Zero to π continuously controllable cross-phase modulation in a Doppler-broadened N-type electromagnetically-induced-transparency medium

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(Received 16 June 2015; published 29 October 2015)

We demonstrate an observation of zero to π continuously controllable cross-phase-modulation based on an N-type electromagnetically-induced-transparency scheme in a room-temperature 87Rb vapor. We theoretically and experimentally show that the signal field acquires a π phase shift compared with the reference light in the presence of the phase-control field. Using the method of the optical Mach-Zehnder interferometer, we demonstrate that a zero to π continuously controllable phase gate can be built by modulating the phase-control field. In addition, our theoretical calculation agrees well with the experimental observation, and the results presented in this work hold potential applications for the orthogonal polarization or vector gate in quantum information processing.

DOI: 10.1103/PhysRevA.92.043838 PACS number(s): 42.50.Gy, 42.65.–k, 32.80.Qk

Quantum computers and quantum-information processing hold the promise to revolutionize information science [1–4]. The effective optical-field manipulation protocols at very low light intensities are critically important to next-generation advanced telecommunications and information processing [5]. Many proposals are provided to build the quantum phase gate based on various phenomena including a magnetic field, a light-field-induced shift, and nonlinear Kerr cross-phase modulations (XPMs) [6–10]. In particular, the nonlinear XPM is the most efficient method to realize quantum phase gate operation. Although the nonlinear XPM is usually weak and requires an extended propagation distance in solid-state media, the distinct energy levels and selection rules allow strong resonant enhancement of nonlinear XPM in gaseous-state media [11]. To this end, many proposals have been investigated theoretically and experimentally in lifetime broadened atomic systems. For instance, a typical four-level scheme using continuous-wave, weakly driven electromagnetically induced transparency (EIT) was conceived [12,13]. In an active Raman gain (ARG) medium, a significantly enhanced XPM was observed where the signal field propagates with a superluminal group velocity [14], and a fast-response phase gate operation was also demonstrated [15]. Recently, the polarization-selective Kerr-phase-shift technique was developed, permitting the fast and complete polarization-gate operations with a weak phase-control light field, and the complete orthogonal polarization rotation has been achieved with a phase-control light intensity as low as 2 mW/cm² in a double N-type ARG scheme [16].

Although the ARG scheme is much more efficient and robust at a low phase-control light level, the EIT scheme has several advantages that cannot be matched by the ARG medium at a weak light level. For instance, the absorptive nature of the EIT process avoids the ambiguity associated with the generation of multiple stimulated signal photons due to the gain process. Recently we have proposed a polarization-selective Kerr phase gate by writing a π phase into one circularly polarized component of a linear polarized signal field in a double N-type EIT medium [17]. Theoretically a controlled-NOT (CNOT) polarization gate operation can be built at weak light level without the requirement of focusing the light field to the diffraction limit [7], which provides a new method to build the vector phase gate for the quantum computer. The advance of such high-fidelity CNOT gate operation naturally raises questions and interests in whether a zero-to-π XPM can be achieved in an EIT medium. Obviously, the larger phase shift, such as a π phase shift, is the key problem for complete orthogonal polarization rotation. Experimentally, a nonlinear phase shift of 0.13 rad induced by a control field with an intensity of 1.8 mW/cm² was observed in an EIT-based XPM scheme [18]. A phase shift of 0.02 rad of a signal pulse modulated by a control-field pulse with a peak intensity of 3 μW/cm² was also demonstrated in an N-type EIT medium [19], and the phase shift of 0.005 rad was observed with a phase-control field at the few-photon level for weak-nonlinearity-based quantum computing [20]. The EIT-based light storage method has achieved a nearly 0.78-rad phase shift by using a control-field intensity of 18 μW/cm² with substantial time delay [21], and a new scheme based on a weak-polarizing optical rotation effect has been studied to demonstrate a discrete zero or π phase jump [22]. As far as we know, the continuously controllable zero-to-π cross-phase modulation has not been demonstrated in an atomic EIT medium.

In this paper, we present the experimental results of a zero to π continuously controllable XPM in a room-temperature N-type EIT medium. In our system, the probe field propagates through the medium with a subluminal group velocity and the attenuation of the signal field is weak due to the quantum interference effect. When the phase-control field is turned on, a nonlinear phase shift is written to the signal field via the nonlinear Kerr effect, and the magnitude of the XPM is

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The theoretical model is developed by considering a lifetime-four level system with a probe field, a phase-control field, and an ambient magnetic field. The system is described by the Hamiltonian:

\[ H = \sum_{i=1}^{4} \left( \omega_i |i\rangle \langle i| - \frac{1}{2} \sum_{j=1}^{4} \omega_{ij} |ij\rangle \langle ij| \right) + \sum_{i=1}^{4} \frac{1}{2} \sum_{j=1}^{4} \gamma_{ij} |ij\rangle \langle ji| \]

where \(\omega_i\) are the transition frequencies, \(\omega_{ij}\) are the two-photon detunings, and \(\gamma_{ij}\) are the decay rates. The equations of motion for the relevant density matrix elements are obtained by solving the Schrödinger equation for the system under the interaction with the probe and phase-control fields. The equations can be written as:

\[ i\dot{\rho} = -\frac{1}{\hbar} [H, \rho] + \sum_{ij} \Gamma_{ij} ({|ij\rangle \langle ji|} - \frac{1}{2} \{ij, ji\}) \]

where \(\Gamma_{ij}\) are the decay rates, and \(\{ij, ji\}\) are the commutators. The steady-state solutions are then obtained by setting the time derivatives to zero and solving the resulting system of equations.

The susceptibility of the signal field is determined by the intensity of the phase-control field. The total decay rate of state \(|i\rangle\) and \(\gamma_{\text{col}}\) is the dephasing rate due to processes such as elastic collisions. The Rabi frequency of the signal field, pump field, and phase-control field, respectively, are \(\Omega_i = p_{2i} E_i / \hbar\), \(\Omega_p = p_{2i} E_p / \hbar\), and \(\Omega_{ph} = p_{4i} E_{ph} / \hbar\). The phase shift is observed by modulating the intensity of the phase-control field, where the group velocity of the probe field depends on the two-photon detuning that is induced by ac Stark shift when the phase-control field is applied. To this end, a controllable phase-gate operation can be built by modulating the phase-control field, which is the fundamental technique for achievements of the scalar or vector gate operations and holds many potential applications in quantum-information processing.

As proposed by Schmidt [12], we consider an ensemble of lifetime-four level N-type EIT scheme outlined in Fig. 1(b), where the signal field \(E_s\) couples the transition between states \(|1\rangle = |5S_{1/2}, F = 1\rangle\) and \(|2\rangle = |5P_{1/2}, F' = 1\rangle\), and the pump field couples the transition between states \(|2\rangle = |5P_{1/2}, F' = 1\rangle\) and \(|3\rangle = |5S_{1/2}, F = 2\rangle\). The phase-control field drives the transition between states \(|3\rangle = |5S_{1/2}, F = 2\rangle\) and \(|4\rangle = |5P_{3/2}, F' = 3\rangle\) as shown in Fig. 1(b). The equations of motion for the relevant density matrix elements in an N-type EIT atomic system can be written as:

\[ i\dot{\sigma}_{21} + d_{21}\sigma_{21} + \Omega_s (\sigma_{11} - \sigma_{22}) + \Omega_p \sigma_{31} = 0, \]

\[ i\dot{\sigma}_{31} + d_{31}\sigma_{31} + \Omega_p^* \sigma_{31} + \Omega_{ph}^* \sigma_{41} - \Omega_s \sigma_{32} = 0, \]

\[ i\dot{\sigma}_{41} + d_{41}\sigma_{41} + \Omega_{ph} \sigma_{31} - \Omega_p^* \sigma_{42} = 0, \]

\[ i\dot{\sigma}_{32} + d_{32}\sigma_{32} + \Omega_p \sigma_{32} - \Omega_{ph}^* \sigma_{42} + \Omega_s^* \sigma_{31} = 0, \]

\[ i\dot{\sigma}_{42} + d_{42}\sigma_{42} + \Omega_{ph} \sigma_{32} - \Omega_s^* \sigma_{41} - \Omega_p^* \sigma_{43} = 0, \]

\[ i\dot{\sigma}_{43} + d_{43}\sigma_{43} + \Omega_{ph} \sigma_{33} - \Omega_{ph} \sigma_{42} = 0, \]

where \(d_{ij} = \Delta_i - \Delta_j + iy_{ij}\) with \(\Delta_i\) being the detuning of state \(|i\rangle\) and \(y_{ij} = (\Gamma_i + \Gamma_j) / 2 + \gamma_{ij}^\text{col} (i, j = 1-4)\). The group velocity depends on two-photon detuning.
induced by the ac Stark shift of the phase-control field. The group velocity will be changed when the phase-control field is considered and given by

\[
\frac{1}{v_g} = \frac{1}{c} + \kappa \left[ \frac{|\Omega_p|^2 - (\gamma_{31} + |\Omega_{ph}|^2/\gamma_{31})^2}{|\gamma_{21}|(\gamma_{31} + |\Omega_{ph}|^2/\gamma_{31}) + |\Omega_p|^2} \right],
\]

(9)

where we have assumed \( \Delta_2 = \Delta_3 = 0 \) and \( \Delta_4 \ll \gamma_{31} \).

Before proceeding further, we comment on the theoretical method used above in treating a Doppler-broadened medium. In general, there are two avenues in treating atomic response in a warm vapor where Doppler broadening is a prominent feature. The first one is to incorporate the Doppler shift in various laser detunings. Consequently, Eqs. (6a) and (6b) must be integrated over all velocity classes using an appropriate Maxwellian velocity distribution (i.e., Doppler averaging). Obviously, this procedure leads to multiple differential-integral equations that can only be evaluated numerically (except for some special cases). Such numerical calculations often yield an opaque picture of the underlying physics of the problem. Our goal, however, is to seek a possible analytical solution that retains most important physics. The second avenue, which is what we adopted in this work, is to introduce “effective” linewidths under a given temperature for various transitions to mimic the effect of Doppler broadening. Clearly, this method will result in some differences when compared with the full numerical treatment where the contributions from the different velocity class are integrated. It allows one, however, to derive analytical expressions from which the most important physics of the problem can be clearly understood. This method is often used when the initial understanding of a warm vapor-based process is the main focus of the study, where the collision broadening is not important at this concentration.

The experiments are performed in a very weak coherent classical field regime. As shown in Fig. 1(a), we use rubidium vapor to observe the low-light-level cross-phase modulation in the four-level system. The \(^{87}\text{Rb}\) vapor cell has a length of 7.5 cm and a diameter of 2 cm. It is filled with about 933-Pa neon buffer gas and it is also shielded from ambient magnetic fields under three layers of \( \mu \) metal. The atomic vapor is actively temperature stabilized at 325 K, corresponding to the atomic density \( N = 6 \times 10^{11}/\text{cm}^3 \). A very weak magnetic field (100 mG) is provided along the light-propagation direction by the axial coils to keep the quantum-axis direction. A strong horizontal-polarized pump laser (6 mW with a 3-mm beam diameter, wavelength 795 nm) drives \( |S_{1/2}, F = 2 \rangle \) and \( |P_{1/2}, F = 1 \rangle \) transitions, and a vertical-polarized signal laser (10 \( \mu \)W with a 1-mm beam diameter, wavelength 795 nm) drives \( |S_{1/2}, F = 1 \rangle \) and \( |P_{1/2}, F = 1 \rangle \) transitions. The signal laser is split by a beam splitter (BS), and one is the signal light while another one is the reference light. They are combined together using another BS to build the optical Mach-Zehnder interferometer. An attenuator plate is used to balance the intensities between two arms of the optical Mach-Zehnder interferometer. The interference fringes can be observed by scanning the PZT voltage. The linear polarized phase-control light (3-mm beam diameter, wavelength 780 nm) and the signal light are overlapped with the opposite propagating direction. Before the detector, a Glan-Taylor prism is used to separate the signal light from the pump light due to the orthogonal polarizations for each other. In the experiment, the two-photon resonant conditions are satisfied between the signal field and the pump light to obtain the maximum transparent intensity of the signal light.

In the absence of the phase-control field, the system is reduced to a typical three-level \( \Lambda \)-type EIT scheme. In the experiment, the pump light is resonant with the transitions \( |S_{1/2}, F = 2 \rangle \) and \( |P_{1/2}, F = 1 \rangle \). The frequency of the signal light is scanned, using an acousto-optic modulator (AOM), around the transitions between \( |S_{1/2}, F = 1 \rangle \) and \( |P_{1/2}, F = 1 \rangle \). After the intensity and one-photon detuning of the pump light are adjusted carefully, the signal light can transmit through the medium with slight attenuation when the two-photon resonance condition is satisfied. The width of the EIT window is about 200 kHz, as shown in Fig. 2. The solid black curve is the transmission spectrum of the signal field which depends on the two-photon detuning between the signal and pump fields. The dashed red curve is the theoretical result obtained from \( \eta_L = I_L(z = L)/I_L(z = 0) = \exp[-2k_s \text{Im}(\chi^{(1)} L)] \). We choose the system parameters as \( \Gamma_L/2\pi = 600 \text{ MHz, } \Gamma_1/2\pi = 10 \text{ kHz, and } \Omega_p = 10 \text{ MHz} \) (the corresponding pump field power is about 6 mW with the 3-mm beam diameter) [23]. The group velocity will be delayed at the exit of the medium. It is delayed about 1.8 \( \mu \)s in the \( \Lambda \)-type three-level EIT medium as shown in Fig. 3 (inset). In the \( N \)-type four-level system, the group velocity delay of the signal light is changed when the phase-control light is applied. The group velocity delay of the signal light depends on the intensity of the phase-control light. As the intensity of the phase-control field (or \( \Omega_{ph}/2\pi \)) is increased, the group velocity delay of the signal light is decreased as shown in Fig. 3. The group velocity depends on the two-photon detuning (\( \Delta_3 \)) induced by the ac Stark shift considering the phase-control light, as in Eq. (9). The black squares are the velocity delay times when the phase-control light is changed. The delay time will be decreased to 800 ns when the power of the phase-control light is increased to 5 mW corresponding to a
FIG. 3. (Color online) The group velocity dependence on the power of the phase-control field. The group velocity delay is observed with 1.8 μs in the Λ-type three-level EIT (see inset). The solid black square represents the experimental data and the dashed red curve denotes the theoretical result.

π phase shift. The dashed red line represents the group velocity delay obtained from $\Delta t_{\text{delay}} = L/V_g - L/c$. From Figs. 2 and 3, the experimental results are completely consistent with the theoretical simulated results.

We are concerned with the cross-phase modulation in the $N$-type four-level system when the phase-control light is considered as shown in Fig. 1(b). The Kerr phase shift of the signal field is induced and controlled by a linear polarized phase-control light field $E_{\text{ph}}$ (0–5 mW with a diameter of 3 mm) that couples the transitions between $|5S_{1/2}, F = 2\rangle$ and $|5P_{3/2}, F = 3\rangle$. Strictly speaking, the medium is not a simple four-level system because of contributions from the transitions of the multiple sublevels (due to Doppler broadening, all $F' = 1, 2, 3$ manifold contributions). Because of this consideration, the phase-control light is propagating anticonlinearly with respect to the probe light. The phase shift is obtained by comparing the phase of the signal light with the reference light, using an unbalanced Mach-Zehnder interferometer, with and without the phase-control light field. The obtained interference fringes are shown in Fig. 4. The solid black line is the interference fringe without the phase-control light, while the dashed blue line and dotted red line are interference fringes for the phase-control lights with 2.5 and 5.0 mW, respectively. Due to the third-order nonlinear absorption, the amplitude of the interference fringes with the phase-control light is lower than the amplitude of the interference fringe without the phase-control light.

In Fig. 5, we show the nonlinear Kerr phase shift due to the cross-phase modulation and the transmittance of signal light due to the third nonlinear absorption for the different powers of the phase-control light in the $N$-type four-level system. Similar to Fig. 4, the interference fringes are obtained by scanning the PZT voltage, and fitted by the sine function for the different powers of the phase-control lights. In order to avoid the phase uncertainty caused by the fluctuation of optical path lengths, the interference fringes are alternately obtained with and without the phase-control light. The phase shift is shown in Fig. 5 (black stars, left axis). The maximum phase shift is observed when the phase-control light is 5.0 mW. The effective intensity of phase-control light can be increased largely when the $N$-type EIT scheme is considered in the hollow-core fiber [20]. From Eq. (8), the third nonlinear absorption is obvious when the phase-control light is increased in the $N$-type four-level system. Indeed, we observed this attenuation of signal light in

FIG. 4. (Color online) Typical optical Mach-Zehnder interferometer output as a function of piezoactuator control voltage for different phase-control light power. The powers of the phase-control lights are 0 mW (solid black line, non-phase-shifted reference), 2.5 mW (dashed blue line, π/2 phase shift), and 5.0 mW (dotted red line, π phase shift), respectively.

FIG. 5. (Color online) Kerr nonlinear phase shift (black stars, left axis) as a function of the phase-control light power (bottom axis) or the phase-control field $\Omega_{\text{ph}}/2\pi$ (top axis). The transmittance $\eta_{\text{NL}} = I_s(\Omega_{\text{ph}})/I_s(\Omega_{\text{ph}} = 0)$ (blue circles, right axis) of the signal light field as a function of the phase-control light power (bottom axis) or the phase-control field $\Omega_{\text{ph}}/2\pi$ (top axis). The phase-control light is nearly resonant with the transitions between $F = 2$ and $F' = 3$. 

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the experiment, and the transmittance of signal light caused by the third nonlinear absorption is shown in Fig. 5 (blue circles, right axis). The solid red curve is the nonlinear phase shift theoretically obtained from Eq. (7). The dashed green curve is the theoretical result corresponding to the transmittance \( \eta_{\text{NL}} = I_{\text{NL}}(\Omega) / I_{\text{NL}}(\Omega = 0) = \exp(-2\alpha_{\text{NL}}) \) of the signal light field, where we take \( \Gamma_s/2\pi = 600 \text{ MHz} \), and other system parameters are the same as those given in Fig. 2. The theoretical calculations are matched well with the experiment results. The transmittance of signal light \( \eta_{\text{NL}} \) is 0.45 in the EIT medium when the phase shift is \( \pi \) [24]. This is different from the active Raman gain medium [15]. Zero to \( \pi \) phase shift provides an important step for building the orthogonal polarization or vector gate as proposed in the EIT medium [17].

In summary, we have demonstrated a zero to \( \pi \) continuously controllable cross-phase modulation in the room-temperature \( N \)-type EIT atomic system. We first investigated the typical three-level \( \Lambda \)-type EIT scheme where a transmission peak was observed with the two-photon resonance satisfied, in which the signal field delayed by 1.8 \( \mu \text{s} \) was also observed at the exit of the medium, corresponding to a subluminal propagation for the signal field. Then, we showed that the delay times of the group velocity can be significantly reduced by increasing the intensity of the phase-control field because the state [3] is shifted when the phase-control field is added. We also showed that the cross-phase modulation can be significantly enhanced due to the nonlinear Kerr effect. By employing the optical Mach-Zehnder interferometer, we demonstrated a zero to \( \pi \) continuously controllable cross-phase modulation in this \( N \)-type EIT scheme at room temperature. Based on this experimental work, it is possible to realize the vector phase-gate operations provided in Ref. [17] in the future, which holds potential applications in quantum computer and information processing.

C.Z. acknowledges the Shanghai Science and Technology Committee (Grant No. 15YF1412400) and the National Natural Science Foundation of China (Grant No. 11504272).