The effects of burial diagenesis on multiscale porosity in the St. Peter Sandstone: An imaging, small-angle, and ultra-small-angle neutron scattering analysis

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1. Introduction

The study of sandstones was one of the earliest efforts in the geological sciences, dating back at least to the work of James Hutton, John Playfair, Roderick Murchison, and Adam Sedgwick on the Old Red Sandstone (Hutton, 1788; Playfair, 1802; Sedgwick and Murchison, 1840). Dott (1964) provided a classic method to characterize sandstones, based in part on the earlier work of Gilbert (in Williams et al., 1954) and Folk (1951, 1961). Although “sand” is formally a term that refers to a range of grain sizes, Pettijohn et al. (2012) noted that the term “sandstone” is typically restricted to “those indurated sands of siliceous character.”

Although one typically thinks of sandstone in terms of its mineral grains, its porosity and permeability makes sandstones important as natural and potential anthropogenic storage sites. Understanding the pore structures of sandstones is, therefore, critical to understanding a wide range of processes: the evolution of aquifers; properties of dimension stone, flagstone, and crushed stone; formation of, and energy recovery from, oil and gas reservoirs; and geological CO2 sequestration. Although much work has been undertaken to understand and quantify sandstone porosity (cf. Freiburg et al., 2016), studies of submicron porosity have typically been limited by the difficulty of quantifying pore structures at the smallest scales. Whereas submicron pore structures are easily observed using transmission electron microscopy (TEM)
and high-resolution scanning electron microscopy (SEM), the large magnifications involved limit the volumes investigated, and thus the ability of such approaches to quantify pore structures at these scales.

Despite the obvious importance of larger-scale pores to sandstone structures available data suggest these smaller pores are also an important component of the overall pore structure. In fact, it is likely these finer-scale features control much of the flow dynamics in sandstones, and can be significantly impacted by diagenetic events. Pettijohn et al. (2012) commented:

The grains in a sand are generally in tangential contact only and thus form an open, three-dimensional network. As a consequence, sands have a high porosity—have a fluid-filled pore system. The unequal distribution of stress along grain boundaries may lead to solution at points of pressure and deposition elsewhere increasing the surfaces of contact and decreasing the pore space. Such action, coupled in some cases with the introduction of cementing materials, leads to the ultimate end-product—a rock with grains in continuous contact and without porosity. In this manner, a sand with tangential contacts and a porosity of 35–40 percent is converted to an interlocking crystalline mosaic with zero porosity. (p. 1)

While this clearly describes the overall evolution of many sandstones, the claim that sandstones progress to zero porosity is somewhat overstated. In our previous studies (Anovitz et al., 2013a, 2015a), for example, we showed that low-pressure silcrete formation the St. Peter Sandstone from southwest Wisconsin, while lowering total porosity, decreasing pore size, and making the remaining pores more isolated, did not eliminate porosity altogether, and tended to make the smaller pores volumetrically more significant. Available data suggest, however, that these samples had never been buried to great depths (Hoholick et al., 1984; Mai and Dott, 1985; Rowan and Goldhaber, 1996; Pitman et al., 1997; Prothero and Dott, 2004; Kelly, 2006; Kelly et al., 2007; Anovitz et al., 2013a). Thus, although these samples allowed us to evaluate the effects of precipitation on porosity, a different sample suite was needed to extend this analysis to basin diagenesis. The focus of this paper, therefore, is to evaluate changes in the pore structure of the St. Peter from different basins at different burial histories.

We have done so by combining imaging analysis at larger scales, with small and ultra-small angle neutron scattering to characterize pore structures over a seven order of magnitude range, from the centimeter to the nanometer scale.

1.1. Geology

The Middle Ordovician St. Peter Sandstone, part of the Ancell Group, covers wide areas of the U.S. Midcontinent (Dake, 1921; Dapples, 1955). The earliest work on the St. Peter was probably that of Owen (1847), who named it for the Minnesota River, which was then called the St. Peter River. It is found in both the Illinois and Michigan Basins, which formed in response to far-field plate tectonic effects (Hinze and Braile, 1988). The St. Peter is a super-mature quartz arenite (< 99% quartz) that consists primarily of well-sorted, H. A. (Hinze and Braile, 1988). The St. Peter is a super-mature quartz arenite (see Kelly, 2006; Kelly et al., 2007; Anovitz et al., 2013a) to nearly 7000 feet (2134 m). In the Michigan Basin, the St. Peter occurs at depths ranging from less than 1000 feet (305 m) in the southeastern Upper Peninsula (Mackinac County) to greater than 10,000 feet (3048 m) near the center of the basin (Zdan, 2013).

Since the deposition of the St. Peter Sandstone, the basins have experienced several episodes of subsidence and uplift (Kolata and Nelson, 2010). By the Late Permian in the Illinois Basin, the St. Peter was buried to a maximum depth of approximately 4500–6000 feet (~1372–1829 m) in southern Illinois, based on data of the top of the Mt. Simon Sandstone (Makowitz et al., 2006). However, burial reconstruction by Pitman et al. (1997) has the St. Peter attaining a burial and temperature maximum (3.3 km and 140 °C) during the Late Pennsylvanian and Early Permian periods, coincident with regional Permian igneous activity and ore-forming events (Mississippi Valley-type [MVT] deposits). Others have suggested a geothermal gradient of 10–110 °C/km (Kelly et al., 2007; Pollington et al., 2011) based on oxygen isotopes and fluid inclusion data of quartz overgrowths in the Mt. Simon Sandstone (Fishman, 1997) and other burial models (Makowitz et al., 2006). In the Michigan Basin near the end of the Carboniferous Period, the St. Peter was buried to a maximum depth of 13,000 feet (~4 km), based on thermal maturity analyses of organics (Cercone, 1984). An elevated thermal gradient of 34–60 °C/km during the Paleozoic has been suggested (Girard and Barnes, 1995; Graham et al., 1996), likely related to a buried Precambrian rift in the center of the basin (Cercone and Pollack, 1991).

The diagenetic history of the St. Peter in the Illinois (Hoholick et al., 1984; Pitman et al., 1997) and Michigan Basins (Barnes et al., 1992; Drzewiecki et al., 1994; Zdan, 2013) has been studied thoroughly. In the Illinois Basin, multiple generations of authigenic minerals occur, contributing to an associated variability in porosity (Pitman et al., 1997). Authigenic K-feldspar emplacement occurred in the Devonian Period at approximately 400 Ma during early diagenesis at relatively shallow depths (< 1500 m) and low temperatures of ~65 °C (Krueger and Woodard, 1972; Marshall et al., 1986; Elliott and Aronson, 1987, 1993; Hay et al., 1988; Sweeney and Burnham, 1989; Lee and Aronson, 1991; Barnes et al., 1992; Hay and Liu, 1994; Pitman and Spötl, 1996; Pitman et al., 1997). Multiple generations of illite formed, and similar generations are observed in older clastic rocks in the Illinois Basin that likely contribute significant microporosity (Freiburg et al., 2014). Compared with K-feldspar emplacement, illite precipitation occurred over a wider time frame (360–215 Ma; Hay et al., 1988; Lee and Aronson, 1991; Grathoff et al., 2001; Elliott and Aronson, 1993). Authigenic quartz forms optically continuous but complex, concentrically zoned (under cathodoluminescent light), synaxial overgrowths on detrital quartz grains and postdates diagenetic illite and feldspar (Pitman et al., 1997; Kelly et al., 2007; Anovitz et al., 2015a). Isotopic studies suggest quartz precipitation at low and relatively homogeneous temperatures (10–30 °C; Graham et al., 1996; Kelly et al., 2007). Other studies of quartz overgrowths in underlying formations such as the Eau Claire (Hyodo et al., 2013) and Mt. Simon (Pollington et al., 2011) suggest that multigenerational quartz cementation formed in response to increased temperatures during burial. Carbonate cements in the St. Peter include dolomite and calcite and are largely segregated to the east-central and southern portions of the Illinois Basin (Hoholick et al., 1984; Pitman et al., 1997). These are attributed to hydrothermal fluids similar to MVT brines based on stable isotopes and fluid inclusion temperatures and salinities. Isotopic analyses in the underlying Eau Claire Formation suggest cements formed in response to progressive sediment burial and heating, with final temperatures consistent with mid-Permian MVT brines (Sliwinski et al., 2016). Anhydrite was the last major diagenetic cement to form in the St. Peter. It is largely segregated to the depocenter of the Illinois Basin in southeastern Illinois and southwestern Indiana, and it is inferred anhydrite was deposited during...
the maximum burial of the St. Peter (Pitman et al., 1997). Other controls on porosity include mechanical and chemical compaction, cement dissolution, framework detrital grain dissolution, and fracturing (Hoholick et al., 1984).

In the Michigan Basin, authigenic minerals in the St. Peter are relatively consistent throughout the central basin (Barnes et al., 1992). Barnes et al. (1992) suggest that no significant variation in diagenetic alteration occurs as a function of burial depth, but that a relationship to primary mineralogy is influenced by variable depositional environments. This differs from the Illinois Basin, where cements are depth dependent and define distinct regions (Hoholick et al., 1984). However, isotopic and fluid inclusion analyses of cathodoluminescent zoned quartz overgrowths in the Michigan Basin suggest precipitation occurred over a range of temperatures (Girard and Barnes, 1995; Graham et al., 1996), implying a relationship to burial depth. Illitization of the St. Peter in the Michigan Basin appears to have some overlap with that in the Illinois Basin but with a shorter window during the Late Devonian to Mississippian periods (367–322 Ma; Girard and Barnes, 1995).

As in the Illinois Basin, porosity is significantly controlled by compaction and dissolution of both authigenic and framework minerals.

1.2. Sample locations

For this study, samples were recovered from whole and sidewall cores from four Illinois wells and one from Michigan. The Illinois wells were drilled in an approximately north–south line. In the first, C-28 (API no. 121670011500), located in Sangamon County, Illinois, township 15N, range 3W, section 1, samples were recovered from 2730 to 2732 feet (820–830 m). In the second, C-13368 (API no. 121350176000), located in Montgomery County, township 9N, range 3W, section 1, samples were recovered from 315 to 3309 feet (962–1009 m). In the third, C-228 (API no. 12027005701), samples were recovered from 4117 to 4119 feet (1254.9–1255.5 m). It was located in Clinton County, Illinois, township 2N, range 3W, section 3. In the fourth, C-4831 (API no. 121210519800), samples were recovered from 5239 to 5250 feet (1590–1600 m). It was located in Marion County, township 1N, range 2E, section 6. In the well from the Michigan Basin, Briggs unit (API no. 20113988900000), located in Arenac County in township 2N, range 4E, section 12, samples were recovered from 10,816 to 10,835 feet (3290–3300 m).

2. Analytical approaches

In our previous work (Anovitz et al., 2013a, 2015a), we have shown that porosity in sandstones is highly hierarchical, with pores ranging in size from nanometers to millimeters or larger. Although traditional optical and electron imaging techniques can provide much information on sandstone porosity, especially when abetted by petrophysical approaches (cf. Anovitz and Cole, 2015), a different approach is required to extend quantitative analyses over a wide range of scales. To do so, we have combined four techniques: small-, very-small and ultra-small-angle neutron scattering (SANS, USANS, VSANS), and backscattered electron imaging (BSE)/SEM to quantify porosity on length scales from ∼1 nm to ∼1 cm, a seven-decade range.

Details of the analytical approaches have been reported elsewhere (Anovitz et al., 2009, 2011, 2013a,b; 2015a,b; Wang et al., 2013) and are, therefore, summarized briefly. SANS, USANS and VSANS were used to probe porosity over length scales from ∼10 nm to 30 µm. This was expanded to ∼1 cm using the autocorrelation functions calculated from BSE images at 14×. Neutron scattering measurements were performed on thin sections (0.15 mm thick) mounted on 1-mm-thick quartz glass slides with the beam incident along the surface normal. SANS measurements were conducted on the general-purpose SANS instrument at the High Flux Isotope Reactor at the Oak Ridge National Laboratory (Littrell et al., 2008) using a 6-mm mask. Three sample-to-detector distances (1.1, 4.7, and 19.3 m) were used with wavelength λ = 4.75 Å at 1.1 and 4.7 m, and 19 Å at 19.3 m, with a wavelength resolution Δλ/λ = 0.14. Measurements were performed with the detector offset to extend the range of the scattering vector Q (Q = (4πλ/Δλ) sin(θ/2)) where θ is the scattering angle) at each setting and provide better overlap. The resultant scattering vector ranged from 0.0009 to 0.9 Å−1, which corresponds to sizes from ∼7 to 7000 Å. Data were correct for empty-beam scattering, background counts, detector uniformity, sample transmission and scattering volume, and reduced to an absolute scale (differential scattering cross-section per unit volume) by normalization to the intensity of the direct beam. The two-dimensional data were always azimuthally symmetric, and were circularly averaged to produce one-dimensional intensity profiles.

Ultra-small-angle neutron scattering measurements were performed on the BT5 instrument at the Center for Neutron Research at the National Institute of Standards and Technology (Barker et al., 2005). Samples were mounted on 1/2- or 5/8-inch cadmium masks. A pair of triple-bounce channel-cut perfect silicon (220) crystals was used for the monochromator and analyzer. The wavelength was 2.38 Å with a wavelength resolution Δλ/λ = 0.059. Data were collected over a Q range from 4.2 × 10−3 to 2.7 × 10−1 Å−1, which corresponds to sizes from 2400 Å to ~15 µm. The horizontal Q resolution (full width at half maximum) was 2.5 × 10−3 Å−1. Scattering intensity data were corrected for empty beam and constant background levels. The corrected data were converted to an absolute scale by measuring the attenuation of the direct beam through the sample. Further data reduction and desmearing were accomplished through the USANS data reduction software provided by NIST/NCNR (Kline, 2006).

Very-small-angle neutron scattering experiments were performed on the KWS-3 instrument operated by Jülich Centre for Neutron Science at the Forschungs-Neutronenquelle Heinz Maier-Leibnitz (FRM II) in Munich, Germany. This instrument uses a focusing mirror geometry to cover a Q range intermediate between SANS and USANS. Samples were run for 1.5 h in the 9.5 m sample-to-detector position with a wavelength of 12.8 Å (wavelength resolution 17%) covering the scattering vector range between 10−4 Å−1 and 2.5 × 10−3 Å−1 (Pipich and Fu, 2015). Absolute intensities were adjusted to fit the SANS and USANS data.

Estimation of the uncertainties for each sample on each instrument is a complex task, as the values vary with Q, sample, counting times and the individual instrument. In addition, further uncertainties arise in modeling and fitting the data. Further discussions of this topic are in Anovitz and Cole (2015), Hammouda (2008), and other references therein.

Backscattered electron images of the pore space were used to extend the scattering curve to larger scales and to provide two-dimensional information on the lacunarity and multifractal spectrum of the pore system (Debye et al., 1957; Anovitz et al., 2013a, 2015a; Wang et al., 2013). Images were obtained using a Hitachi S3400 environmental SEM. The samples were not coated, and a pressure of 50 Pa was used to eliminate charging. Images were obtained at 14×. Each image consisted of 5120 × 3840 pixels, and the pixel edge length was 1.772 µm. Thus, the pixel size overlapped the maximum USANS scale by approximately one order of magnitude.

3. Results

3.1. Sample petrography

Ten sections were prepared from cores from the wells described above for petrographic analysis. Thin sections were impregnated with blue epoxy to identify porosity. Petrographic analyses included plane-light and cathodoluminescence microscopy. Mineral composition and porosity were quantified using standard point-counting methods (typically 300 points per thin sections) and image analysis software (Olympus Stream). Results are shown in Table 1.

Petrography and diagenesis vary significantly in the samples examined. Samples from well C-28 in Sangamon County, Illinois were well-sorted
<table>
<thead>
<tr>
<th>Sample-Depth (Ft)</th>
<th>Location</th>
<th>Detrital and Authigenic Porosity (%)</th>
<th>Detrital Grain Size</th>
<th>Sorting</th>
<th>Rounding</th>
<th>Grain-Grain Contacts</th>
<th>Grain/matrix Texture</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Quartz (%)</td>
<td>Dolomite (%)</td>
<td>Clay (%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C13638-3208</td>
<td>Montgomery Co., Illinois</td>
<td>78.6</td>
<td>0</td>
<td>4.5</td>
<td>16.8</td>
<td>medium to fine</td>
<td>Moderately Well</td>
</tr>
<tr>
<td>C13638-3304</td>
<td>Montgomery Co., Illinois</td>
<td>64.3</td>
<td>5.9</td>
<td>0</td>
<td>29.7</td>
<td>Medium to Fine</td>
<td>Moderately Well</td>
</tr>
<tr>
<td>C228-4117</td>
<td>Clinton Co., Illinois</td>
<td>78.7</td>
<td>6.3</td>
<td>0</td>
<td>14.8</td>
<td>Medium to silt</td>
<td>Moderately to poor</td>
</tr>
<tr>
<td>C228-4118</td>
<td>Clinton Co., Illinois</td>
<td>76.6</td>
<td>0</td>
<td>1.8</td>
<td>21.5</td>
<td>coarse lower to very fine upper</td>
<td>Moderate</td>
</tr>
<tr>
<td>C4831-5246</td>
<td>Marion Co., Illinois</td>
<td>91.3</td>
<td>0</td>
<td>0.3</td>
<td>8.2</td>
<td>Medium to Fine</td>
<td>Moderately Well</td>
</tr>
<tr>
<td>C4831-5250</td>
<td>Marion Co., Illinois</td>
<td>94.4</td>
<td>0</td>
<td>0.3</td>
<td>5.1</td>
<td>Medium to Fine</td>
<td>Moderately Well</td>
</tr>
<tr>
<td>C28-2720</td>
<td>Sangamon Co., Illinois</td>
<td>67.9</td>
<td>0</td>
<td>1.9</td>
<td>30.2</td>
<td>medium to fine</td>
<td>well</td>
</tr>
<tr>
<td>C28-2706</td>
<td>Sangamon Co., Illinois</td>
<td>81.3</td>
<td>0</td>
<td>12.6</td>
<td>6.2</td>
<td>medium to fine</td>
<td>well</td>
</tr>
<tr>
<td>Briggs-10816</td>
<td>Arenac Co., Michigan</td>
<td>71.8</td>
<td>0</td>
<td>6.7</td>
<td>21.5</td>
<td>fine to coarse</td>
<td>moderate</td>
</tr>
<tr>
<td>Briggs-10835</td>
<td>Arenac Co., Michigan</td>
<td>79</td>
<td>20</td>
<td>1</td>
<td>0</td>
<td>fine to coarse</td>
<td>moderate</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sample-Depth (Ft)</th>
<th>Maturity</th>
<th>Porosity Type</th>
<th>Authigenic Cements</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Quartz</td>
<td>Dolomite</td>
</tr>
<tr>
<td>C13638-3208</td>
<td>High</td>
<td>Primary interparticle and residual interparticle</td>
<td>Common</td>
<td>Very rare</td>
</tr>
<tr>
<td>C13638-3304</td>
<td>High</td>
<td>Primary interparticle and residual interparticle</td>
<td>Common</td>
<td>Very rare</td>
</tr>
<tr>
<td>C228-4117</td>
<td>High</td>
<td>Primary interparticle and residual interparticle</td>
<td>rare</td>
<td>rare</td>
</tr>
<tr>
<td>C228-4118</td>
<td>High</td>
<td>Primary interparticle and residual interparticle</td>
<td>rare</td>
<td>common</td>
</tr>
<tr>
<td>C4831-5246</td>
<td>High</td>
<td>Primary interparticle and residual interparticle</td>
<td>rare</td>
<td>Very rare</td>
</tr>
<tr>
<td>C4831-5250</td>
<td>High</td>
<td>Primary interparticle and residual interparticle</td>
<td>common</td>
<td>Very rare</td>
</tr>
<tr>
<td>C28-2720</td>
<td>High</td>
<td>Stable framework</td>
<td>rare</td>
<td>none</td>
</tr>
<tr>
<td>C28-2706</td>
<td>High</td>
<td>Stable framework, leached cement</td>
<td>rare</td>
<td>none</td>
</tr>
<tr>
<td>Briggs-10816</td>
<td>high</td>
<td>Stable framework, leached cement</td>
<td>rare</td>
<td>trace</td>
</tr>
<tr>
<td>Briggs-10835</td>
<td>High</td>
<td>Early cement, none</td>
<td>rare</td>
<td>predominant</td>
</tr>
</tbody>
</table>
quartz arenites predominantly composed of lightly consolidated and compacted, medium to fine, well-rounded quartz grains. Authigenic cement was partially abundant throughout with rare authigenic quartz and dolomite cements. Porosity was largely a primary framework with minor secondary cement and complete grain dissolution. Porosity was measured from 6.2 to 30.2% in thin section. Samples from well C-13368 in Montgomery County, Illinois were moderately well-sorted quartz arenites predominantly composed of moderately consolidated, lightly compacted, medium to fine, rounded to well-rounded quartz grains. Authigenic quartz overgrowths were common, and dolomite and clay cements were rare. Porosity was largely a primary framework with significant secondary contribution from unknown dissolved detrital grains and ranged from 16.8 to 29.7% in thin section. Samples from well C-228 in Clinton County, Illinois were moderately to poorly sorted quartz arenites predominantly composed of moderately consolidated, lightly compacted, coarse to silt-size, rounded to subangular quartz grains. The sample recovered from 4118 feet (1.26 km) had abundant fine-grained dolomite cement with rare, later, coarser euhedral dolomite rhombohedra. Authigenic quartz overgrowths were common, and dolomite and clay cements were rare. Porosity was largely a primary framework with moderate secondary leached dolomite cement and detrital grain dissolution. Samples from well C-4831 in Marion County, Illinois were moderately well-sorted quartz arenites predominantly composed of well-consolidated, highly-compacted, medium to fine, well-rounded to sub-rounded quartz grains. Authigenic quartz, dolomite, and clay were rare. Quartz grains were highly compacted with sutured grain boundaries. Porosity appeared largely a primary framework; however, secondary complete grain dissolution is possible. Porosity was measured from 5.1 to 8.2% in thin section. The relative mineralogical purity of most Illinois samples is consistent with regional mapping of diagenetic mineralogy (Pitman et al., 1997).

Samples from Arenac County, Michigan were moderately sorted quartz arenites composed of moderately consolidated and compacted, fine to coarse, rounded to well-rounded quartz grains. Primary grain-supported pore space was completely cemented by authigenic dolomite cement with rare, later, coarser euhedral dolomite rhombohedra. Authigenic quartz overgrowths were common, and dolomite and clay cements were rare. Porosity was largely a primary framework with significant secondary contribution from unknown dissolved detrital grains and ranged from 16.8 to 29.7% in thin section. Samples from well C-13368 in Montgomery County, Illinois were moderately well-sorted quartz arenites predominantly composed of moderately consolidated, lightly compacted, medium to fine, rounded to well-rounded quartz grains. Authigenic quartz overgrowths were common, and dolomite and clay cements were rare. Porosity was largely a primary framework with minor secondary cement and complete grain dissolution. Porosity was measured from 6.2 to 30.2% in thin section. Samples from well C-228 in Clinton County, Illinois were moderately to poorly sorted quartz arenites predominantly composed of moderately consolidated, lightly compacted, coarse to silt-size, rounded to subangular quartz grains. The sample recovered from 4118 feet (1.26 km) had abundant fine-grained dolomite cement with rare, later, coarser euhedral dolomite rhombohedra. Authigenic quartz overgrowths were common, and dolomite and clay cements were rare. Porosity was largely a primary framework with moderate secondary leached dolomite cement and detrital grain dissolution. Samples from well C-4831 in Marion County, Illinois were moderately well-sorted quartz arenites predominantly composed of well-consolidated, highly-compacted, medium to fine, well-rounded to sub-rounded quartz grains. Authigenic quartz, dolomite, and clay were rare. Quartz grains were highly compacted with sutured grain boundaries. Porosity appeared largely a primary framework; however, secondary complete grain dissolution is possible. Porosity was measured from 5.1 to 8.2% in thin section. The relative mineralogical purity of most Illinois samples is consistent with regional mapping of diagenetic mineralogy (Pitman et al., 1997).

Fig. 1 shows BSE images of four of the samples studied. These images clearly indicate the decrease in porosity attributable to compaction and cementation. Most grains are at least moderately well-sorted and rounded. A transition occurs with depth from a grain-supported to a more cement-supported structure, although this only dominates one of the two deep samples from the Michigan Basin. Sutured grain boundaries also become more common with depth.

### 3.2. Analysis of porosity in back-scattered electron imagery

To better quantify porosity at the image scale, the BSE images were first converted to binaries using ImageJ (Abramoff et al., 2004; Rasband, 1997–2016; Schneider et al., 2012; Anovitz et al., 2013a). Several autothresholding approaches (cf. Anovitz and Cole, 2015) were applied to each image to determine the approximate threshold setting, which was then adjusted as needed. More complex segmentation algorithms (e.g., watershed, edge finding) were not utilized. 

Fig. 2 shows the image-scale porosity of each of the 36 samples examined as a function of depth, along with the average for all samples from each well. The number of samples from each core differs significantly, but average and one sigma values for percentage of image-scale porosity for each of the cores are as follows: 24.26 ± 4.44 (C-28), 17.91 ± 3.64 (C-13368), 18.88 ± 3.46 (C-228), 15.20 ± 3.81 (C-4831), and 9.48 ± 3.77 (Briggs unit). As expected, average porosity decreases with depth. This change is approximately logarithmic. Although relatively little difference in depth occurs between the samples from each well, considerable variation in porosity is observed at each depth. Thus, trends in these data can easily be lost in this background. To counteract this difficulty, comparisons are made later in this article for the highest and lowest porosity samples from each well. The ~4100-foot (~1250 m) samples are excluded from the low-porosity set because the few samples from well C-228 have image porosities near the high end of the other distributions.

The observed image-scale porosities (Fig. 2) are somewhat higher than those reported by Hoholick et al. (1984). This may reflect sampling bias as the samples obtained by Hoholick et al. (1984) were more widely distributed but generally included fewer samples per core than ours. On the other hand, the difference may be methodological. Hoholick et al. (1984) determined the percentage of porosity by using point-counting techniques. Magnifications and resolution were not provided. Our results, however, were determined by analysis of SEM images.
images with a pixel edge length of 1.24 μm. It is therefore likely that we analyzed many more points, leading to better statistics. Our ability to resolve and include finer pore sizes may also have been superior, although, grain-boundary pixels add to the uncertainty in image-based analyses.

Analysis of these results suggests that image-scale pore volumes may not be distributed normally. Fig. 3 shows a histogram of the difference between the percentage porosity in each sample and the average value for that core. As can be seen, although the numbers remain small, the histogram suggests that the porosity is bimodal, with one group below and the other at or above the mean. It is unclear whether this is real or a function of the limited sample size and, if it is real, what its origins might be.

3.3. Combination of small-angle scattering and backscattered electron imagery

As shown by our work to date (Anovitz et al., 2009, 2011, 2013a,b; 2015a,b; Wang et al., 2013), the pore structures of most rock types are inherently multiscalar, with pore sizes ranging from the nanoscale up. The combination of SANS and USANS covers a pore-size range from approximately 7 Å to 15 μm, although the smallest scales are often obscured by background because of incoherent scattering. This range can be extended to larger scales by using a two-point autocorrelation analysis to combine two- or three-dimensional imaging data with the results of the scattering experiments, covering ranges up to 1 cm or larger (Anovitz et al., 2013a, 2015a; Wang et al., 2013).

Fig. 4 shows an example of a background-subtracted scattering pattern (sample C13368-3276), as the log of the intensity I(Q) in inverse centimeters as a function of the log of the scattering vector Q.
(Å⁻¹). Although many details are hidden in these data (cf. Anovitz et al., 2013a, 2015a), to a first approximation they can be modeled as two straight lines with an intersection (R) calculated as 2π/Q, a correlation length that represents the largest pore size. For all samples, the log-log slope of the curve (n) at higher Q (smaller scales) falls between −3 and −4, reflective of a surface fractal dimension Ds with a value of 6 + n, reflecting both surface roughness and pore-size distribution (Table 2).

Fig. 5 shows the distribution of Ds as a function of depth for the St. Peter Sandstone samples examined. Although the scatter is significant, a clear increase appears as a function of depth, especially for the averaged values of 2.34 at the surface to 2.67 at 12,000 feet (3.66 km). This result is consistent with the presence of grain suturing in the more compacted materials and the observation by Kruhl and Nega (1996) that such boundaries are fractal (see also Majumder and Mamtani, 2009; Mamtani and Greiling, 2010). They found, however, that at optical scales Ds decreases with temperature in igneous and metamorphic rocks. This contrasts with the increase in Ds with depth (increasing pressure and temperature) observed in our samples of sedimentary rocks undergoing diagenesis.

The change in Ds also reflects changes in the pore-size distribution. Since the intensity I(Q) ∼ cQDs−6 (Bale and Schmidt, 1984), then dI/dQ = Ilog Q. This, in turn, is necessarily negative since logQ is negative over the range of measurements. That is as Ds increases, I(Q) decreases and, therefore, an increase in the surface fractal dimension reflects a decrease in the pore size number distribution.

As exemplified by Fig. 4, in most samples a sharp transition can be seen between the low-Q and high-Q parts of the scattering curve. The slope of the scattering curve at low-Q (larger scales) lies between 0 and −1 for all the samples examined. It therefore does not reflect mass-fractal scattering, for which the slope should be between −2 and −3, but an upper limit to the sizes of the observed pores. Fig. 6 shows changes in this correlation length (R) as a function of depth. As before, significant scatter can be seen at any given depth, but the average values suggest a decrease in R from a value of approximately 300 µm near the surface to approximately 190 µm at 12,000 feet (3.66 km, especially if the limited data set near 4000 feet [1.22 km] is ignored). This result is consistent with the general compaction and cementation of the sample and decrease in the larger pore sizes observed in the SEM imagery (Fig. 1).

The apparent difference between the evolution of our Ds values and those of Kruhl and Nega (1996) may be a factor of scale. Anovitz et al. (2013a, 2015a, and see below), found that, for silcrete formation in St. Peter samples from southwest Wisconsin, Ds decreased at coarser SEM image scales in the same rocks in which it increased at smaller scales in the scattering data. Thus, larger scale images must also be considered for the samples from the Illinois and Michigan Basins.

Fractal dimensions can be obtained from SEM images by direct box-counting monofractal (Block et al., 1991) or multifractal analysis (cf. Anovitz et al., 2013a, 2015a; Anovitz and Cole, 2015). The multifractal approach (Mandelbrot, 1989; Evertsz and Mandelbrot, 1992) includes, and is an expansion of, the original monofractal description (Mandelbrot, 1977, 1982) that considers nonuniform variations in the overall density of the image. In a multifractal description, self-similarity is scale dependent, and a single exponent is not sufficient to describe the system. Rather, an array of exponents, known as the singularity spectrum, is used. Following our previous work, we have represented the multifractal spectrum using the Rényi dimension D(q), where q, a positive or negative integer, is a deformation parameter that serves to emphasize high- or low-concentration parts of the image (for a fuller description, see Anovitz et al., 2013a, and references therein). Negative values of q emphasize regions of low concentration of the measure (porosity), and positive values indicate regions of high concentration. Thus, the multifractal spectrum shows how fractal behavior depends on regional variations in pore density. When q is zero, D(0) is the monofractal dimension and, as shown in our previous work (Anovitz et al., 2013a, 2015a), the extent of multifractality can be described simply by showing the difference between D(0) and D at some other value of q.

Fig. 7 shows the monofractal value [D(0)] and the multifractality [D(0) − D(10.75)] of the highest and lowest porosity samples from each drill hole as a function of the image-scale porosity. As can be seen, the values for D(0) decrease with decreasing porosity, and therefore with depth. This is opposite the trend observed for the averaged D values obtained at smaller scales from the scattering data (Fig. 5), but similar to that observed at larger scales by Kruhl and Nega (1996) with increasing temperature. The multifractality of the pore structures, however, increases with decreasing porosity. Thus, the differences in fractal
dimension between the low-concentration and high-concentration parts of the image increase as porosity decreases with depth, but the fractal-ty of the system decreases overall (the pore system becomes smoother).

The SEM images also allow the lacunarity of the system to be calculated. Lacunarity describes the clustering of features in an image (Anovitz and Cole, 2015), and can also be viewed as a function of translational homogeneity. It provides a quantification of how isolated each pore, or group of pores, is from others. Analysis of lacunarity as a function of a given geological process therefore provides a measure of whether that process tends to isolate or homogenize the pore structure, a likely indicator of the relationship between changes in porosity and those in permeability.

**Fig. 8** shows the relationship of both porosity and multifractality to lacunarity. Lacunarity increases with decreasing porosity (increasing depth) and increasing multifractality. Although the absolute lacunarity values are relatively small, the relative change with porosity or multifractality is approximately a factor of two. Therefore, at the scale observed in these images (approximately 1.24 μm–6.35 mm) burial diagenesis tends to make the pore structures smoother but more isolated and more scale dependent.

### 3.4. Detailed (U)SANS analysis

Despite the apparent regularity of the slope and R versus depth data presented in Figs. 5 and 6, as we have pointed out previously (Anovitz et al., 2013a, 2015a), the data do not, in detail, consist of a simple, straight line reflecting a pure surface fractal over a wide range of scattering vectors and intensities. Instead, the scattering curves consist of a series of subtle steps, reflecting individual pore-scale distributions that may, or may not, be overlain on a fractal background. This is best seen by replottting the data in terms of Q^2 as a function of Q, which effectively flattens a scattering curve with a slope of \(-4\) in log Q versus log(I/Q) space (the slope expected for scattering from a smooth surface), allowing the curve to be seen in greater detail by expanding the vertical axis of the graph. Figs. 9 and 10 show this "Porod transform" of the background-subtracted data for the lowest porosity samples at four depths, and the highest porosity samples at five. The data have been slightly truncated primarily to show the higher-Q data, although the intersection between those and the low-Q results is also included.
Several features are apparent in Figs. 9 and 10. Again, the high-\(Q\) portion of the data cannot be completely described by a single slope defining a single \(D_s\) value, but is composed of a series of steps or peaks, often at similar \(Q\) values. Anovitz et al. (2013a) considered and disproved the possibility that these steps reflect the intersections between detector positions in the SANS instrument. That conclusion is further supported in the present data set by the results near \(Q = 10^{-3}\ \text{Å}^{-1}\), which were obtained on the VSANS instrument (not utilized by Anovitz et al., 2013a) at a single detector position, and near \(Q = 10^{-4}\ \text{Å}^{-1}\), which were obtained on a USANS instrument, also at a single detector position.

![Graph showing \(D_s\) average as a function of depth](image1)

**Fig. 5.** The surface fractal dimension \((D_s)\), derived from the overall slope of the high-\(Q\) portion of the scattering diagram as a function of depth. Averages for each depth are shown as open squares. The highest porosity samples for each depth are shown as gray triangles, the lowest porosity samples for each depth are shown as gray circles.

![Graph showing \(R\) as a function of depth](image2)

**Fig. 6.** Correlation length \((R)\) as a function of depth. Averages for each depth are shown as open squares. The highest porosity samples for each depth are shown as gray triangles, and the lowest porosity samples for each depth are shown as gray circles. The trend line is fitted to the average values.
position. Both instruments show similar steps.

The data also show a distinct pattern of changes as a function of depth. At larger scales (near $Q = 10^{-5}$ Å$^{-1}$), the intensity of the scattering curve, and therefore the porosity of the sample in approximately that size range, decreases with depth. This result is as expected and reflects the loss of the larger pores with depth due to compaction and cementation. At smaller scales (near $Q = 10^{-2}$ Å$^{-1}$), however, the deepest samples have the largest intensity, implying increased nanoscale porosity with increasing depth. This pattern is, however, clearer for the lowest porosity samples at each depth (Fig. 9) than for those with the greatest image-scale porosity (Fig. 10).

Figs. 11 and 12 show calculated pore volume distributions for the lowest and highest porosity samples from each drill core, respectively. These distributions were calculated by using the total non-negative least squares approach coded in the Irena plugin for IGOR (Ilavsky and Jemian, 2009). Each was run 10 times to estimate uncertainties, assuming, in each case, spherical pores and that only quartz and pores were present. Data for 04Wi17bPRL and 05Wi02, the lowest and

![Graph showing changes in the monofractal dimension $D(0)$ (squares) and multifractality $D(0) - D(10.75)$ (triangles) as a function of porosity for both the highest porosity (open symbols) and lowest porosity (filled symbols) samples at each depth. Trends showing both decreasing overall fractality and increasing multifractality with increasing depth are well defined.](image1)

![Graph showing relationships between average lacunarity, imagescale porosity (blue diamonds) and multifractality expressed as $D(0) - D(10.75)$ (orange circles). Values for the highest porosity (green circles), lowest porosity (purple circles), and mean values (red squares) for each depth are highlighted. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)](image2)
highest porosity samples from the St. Peter Sandstone at the surface in southwestern Wisconsin (Anovitz et al., 2013a), have been included for comparison, although the total porosity for 04Wi17bPRL is much lower than expected from the trend shown in Fig. 2 because of the large amount of silcrete cement it contains.

It is clear from all pore distributions shown that, rather than consisting of a continuous distribution of pore sizes, the porosity distribution for each sample is dominated by a series of log-normal-like distributions, the apparent intensity of which is somewhat visually suppressed in Figs. 11 and 12 by the logarithmic intensity scale. The calculated pore volume distributions are, however, somewhat dependent on the approach used for the calculation and on the assumed geometry of the scatterers. The latter is illustrated in Fig. 12, which shows the total non-negative least squares solution for sample 2720 for spheroids with aspect ratios of 2 (prolate), 1, and 0.5 (oblate). Changing from a prolate to an oblate spheroid tends to narrow the individual pore distributions, although the total pore volume of necessity remains constant.

Although the intensities of the larger pore diameter distributions appear small, because the total volume of each distribution is the...
integral of each, the volumes of these distributions are significantly larger than those representing smaller pore sizes. In addition, the uncertainties of many of the smallest pore distributions are relatively large. This result is caused by the proximity in both intensity and Q of the scattering data at these scales to the incoherent background.

Many of the calculated pore-size distributions in Figs. 11 and 12 show a minimum near $4 \times 10^4$ Å (4 μm), or $Q \sim 1.6 \times 10^{-4}$. This is well within the range of the USANS data in the area where $Q^2I$ is dropping rapidly in Figs. 9 and 10. This gap is not, however, present in the low-pressure samples from southwest Wisconsin. This result suggests that, within the uncertainties in the total non-negative least squares approach, this is a real break in the pore sizes, not an analytical artifact, and that the pore-size distributions might be separable into two generalized groups of larger and smaller pores, perhaps implying a different genetic origin. The larger of these two distribution groups is, in most cases, composed of three to four relatively regularly spaced individual distributions, the volume of which decreases with increasing burial depth. The opposite appears to occur for the smaller pore distributions. This result is consistent with changes in the scattering curve intensities shown in Figs. 9 and 10.

4. Discussion

4.1. Pore size evolution with depth

From the data summarized above, it is clear that there are significant changes in the pore structure of the St. Peter Sandstone with depth in the Illinois and Michigan Basins. As discussed by Stack (2015), however, a primary question is how overall porosity effects pore size distributions. Available data suggest that different geological processes can have significantly different effects on pore-size distributions. For instance, silcrete formation in the St. Peter Sandstone in southwest Wisconsin tended to fill larger pores first (Anovitz et al., 2013a, 2015a). By contrast, high-temperature, low-pressure metamorphism in the Hatrurim Formation in Israel (Wang et al., 2013) had the opposite effect: smaller pores were filled first. Figs. 14–16 show changes in the overall void ratio $[\phi(\text{pore})/\phi(\text{rock})$, image plus scattering] and in the absolute and normalized pore-size fractions as a function of depth. These values are summarized in Table 2. The total pore volume decreases logarithmically as a function of depth, as does the image-scale porosity (Fig. 2). This is consistent with the logarithmic relation first proposed by Terzaghi (1923; see also Athy, 1930; Biot, 1941; Skempton, 1944; Parasnis, 1960; Fowler and Yang, 1998; Bahr et al., 2001). Although more recent work has included the effects of overpressures (Rubey and Hubbert, 1959; Smith, 1971), elastoplastic behavior (Shi and Wang, 1986; Audet and Fowler, 1992; Schneider, 1993; Schneider et al., 1996), and three-dimensional effects (Pouya et al., 1998), this correspondence suggests that most of the volume change observed is due to compaction. This logarithmic behavior is not, however, consistent across all pore scales. As can be seen in Fig. 15, most of the absolute change in volume occurs through the loss of the larger pores. Little or no change occurs in the absolute volume of the smallest pores ($\log(r(Å)) < 4$), although the fraction of intermediate-sized pores ($5 > \log(r(Å)) > 4$) does appear to decrease (weakly) with depth. Nonetheless, as a percentage of the total porosity (Fig. 16), smaller pores ($\log(r(Å)) < 10^3$ Å) increase significantly. To the extent that these changes can be determined from our data, they do not appear to be logarithmic, but rather nearly linear as a function of depth.

The relative changes in pore volumes at different scales can be seen more clearly by plotting the ratio of the pore volumes (either normalized or not) at each scale of the most porous sample (05Wi02 from southwest Wisconsin) to those of the least porous sample (10835 from the Briggs Unit well in the Michigan Basin), as shown in Fig. 17. Increasing depth and decreasing overall porosity decreases the absolute
porosity only at scales larger than $10^6$ Å; when normalized to 100%, however, all fractions at scales larger than $10^4$ Å increase, although only a slight decrease occurs in the absolute porosity between $10^4$ and $10^6$ Å. In both the normalized and the absolute cases, the fractions of pores with radii below $10^4$ Å increase with depth. Although changes in the absolute pore fraction are small, this ratio increases significantly, by a factor of nearly seven. This result implies a large increase in the number of such small pores. Assuming this porosity is connected (cf. Hu et al., 2012) or that these pores represent pore throats, or both, this result may have a dramatic impact on the available reactive surface area in the rock. These results are consistent with the qualitative conclusions drawn above from Figs. 9 and 10.

4.2. Changes in surface area

The surface area per unit volume is a critical variable of the pore structure because of the limits it places on reactivity, although the total, accessible, and reactive surface areas are not necessarily identical. The effects of compaction and cement overgrowth on the surface area of a grain can be estimated by considering some simple geometry. If we assume that compaction merely reforms grain shapes to fill space, then its effects on the surface area can be estimated by comparing the surface area of a sphere to that of a space-filling regular dodecahedron of the same volume. A sphere with a volume of 1 cm$^3$ has a surface area of 4.84 cm$^2$. If that same sphere is re-formed into a regular dodecahedron, the ratio increases significantly, by a factor of nearly seven. This result implies a large increase in the number of such small pores. Assuming this porosity is connected (cf. Hu et al., 2012) or that these pores represent pore throats, or both, this result may have a dramatic impact on the available reactive surface area in the rock. These results are consistent with the qualitative conclusions drawn above from Figs. 9 and 10.

Similarly, we can estimate the effects of flattening or elongation of pores on surface area. Fig. 18 shows the surface area of a spheroid of rotation as a function of its axial ratio under two conditions: constant volume and constant diameter (of the circular section). For a constant volume spheroid with a volume of 1 cm$^3$, the surface areas are 5.30 cm$^2$ for a 0.5:1 oblate spheroid, 4.84 cm$^2$ for a sphere, 6.43 cm$^2$ for a 2:1 prolate spheroid, and 33.07 cm$^2$ for a 100:1 prolate spheroid, relative changes of −8.7, 32.9, and 414.3%, respectively. For a constant diameter spheroid with a diameter of 1 cm, the surface areas are 2.17 cm$^2$ for a 0.5:1 oblate spheroid, 3.14 cm$^2$ for a sphere, 6.63 cm$^2$ for a 2:1 prolate spheroid, and 462.84 cm$^2$ for a 100:1 prolate spheroid, for relative increases of 44.7, 111.1, and 6881.0%, respectively.

Pore distributions calculated using the total nonnegative least squares approach allow us to make similar calculations for the samples of St. Peter Sandstone. As shown in Fig. 13, the more oblate the pores, the more separated and narrow the estimated pore distributions become, which tends to yield smaller surface areas. These calculations were made for a constant total pore volume, although the number of pores at any given size is not necessarily constant. Surface areas estimated for sample 2720 (Fig. 18) are 1734 cm$^2$/cm$^3$ for a 0.5:1 oblate spheroid, 1796 cm$^2$/cm$^3$ for spherical pores, 2118 cm$^2$/cm$^3$ for a 2:1 prolate spheroid, and 3326 cm$^2$/cm$^3$ for a 100:1 prolate spheroid, for relative increases of 3.6, 17.9, and 57.0%, respectively. For sample 10835, the lowest porosity sample, the relative surface areas are 3452 cm$^2$/cm$^3$ for a 0.5:1 oblate spheroid, 3993 cm$^2$/cm$^3$ for spherical pores, 4328 cm$^2$/cm$^3$ for a 2:1 prolate spheroid, and 7124 cm$^2$/cm$^3$ for a 100:1 prolate spheroid, for relative increases of 12.8, 11.8, and 64.6%, respectively. Even for oblate spheroids, the surface areas for the natural samples calculated using this approach are quite high. This result reflects the larger numbers of very small pores modeled in each sample. The extent to which this represents accessible or reactive
porosity, however, remains uncertain.

When the results of the fits to our scattering data are compared with Fig. 18 on the prolate side, model changes for both constant volume and constant diameter compression are larger than observed. On the oblate side, however, the observed change lies between the two alternatives and decreases with a decreasing axial ratio. This suggests that the majority of pores composing the measured surface area (i.e., the smaller pores) are slit- or discus-like and more consistent with interfacial spaces than with channel-like pores along intergrain triple junctions. Lee et al. (1991) suggested, following Toramaru and Fujii (1986) and Watson and Brenan (1987; see also Bulau et al., 1979; Waff and Bulau, 1979; Von Bargen and Waff, 1986), that for wetting angles less than 60°, “fluid-filled triple junction channels interconnect at four-grain junction tetrahedra having concave faces. Grain boundaries are nonwetted; any fluid present there occurs as discus-shaped fluid inclusions” (p. 10,025), whereas for larger wetting angles, “the bulk of the fluid is isolated at four-grain junction tetrahedra having flat to convex faces. Fluid present in triple junctions is segregated into discrete, football-shaped fluid inclusions. Similarly, in grain boundaries, fluid is segregated into discus-shaped fluid inclusions” (p. 10,025).

Thus, our results suggest that these “discus-shaped fluid inclusions” form a significant portion of the overall pore structure. Laporte and Watson (1991) obtained similar experimental results, including evidence of discus-shaped pores on grain faces. In SEM images, many of these grain face pores appear to be isolated, what Hiraga et al. (2001) called a “necklace microstructure” when observed by transmission electron microscopy.

Such grain boundary pores may or may not remain connected to the rest of the pore system, but open connectivity may occur at the nanoscale (see Royne and Jamtveit, 2015). For instance, Kruhl et al. (2013) found that quartz grain boundaries in contact and regional metamorphic rocks were open on the nanometer scale. It is unclear, however, whether these surfaces are wetted (see Farver and Yund, 1992, 1995). In addition, Takei (2010) found that under nonhydrostatic stresses, additional grain boundary wetting occurred on surfaces normal to the maximum tensile stress direction ($\sigma_3$; see also Daines and Kohlstedt, 1997; Zimmerman et al., 1999; Takei and Katz, 2013). Similar complexities also have been found on grain surfaces in quartz mylonites (Mancktelow et al., 1998). These conclusions are clearly consistent with the observation above that oblate or discus-shaped pores are dominant when the overall surface area is considered. The extent to which such porosity is connected to, or isolated from the remainder will, therefore, have a significant effect on transport and reactivity.

**Fig. 13.** Pore volume distributions and scattering curves for sample 2720 calculated for spherical aspect ratios of 100 (prolate), 1, and 0.5 (oblate). Note that in all cases, the pore structure consists of a series of log-normal-like distributions and that the sizes of each distribution are similar for all three solutions.

**Fig. 14.** Change in the void ratio [$\phi$(pore)/$\phi$(rock)] as a function of depth.
Fig. 19 (triangles) shows the changes in surface area as a function of depth calculated from the pore volume distributions. As the above analysis suggests, the pores controlling the total surface area are largely oblate, whether connected or not. Because variations in axial ratio make relatively small differences in the surface area calculated for oblate systems, we can approximate this variation by calculating the surface area for spherical pores. The results show a general increase in surface area with depth. The surface area calculated in this manner for sample 10835 is approximately a factor of two larger than that of sample 2720 (ratios range from 1.99 to 2.17, depending on the axial ratio). In addition, a large percentage of this surface area is from the smallest pores. For sample 2702, 44% of the surface area is on pores.

Fig. 15. Average pore fractions as a function of depth for various pore-size ranges: log(\(r(\text{Å})\)) > 1.4 (orange circles), log(\(r(\text{Å})\)) > 5 (purple squares), 5 > log(\(r(\text{Å})\)) > 4 (green triangles), 4 > log(\(r(\text{Å})\)) > 1.4 (red squares), log(\(r(\text{Å})\)) < 1.4 (blue squares). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Fig. 16. Volume fraction normalized to 100% total. Pores with radii greater than 10^4 Å are shown as solid circles, and pores with radii smaller than 10^4 Å are shown as open circles.
with diameters less than 1000 Å, and for sample 10835, 67% is from pores of this size range. The surface areas for some of the 3000- and 4000-foot-depth (0.91- and 1.22-km-depth) samples increased by significantly larger ratios (up to about 5.5), implying that a process other than compaction was involved. This result may be explained by the fact that the 4118-foot (1.26-km) sample contained abundant fine-
grained dolomite cement with rare, later, coarser euhedral dolomite rhombohedra, which appears to have yielded an R value well below the overall trend (Fig. 6). The relatively large value for sample 10816 (Table 2) may be due to its significant clay content.

The proportional increases in surface area calculated from the pore volume distributions (Fig. 19) are significantly larger than those predicted for either compaction or cement overgrowth precipitation. The pore shape assumed does not, however, account for the observed increase in the surface fractal dimension (Fig. 5), which suggests a roughening of the pore–grain interface, and therefore predicts an increase in the surface area larger than that for polyhedral compression or overgrowth formation. Where spheroidal pore shapes are assumed, such roughening is reflected by an increase in the number of smaller diameter pores. This may be related to the various dissolution events experienced by these samples (Hoholick et al., 1984; Fishman, 1997; Freiburg et al., 2016). As discussed by Anovitz et al. (2015a), Cohen (1987) noted that, unlike sintered materials, sedimentary rocks tend to form fractally-rough grain boundaries. He suggested this “antisintering” behavior was due to reduction of grain or pore free energy by interaction with pore water. Wong et al. (1986) noted that in clean water, roughness is controlled by the competition between thermal fluctuations and surface tension; therefore, a roughening transition temperature, $T_n$, occurs above which surfaces become rougher with time. In addition, random contamination of atomic-scale growth sites by impurities lowers the $T_n$, making its occurrence in sedimentary environments more likely (Wong, 1985; Barabási and Stanley, 1995).

Following Allen (1991) and Anovitz et al. (2009), surface areas can be calculated from the surface fractal dimension $D_s$ as

\[
\left(\frac{S}{V}\right) = \left(\frac{S}{V}_0\right) \left(\frac{r}{d_0}\right)^{2-D_s}
\]  

where $(S/V)_0$ is the surface area-to-volume ratio for a smooth particle, $r$ is the fractal “ruler” length, $d_0$ is the correlation length representing the upper limit of surface fractal behavior, and $D_s$ is the surface fractal dimension. Thus, if $D_s = 2$, the particle is smooth and $(S/V)$ is equal to $(S/V)_0$, and if $D_s > 2$, $(S/V)$ is greater than $(S/V)_0$. It should be noted, however, that these surface areas represent only those surfaces that scatter neutrons. Tight quartz/quartz grain boundaries, for instance, will not be represented because no scattering length contrast exists. Because the rocks consist largely of quartz and are essentially isotropic, a value of 4.834 Å, calculated as the cube root of the quartz unit-cell volume (113.0 Å$^3$), was selected for the ruler length. This is, however, an arbitrary selection. The larger the value chosen for $r$, the smaller the values of $(S/V)/(S/V)_0$. In addition, because $r$ is raised to the $2-D_s$ power, the closer the value of $r$ becomes to those of $d_0$, the smaller the effect of changes in $D_s$ on $(S/V)/(S/V)_0$. For these calculations, the values of $R$ shown in Fig. 6 and Table 2 were used for $d_0$. These are larger than the values for 2.1, the surface fractal/mass fractal boundary (cf. Wang et al., 2013), because no such boundary was observed in our data.

The results of this calculation are shown as circles in Fig. 19. Data are not shown for sample Briggs-10816, which has a much higher surface area, probably because of its clay content. The $(S/V)/(S/V)_0$ values for both the lowest and highest porosity samples increase significantly between about 3000 and 5000 feet (910 and 1520 m), although this is at somewhat deeper depths for the lower porosity samples, which may be due to dolomite cement in sample 4118. This correlates to porosities of approximately 0.1 for the lowest porosity samples and 0.3 for the highest, and correlate with a change in the porosity type. Hoholick et al. (1984) noted that primary porosity is dominant at depths to 4000 feet (1220 m), but secondary porosity becomes dominant for deeper samples. The single available data point from samples below 6000 feet (1830 m) from Briggs-10835 suggests that this increase continues with increasing depth. This result reflects increases in the $D_s$ values with depth as well, and probably the increasing presence of sutured grain boundaries, and possibly multiple generations of dolomite cementation and other fractal roughening. It must be remembered, however, that $D_s$ in this case is an average over the whole scale or intensity range of the scattering data and does not necessarily reflect the multiple log-normal-like pore distributions seen when the data are examined in detail.
4.3. Comparisons with the St. Peter in southwestern Wisconsin

Contrasting the data obtained in this study with those from the St. Peter in southwestern Wisconsin (Anovitz et al., 2013a), which has never been buried to any significant depth, allows a comparison to be made between the effects of overgrowth formation in samples that have not been deeply buried and burial diagenesis on the pore structure of a sandstone. As noted above, the two data sets show similar results in terms of the evolution of the pore-size distribution, fractal and multifractal behavior, and lacunarity with depth. In addition, in both data sets the pore distribution is noncontinuous. Anovitz et al. (2013a, 2015a) suggested that the multiple distributions observed in the St. Peter Sandstones were due to packing of polydispersed particles following a nondeterministic form of an Apollonian gasket, with smaller particles packing into the spaces between the larger ones. A similar model has been suggested for soils (Hillel, 1980) and dense granular systems (Anishchik and Medvedev, 1995). In fact, as noted by Desmond and Weeks (2014), such packings have attracted a great deal of interest because of their applications to a wider variety of materials and processes, including living cells, granular media, emulsions or colloids, glasses, amorphous solids, jamming, and suspension viscosity, and a large number of articles have been published on the topic. Sohn and Moreland (1968) investigated the effect of particle-size distributions on the packing efficiency of natural sands, in part using Ottawa Sand, mined in Ottawa, Illinois, from the St. Peter Sandstone, as a starting material. They found that for both Gaussian and log-normal distributions packing density increased with the polydispersivity of the sand distribution. Such a result is likely due to packing of smaller grains between larger ones, and thus will generate a range of smaller pore sizes, which will also be controlled by grain size, sorting and shape, both initially and as the rock evolves (Beard and Weyl, 1973). In fact, Amirjanov and Sobolev (2006, 2012) showed that pore fraction is an inverse power-law function of the number of particles for both Apollonian and random Apollonian packings. This is also true as a function of the number of grains with a radius larger than some cutoff value (C; Herrmann et al., 1990; Anishchik and Medvedev, 1995). In neither case, however, was the effect on the pore-size distribution considered, nor is it immediately clear whether such scalings would give rise to a stepped pore-size function. In part, this is because the definition of pore sizes depends on pore shape, which is not spherical for packings of spherical particles. For the St. Peter, this analysis is made more uncertain by the relatively high sorting of the sand, which reduces the number of grains able to fit into the smaller interparticle spaces, although Fig. 1 suggests this may be less true for the deeper material from the Michigan Basin.

The relationship between multifractality and lacunarity shown in Fig. 8 is also consistent with that observed in the St. Peter Sandstone in southwest Wisconsin (Anovitz et al., 2013a), where silcrete precipitation with little or no compaction is the primary cause of changes in porosity, and with our observations of experimental precipitation of quartz in samples of St. Peter Sandstone (Anovitz et al., 2015a). Although the multifractals are similar, the absolute change in $D(0)$ is somewhat larger for the silcretes than for the basinal sandstones over a similar range of porosities; thus, $D(0)$ values for low-porosity samples tend to be smaller for silcretes than for basinal sandstones. The same is true at finer scales (Fig. 5) in which, as depth increases and average porosity decreases, the surface fractal dimension $D_s$ increases. As noted above, this is in contrast to what is observed at larger scales. The $D_s$ values observed in the silcretes (2.19–2.56) are also smaller than those in the basinal sandstones (2.35–2.68), although the extent of the latter is primarily a function of samples from the Briggs Unit core. A reasonable positive correlation also exists between the lacunarity and $D(0) − D(10.75)$ (Fig. 8), suggesting that, to some extent, lacunarity and multifractality are recording similar phenomena. It is, therefore, clear that several qualitative similarities exist between changes in the pore structure due to simple cementation and those observed with burial-related processes. It is uncertain, however, whether this result implies that the overall effects of cementation and compaction on the pore structures of sandstone are similar, or whether the current pore structures of the samples from the Illinois and Michigan Basins are dominated by precipitatve or dissolution–reprecipitation processes that are qualitatively similar to those observed during silcrete formation.

4.4. Effects of compression

The changes in pore-size distributions observed in the St. Peter Sandstone are also consistent with other predictions from soil research (cf. Nimmo, 2004, 2013), which suggest mechanical compression can decrease the size of, or even close macropores. The number of smaller pores may increase because of the size reduction of the larger, or decrease because of disaggregation, reducing the number of intra-aggregate pores, although the latter is less likely in a clean sandstone like the St. Peter. For example, in an experimental study of soil compression, Kutilek et al. (2006) found a decrease in larger scale porosity ($> 10 \mu m$) and, in some cases, an increase in the fraction of smaller pores ($< 10 \mu m$), suggesting the pore-size evolution observed could be largely a physical, rather than a chemical, effect, although the same result was observed in the relatively uncompressed material from southwestern Wisconsin. Anovitz et al. (2013a), however, argued for a chemical or flow-dominated origin. Several authors (e.g., Hedges and Whitelam, 2012; Wang et al., 2013) have argued that precipitation may be enhanced in smaller pores because of increased surface area of the pore wall, whereas others have shown that precipitation can also be enhanced in larger pores, either due to enhanced flow and transport or because precipitation in smaller pores is inhibited owing to increased surface energy of the precipitate (Prieto et al., 1990; Putnis et al., 1995; Putnis and Maathe, 2001; Emmanuel and Berkowitz, 2007; Emmanuel and Ague, 2009; Emmanuel et al., 2010; Cook et al., 2011). Available data suggest that this difference may be process related and that both options or neither (Borgia et al., 2012) can occur. Nonetheless, data obtained in this study clearly indicate that pore-size dependence needs to be considered in terms of the physical, as well as the chemical, processes to which a given rock unit is subjected.

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References

Abramoff, M.D., Magalhaes, P.J., Ram, S.J., 2004. Image processing with ImageJ.