac{C \mu_0^2 \delta^3 \mu_0 W}{8 \rho} |(\hat{\partial}_t \vec{m})_x|^2, \quad \text{(B3)}$$

with $\delta_{\text{eff}} := C\delta/2$, where $0 \leq C \leq 1$ is a phenomenological parameter that accounts for details of the nonuniform eddy-current distribution in the ferromagnet. Analogous to the derivation of the radiative damping, we now need the energy of the magnetic excitations. The number of magnons in the
system is given by

$$N_{\text{mag}} = \frac{M_s}{g \mu_B \omega^2} |\delta \vec{m}|^2. \quad (B4)$$

Thus, the total magnon energy is

$$E_{\text{mag}} = \hbar \omega N_{\text{mag}} W \delta$$

$$= \frac{M_s}{\gamma \omega} |\delta \vec{m}|^2 W \delta. \quad (B6)$$

The rate of energy dissipation is then given by

$$\frac{1}{T_1} = \frac{P_{\text{ind}}}{E_{\text{mag}}} = \frac{C \gamma \omega \mu_0^2 M_s \delta^2 |\delta \vec{m}|^2}{8 \rho} \frac{1}{|\delta \vec{m}|}. \quad (B7)$$

The maximum energy decay rate occurs when $$|\delta \vec{m}| = |\delta \vec{m}|$$, in which case

$$\left( \frac{1}{T_1} \right)^x = \frac{C \gamma \omega \mu_0^2 M_s \delta^2}{8 \rho}, \quad (B8)$$

where the superscript indicates that this is the maximum decay rate for magnetization oscillations along the x axis. For the case of a perpendicular applied field sufficient to saturate the static magnetization out of the film plane, the damping process is isotropic, i.e.,

$$\left( \frac{1}{T_1} \right)^y = \frac{C \gamma \omega \mu_0^2 M_s \delta^2}{8 \rho}. \quad (B9)$$

Therefore, analogous to Eq. (A16), the frequency-swept linewidth $$\Delta \omega$$ is simply

$$\Delta \omega = \left( \frac{1}{T_1} \right)^x + \left( \frac{1}{T_1} \right)^y \quad (B10)$$

$$= 2 \alpha_{\text{eddy}} \omega, \quad (B11)$$

$$\alpha_{\text{eddy}} = \frac{C \gamma \omega \mu_0^2 M_s \delta^2}{16 \rho}. \quad (B12)$$