Improved spectral aberration in Johnson Noise Thermometry

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Abstract — Spectral aberration has been the main source of uncertainty in Johnson Noise Thermometry approach to measuring the Boltzmann constant. Recently, with newly developed hardware and the introduction of a novel fitting algorithm for analyzing the data, we have achieved a frequency independent spectral aberration for measurements with the NIST JNT system. Consequently, we performed a data drift analysis and an electromagnetic interference investigation to explain a residual offset that affects the Boltzmann constant.

Index Terms — Boltzmann equation, Josephson junction, measurement units, noise measurement, standards, temperature measurement.

I. INTRODUCTION

Johnson noise is the thermal fluctuation of the electrons in an electrical resistor. The Nyquist relation describes this phenomenon such that the time-averaged mean-square voltage \( \langle V^2 \rangle \) across a resistance \( R \) is

\[
\langle V^2 \rangle = 4kTR\Delta f.
\]

Here \( k \) is the Boltzmann constant, \( T \) is the resistor temperature, and \( \Delta f \) is the measurement bandwidth.

Since 2002, NIST has been operating and developing Johnson Noise Thermometry systems based on a Quantum Voltage Noise Source (QVNS) [1]. These systems allow a direct electronic method to determine the Boltzmann constant when they are optimized to measure the noise of a sense resistor at the triple point of water (TPW).

A cross-correlation measurement is necessary to minimize the effect of uncorrelated noise due to voltage amplifiers. The voltage noise of each source, namely, the QVNS and the sense resistor, is alternatively measured for 100 s with two low-noise amplifier channels, from which is calculated the auto- and cross-correlation.

II. FITTING SELECTION MODEL

The major system limitation has been the bandwidth dependence of the ratio of the measured power spectral density of the two noise sources. For comparison, we calculate the “offset” \( (a_0 - a_{\text{calc}}) \), which correspond to the \( k \), for different bandwidths, namely, the difference of the \( a_0 \) intercept of the cross-correlation ratio of the spectral density of the two noise sources (after fitting over a chosen bandwidth) from the calculated value.

Recently, we introduced a new approach to determine the Boltzmann constant based on a cross-validation method [2].

Fig. 1. Cross-correlation ratio fitting. (a) Spectral aberration obtained with higher order “d” polynomials as a function of measurement bandwidth for data “set 2”. (b) Model selection results for an older data “set 1” (gray) and the newest data “set 2” (black).

The cross-correlation ratio is fit with an even polynomial function whose complexity (polynomial order) is selected after determining whether the given model is consistent with validation data and training data. The model selection algorithm determines the optimum model or order as a function of the fitting bandwidth. This method was applied to various data sets that were obtained under different conditions.

In Fig. 1, we report the results of the method applied to our newest data “set 2,” which was optimally obtained under stable conditions and in a low electromagnetic interference (EMI) environment, whereas “set 1” was obtained in a common laboratory. Fig. 1(a) reports the offsets vs. different bandwidths that were obtained for five different models with fixed complexity. Fig. 1(b) shows for old and new data sets the selected offsets for different bandwidths that were determined using the model selection algorithm Thanks to hardware improvements [3] and the new model selection method, we achieved (“set 2”) for the first time nearly bandwidth independent results between 200 kHz and 700 kHz for the determination of \( k \), given the reported uncertainty.

III. DATA DRIFT AND EMI INVESTIGATION

In order to search for systematic errors, we applied the model selection method to the daily single measurements that made up the above data set and fixed the analysis bandwidth
Fig. 2. Values of $a_0 - a_{0\text{calc}}$ as function of measurement number or day (black) and the linear trend (gray).

Fig. 3. Cross-correlated measured noise power spectra for the sense resistor (black) and QVNS (gray) vs. frequency. The higher points represent two EMI tones coupling to the apparatus.

Frequency independent $(a_0-a_{0\text{calc}})$ was demonstrated through a combination of new hardware and a new fitting model selection method. Preliminary EMI measurements were shown that may potentially explain the remaining offset of data “set 2,” which is still under analysis.

VI. CONCLUSION

Frequency independent $(a_0-a_{0\text{calc}})$ was demonstrated through a combination of new hardware and a new fitting model selection method. Preliminary EMI measurements were shown that may potentially explain the remaining offset of data “set 2,” which is still under analysis.

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REFERENCES


