Transient Analysis of a Circular Foil Gage in a Convective and Radiative Environment

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ABSTRACT

An analysis of a circular thin-foil gage is presented that includes transient effects, convective heat transfer, and an arbitrary time-varying boundary condition at the foil edge to account for fluctuations in cooling water temperature. The governing energy equation is solved using Laplace transform to obtain the temporal and spatial temperature distributions of the foil. Under constant temperature at the foil edge and constant thermal radiative heat flux, closed-form response curves for the gage under various modes of heat transfer are provided. Steady-state results are also presented as limiting cases.

Keywords: Heat conduction; heat convection, heat flux gage; thermal radiation

Introduction

In 1953, Gardon [1] developed a circular thin-foil gage to measure thermal radiation and provided a simple steady state heat conduction analysis to describe the operation of the gage. Since then circular thin-foil gages, sometimes referred to as Gardon gages named in honor of Gardon, have been used extensively in fire research and testing [2], solar radiation measurements [3], and other aerospace applications [4]. A cross section of the gage is depicted in Figure 1. The gage sensing element is made from constantan metal foil in the form of a circular disk and is attached to a water-cooled constant-temperature copper ring. When the incident heat flux strikes the foil, the energy absorbed by the foil diffuses radially toward the outer circumference of the foil and into the copper ring acting as a heat sink, causing a temperature difference between the foil center and the edge. The temperature difference, which is measured using a differential thermocouple, can be related to the incident heat flux.

Analytical studies have been reported in the literature to examine various aspects of the thin-foil gage operation and performance. Gardon [1] originally modeled the gage by treating it as a one-dimensional steady-state heat conduction through the foil in the radial direction to obtain the temperature distribution in the foil with an applied uniform thermal radiative heat flux on the foil. Malone [4] used a one dimensional steady-state formulation to analyze convective heat transfer to or from a radiantly heated foil and heat loss down the center wire. Ash [5] studied the transient response characteristics of thin-foil heat flux sensors under thermal radiative or convective environment. Kirchoff [6] provided a two-dimensional transient heat conduction analysis on the foil to study the effect of foil thickness on the gage response without considering...
heat convection. A one-dimensional transient heat conduction model was developed by Keltner and Wildin [7] to examine transient response of the gages. Borell and Diller [8], Kuo and Kulkarni [9], and Fu et al. [10] expanded the steady-state analysis of Gardon [1] by including convective heat transfer at the foil surface and provided calibration corrections for convective heat transfer. Recently, Fu et al. [11] applied the Duhamel theorem to study Gardon gage exposed to fast heat flux transients.

Here the analysis is expanded to include transient effects, convective heat transfer, and the time-varying heat-sink temperature to reflect fluctuations in cooling water temperature. However, we still limit our analysis to one dimension in the radial direction of the foil exposed to a constant radiative heat flux.

**Formulation and Analysis**

Assuming no heat conduction loss along the copper wire at the center of the foil, no convective heat transfer on the backside of the gage, and constant thermophysical properties of the foil, the energy equation for the foil is

\[
\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{q_i}{k_H} + \frac{h}{k_H} (T_\infty - T) = \frac{1}{\alpha} \frac{\partial T}{\partial t}
\]

with the following initial and boundary conditions

I.C.: at \( t = 0 \), \( T = T_0 \) for \( 0 \leq r \leq R \)

B.C. 1: at \( r = 0 \), \( \frac{\partial T}{\partial r} = 0 \) or \( T \) is finite

B.C. 2: at \( r = R \) \( T = f(t) \)

The boundary condition (B.C. 2) at \( r = R \) reflects an arbitrary time-varying heat-sink temperature.

Introducing the following dimensionless variables:

\[
\xi = r / R \quad \tau = \alpha t / R^2 \quad \theta = T / T_0
\]

With \( Nu = hR / k_f \), \( Q^* = q_iR^2 / kHT_0 \), \( \kappa = k_f / k \), \( \delta = R / H \), and \( Nu^* = \kappa \delta Nu \), Eq. (1) can be non-dimensionalized to

\[
\frac{\partial^2 \theta}{\partial \xi^2} + \frac{1}{\xi} \frac{\partial \theta}{\partial \xi} - Nu^* \theta + (Q^* + Nu^* \theta) = \frac{\partial \theta}{\partial \tau}
\]

I.C.: at \( \tau = 0 \), \( \theta = \theta_0 \) for \( 0 \leq \xi \leq 1 \)

B.C. 1: at \( \xi = 0 \), \( \partial \theta / \partial \xi = 0 \)
B.C. 2: at \( \xi = 1 \) \quad \theta = \theta_f = f(\tau)/T_o

Equation (2), together with the non-dimensionalized initial and boundary conditions, can be solved using Laplace transform. In the following, the Laplace transform and its inverse transform operators are represented by \( L \) and \( L^{-1} \) respectively, and we use \( \hat{\theta}(\xi, s) \) to denote the transformed variable of \( \theta(\xi, \tau) \) with respect to \( \tau \).

Taking Laplace transform of Eq. (2) with respect to \( \tau \),

\[
L \left\{ \frac{\partial^2 \theta}{\partial \xi^2} + \frac{1}{\xi} \frac{\partial \theta}{\partial \xi} - \text{Nu}^* \theta + (\text{Q}^* + \text{Nu}^* \theta_\infty) \right\} = L \left\{ \frac{\partial \theta}{\partial \tau} \right\}
\]

\[
\frac{\partial^2 \hat{\theta}(\xi, s)}{\partial \xi^2} + \frac{1}{\xi} \frac{\partial \hat{\theta}(\xi, s)}{\partial \xi} - \text{Nu}^* \hat{\theta}(\xi, s) + \frac{(\text{Q}^* + \text{Nu}^* \theta_\infty)}{s} = s\hat{\theta}(\xi, s) - \theta(\xi, \tau = 0)
\]

For clarity, we simply use \( \hat{\theta} \) instead of \( \hat{\theta}(\xi, s) \) in what follows.

\[
\frac{d^2 \hat{\theta}}{d\xi^2} + \frac{1}{\xi} \frac{d \hat{\theta}}{d\xi} - \text{Nu}^* \hat{\theta} + \frac{(\text{Q}^* + \text{Nu}^* \theta_\infty)}{s} = s\hat{\theta} - \theta_o
\]

\[
\frac{d^2 \hat{\theta}}{d\xi^2} + \frac{1}{\xi} \frac{d \hat{\theta}}{d\xi} - (\text{Nu}^* + s) \hat{\theta} = -[\theta_o + \frac{(\text{Q}^* + \text{Nu}^* \theta_\infty)}{s}]
\]

B.C. 1: at \( \xi = 0 \), \( L \left\{ \frac{\partial \theta}{\partial \xi} \right\} = \frac{\partial \hat{\theta}}{\partial \xi} = 0 \)

B.C. 2: at \( \xi = 1 \) \quad \hat{\theta} = \hat{\theta}_f

The general solution to Eq. (3) is

\[
\hat{\theta}(\xi, s) = \frac{[\theta_o + (\text{Q}^* + \text{Nu}^* \theta_\infty)]}{\text{Nu}^* + s} + c_1 J_0(i\xi \sqrt{\text{Nu}^* + s}) + c_2 Y_0(i\xi \sqrt{\text{Nu}^* + s})
\]

(4)

where \( c_1 \) and \( c_2 \) are constants and \( J_0 \) and \( Y_0 \) are the Bessel functions of the first and second kinds of zero order respectively. From B.C. 1, for \( \hat{\theta}(\xi, s) \) to be finite at \( \xi = 0 \), \( c_2 = 0 \) since \( Y_0(0) \) approaches minus infinity.

From B.C. 2,
\[ c_1 = \frac{\hat{\theta}_f (NU^* + s) - [\theta_0 + (Q^* + NU^* \theta_\infty)]}{(NU^* + s)J_0(i\sqrt{NU^* + s})} \]

Equation (4) then becomes

\[
\hat{\theta}(\xi, s) = \frac{\theta_0}{(NU^* + s)} + \frac{(Q^* + NU^* \theta_\infty)}{s(NU^* + s)} \]

\[ + \left\{ \hat{\theta}_f - \frac{\theta_0}{(NU^* + s)} - \frac{(Q^* + NU^* \theta_\infty)}{s(NU^* + s)} \right\} \]

\[ J_0(i\xi \sqrt{NU^* + s}) \]

\[ J_0(i\sqrt{NU^* + s}) \]

(5)

Taking the inverse Laplace transform of Eq. (5),

\[
\theta(\xi, \tau) = L^{-1} \left\{ \frac{\theta_0}{(NU^* + s)} \right\} + L^{-1} \left\{ \frac{(Q^* + NU^* \theta_\infty)}{s(NU^* + s)} \right\} \]

\[ + L^{-1} \left\{ \hat{\theta}_f - \frac{\theta_0}{(NU^* + s)} - \frac{(Q^* + NU^* \theta_\infty)}{s(NU^* + s)} \right\} \frac{J_0(i\xi \sqrt{NU^* + s})}{J_0(i\sqrt{NU^* + s})} \]

\[ - L^{-1} \left\{ \frac{(Q^* + NU^* \theta_\infty)}{s(NU^* + s)} \right\} \frac{J_0(i\xi \sqrt{NU^* + s})}{J_0(i\sqrt{NU^* + s})} \]

(6)

The first two inverse Laplace transform terms on the right-hand side of Eq. (6) can be easily obtained.

\[ L^{-1} \left\{ \frac{\theta_0}{(NU^* + s)} \right\} = \theta_0 \exp(-NU^* \tau) \]

(7)

\[ L^{-1} \left\{ \frac{(Q^* + NU^* \theta_\infty)}{s(NU^* + s)} \right\} = \frac{(Q^* + NU^* \theta_\infty)}{NU^*} \{1 - \exp(-NU^* \tau)\} \]

(8)

The fourth term on the right-hand side of Eq. (6) can be evaluated as follows. From Stephenson [12],

\[ L^{-1} \left\{ \frac{1}{s} J_0(i\xi \sqrt{s}) \right\} = 1 - 2 \sum_{n=1}^{\infty} \frac{\exp(-\lambda_n^2 \tau) J_0(\lambda_n \xi)}{\lambda_n J_1(\lambda_n)} \]

(9)

where \( \lambda_1, \lambda_2, \lambda_3, \lambda_4, \ldots \) are the positive roots of the equation \( J_0(\lambda_n) = 0 \). An application of the shift theorem [13] results in
The last term on the right-hand side of Eq. (6) can now be obtained using the convolution theorem [13] and Eq. (10).

\[
L^{-1}\left\{ \frac{\theta_0}{(Nu^* + s)} \right\} = \theta_0 \exp(-Nu^*\tau) \left\{ 1 - 2 \sum_{n=1}^{\infty} \frac{\exp(-\lambda_n^2\tau) J_0(\lambda_n\xi)}{\lambda_n J_1(\lambda_n)} \right\}
\]

The third term on the right-hand side of Eq. (6) can be obtained as follows. First, we re-write this term as

\[
L^{-1}\left\{ \frac{(Q^* + Nu^*\theta_\infty)}{s(Nu^* + s)} \right\} =
\]

\[
(Q^* + Nu^*\theta_\infty) \int_0^\infty \exp(-Nu^*\tau) \left\{ 1 - 2 \sum_{n=1}^{\infty} \frac{\exp(-\lambda_n^2\tau) J_0(\lambda_n\xi)}{\lambda_n J_1(\lambda_n)} \right\} d\tau^*
\]
Then an application of the shift and convolution theorems [13] results in

Substituting Eqs. (7), (8), (10), (11), and (12) into Eq. (6) and after a few algebraic manipulations, the following dimensionless temperature distribution \( \theta(\xi, \tau) \) of the foil is obtained.
\[
\theta(\xi, \tau) = \theta_0 \exp(-Nu^* \tau) \\
+ \int_0^\tau \exp[-Nu^*(\tau - \tau^*)] \left\{ 1 - 2 \sum_{n=1}^{\infty} \frac{\exp[-\lambda_n^2 (\tau - \tau^*)]}{\lambda_n} \frac{J_0(\lambda_n \xi)}{J_1(\lambda_n)} \left[ \frac{d\theta_f}{d\tau^*} \right] \right\} d\tau^* \\
+ Nu^* \int_0^\tau \exp[-Nu^*(\tau - \tau^*)] \left\{ 1 - 2 \sum_{n=1}^{\infty} \frac{\exp[-\lambda_n^2 (\tau - \tau^*)]}{\lambda_n} \frac{J_0(\lambda_n \xi)}{J_1(\lambda_n)} \right\} \theta_f(\tau^*) d\tau^* \\
+ \theta_f (\tau = 0) \exp(-Nu^* \tau) \left\{ 1 - 2 \sum_{n=1}^{\infty} \frac{\exp[-\lambda_n^2 \tau]}{\lambda_n} \frac{J_0(\lambda_n \xi)}{J_1(\lambda_n)} \right\} \\
+ (2Q' + 2Nu^* \theta_0) \sum_{n=1}^{\infty} \frac{J_0(\lambda_n \xi)}{\lambda_n J_1(\lambda_n)} \frac{1 - \exp[-\tau(Nu^* + \lambda_n^2)]}{Nu^* + \lambda_n^2} 
\]

(13)

Since the temperature difference between the center of the gage (\( \xi = 0 \)) and the edge (heat sink) of the gage (\( \xi = 1 \)) is directly related to the thin-foil gage operation and measurements, the general expressions for \( \theta(\xi = 0, \tau) \) and \( \theta(\xi = 1, \tau) \) with \( J_0(0) = 1 \) and \( J_0(\lambda_n) = 0 \) for all \( \lambda_n \) are

\[
\theta(\xi = 0, \tau) = 2\theta_0 \exp(-Nu^* \tau) \sum_{n=1}^{\infty} \frac{\exp(-\lambda_n^2 \tau)}{\lambda_n} \frac{1}{J_1(\lambda_n)} \\
+ \int_0^\tau \exp[-Nu^*(\tau - \tau^*)] \left\{ 1 - 2 \sum_{n=1}^{\infty} \frac{\exp[-\lambda_n^2 (\tau - \tau^*)]}{\lambda_n} \frac{1}{J_1(\lambda_n)} \left[ \frac{d\theta_f}{d\tau^*} \right] \right\} d\tau^* \\
+ Nu^* \int_0^\tau \exp[-Nu^*(\tau - \tau^*)] \left\{ 1 - 2 \sum_{n=1}^{\infty} \frac{\exp[-\lambda_n^2 (\tau - \tau^*)]}{\lambda_n} \frac{1}{J_1(\lambda_n)} \right\} \theta_f(\tau^*) d\tau^* \\
+ \theta_f (\tau = 0) \exp(-Nu^* \tau) \left\{ 1 - 2 \sum_{n=1}^{\infty} \frac{\exp(-\lambda_n^2 \tau)}{\lambda_n} \frac{1}{J_1(\lambda_n)} \right\} \\
+ (2Q' + 2Nu^* \theta_0) \sum_{n=1}^{\infty} \frac{1}{\lambda_n J_1(\lambda_n)} \frac{1 - \exp[-\tau(Nu^* + \lambda_n^2)]}{Nu^* + \lambda_n^2} 
\]

(14)
\[ \theta(\xi = 1, \tau) = \int_0^\tau \exp[-Nu^*(\tau - \tau^*)] \left[ \frac{d\theta}{d\tau^*} \right] d\tau^* + Nu^* \int_0^\tau \exp[-Nu^*(\tau - \tau^*)] \theta_f(\tau^*) d\tau^* + \theta_f(\tau = 0) \exp(-Nu^*\tau) \]
Figure 2 shows the response curve of \( \{\theta(\xi = 0, \tau) - \theta(\xi = 1, \tau)\} \) as a function of \( \tau \) at various \( Q^* \) and \( Nu^* = 0 \). In obtaining the results in Figure 2, only the sum of the first five terms in the series is needed to achieve convergence.

As \( \tau \to \infty \) (steady state), Eq. (18) becomes

\[
\theta(\xi, \tau \to \infty) = \theta_o + \frac{Q^*}{4}(1 - \xi^2)
\]  

(20)

The above equation was first derived by Gardon [1]. From Eq. (19),

\[
\theta(\xi = 0, \tau \to \infty) - \theta(\xi = 1, \tau \to \infty) = 2Q^* \sum_{n=1}^{\infty} \frac{1}{\lambda_n^3 J_1(\lambda_n)}
\]

From Eq. (17) and with \( \xi = 0 \) and \( J_0(0) = 1, \sum_{n=1}^{\infty} \frac{1}{J_1(\lambda_n)} \left[ \frac{1}{\lambda_n^3} \right] = \frac{1}{8} \)

\[
\theta(\xi = 0, \tau \to \infty) - \theta(\xi = 1, \tau \to \infty) = \frac{Q^*}{4}
\]  

(21)

Equation (21) was used by Gardon [1] to obtain thermal radiative heat flux from temperature difference measurements between the foil center and edge and to analyze the performance of the gage.

**Case 2. No radiative heat transfer with \( \theta_f = \theta_o = \) constant**

If \( Q^* = 0 \) and \( \theta_f(\tau) = \theta_o = \) constant, it can be shown that Eq. (13) reduces to

\[
\theta(\xi, \tau) = \theta_0 + 2Nu^*(\theta_\infty - \theta_0) \sum_{n=1}^{\infty} \frac{J_0(\lambda_n \xi)}{\lambda_n J_1(\lambda_n)} \left[ 1 - \exp[-\tau(Nu^* + \lambda_n^2)] \right] \left( \frac{1}{Nu^* + \lambda_n^2} \right)
\]  

(22)

and

\[
\theta(\xi = 0, \tau) - \theta(\xi = 1, \tau) = 2Nu^*(\theta_\infty - \theta_0) \sum_{n=1}^{\infty} \frac{1}{\lambda_n J_1(\lambda_n)} \left[ 1 - \exp[-\tau(Nu^* + \lambda_n^2)] \right] \left( \frac{1}{Nu^* + \lambda_n^2} \right)
\]  

(23)

It is interesting to compare our results to Ash’s work [5]. The corresponding equation derived by Ash [5] and expressed in terms of the dimensionless variables used in the current work is

\[
\theta(\xi = 0, \tau) - \theta(\xi = 1, \tau) = (\theta_\infty - \theta_0) \left( 1 - \frac{1}{I_0(\sqrt{Nu^*})} - 2Nu^* \sum_{n=1}^{\infty} \frac{\exp[-(Nu^* + \lambda_n^2)\tau]}{\lambda_n J_1(\lambda_n)(Nu^* + \lambda_n^2)} \right)
\]  

(24)
Figure 3 shows the response curves of \( \{ \theta(\xi = 0, \tau) - \theta(\xi = 1, \tau) \} \) as a function of \( \tau \) at various \( Nu^* \) with \( Q^* = 0 \) and \( \theta_f = 2 \) calculated using Eq. (23) from our work and Eq. (24) from Ash (1969). Since the results obtained from Eq. (23) is identical to those from Eq. (24), as demonstrated in Figure 3, we obtain the following identity.

\[
\left\{ 1 - \frac{1}{I_0(\sqrt{Nu^*})} \right\} = 2Nu^* \sum_{n=1}^{\infty} \frac{1}{\lambda_n J_1(\lambda_n)(Nu^* + \lambda_n^2)}
\]  

(25)

As \( \tau \to \infty \) (steady state), Eq. (23) becomes

\[
\theta(\xi = 0, \tau \to \infty) - \theta(\xi = 1, \tau \to \infty) = 2Nu^* (\theta_\infty - \theta_0) \sum_{n=1}^{\infty} \frac{1}{\lambda_n J_1(\lambda_n)(Nu^* + \lambda_n^2)}
\]

(26)

It is worth pointing out that the corresponding equation derived by Kuo and Kulkarin [9] and expressed in terms of the dimensionless variables used in the current work is

\[
\theta_{\xi=0} - \theta_{\xi=1} = (\theta_\infty - \theta_0)[1 - \frac{1}{I_0(\sqrt{Nu^*})}]
\]

(27)

Based on Eq. (25), Eq. (26) is identical to Eq. (27), which can also be deduced from Eq. (24) when \( \tau \to \infty \).

**Case 3. Pure heat conduction with \( \theta_f = \theta_0 = \text{constant} \)**

In this case, \( Q^* = 0 \), \( Nu^* = 0 \), and \( \theta_f = \theta_0 \), then Eq. (13) trivially becomes

\[
\theta(\xi, \tau) = \theta_0
\]

(28)

**Case 4. Radiative and convective heat transfer with \( \theta_f = \theta_0 = \text{constant} \)**

In this case, \( Q^* \neq 0 \), \( Nu^* \neq 0 \), and \( \theta_f(\tau) = \theta_0 = \text{constant} \). Eq. (13) becomes
\[ \theta(\xi, \tau) = \theta_0 \exp(-Nu^* \tau) \]
\[ + \theta_0 Nu^* \int_0^\tau \exp[-Nu^*(\tau - \tau^*)] \frac{\exp[-\lambda_n^2 (\tau - \tau^*)]}{J_n(\lambda_n \xi)} d\tau^* \]
\[ + \theta_0 \exp(-Nu^* \tau) \left\{ 1 - 2 \sum_{n=1}^{\infty} \frac{\exp(-\lambda_n^2 \tau)}{J_n(\lambda_n \xi)} \frac{J_0(\lambda_n \xi)}{J_1(\lambda_n \xi)} \right\} \]
\[ - \theta_0 \exp(-Nu^* \tau) \left\{ 1 - 2 \sum_{n=1}^{\infty} \frac{\exp(-\lambda_n^2 \tau)}{J_n(\lambda_n \xi)} \frac{J_0(\lambda_n \xi)}{J_1(\lambda_n \xi)} \right\} \]
\[ + (2Q^* + 2Nu^* \theta_\infty) \sum_{n=1}^{\infty} \frac{J_0(\lambda_n \xi)}{\lambda_n J_1(\lambda_n \xi)} \left\{ 1 - \frac{\exp[-\tau(Nu^* + \lambda_n^2 \tau)]}{Nu^* + \lambda_n^2} \right\} \]
\[ (29) \]

and

\[ \theta(\xi = 0, \tau) - \theta(\xi = 1, \tau) = 2[Q^* + Nu^*(\theta_\infty - \theta_0)] \sum_{n=1}^{\infty} \frac{1}{\lambda_n J_1(\lambda_n \xi)} \left\{ 1 - \frac{\exp[-\tau(Nu^* + \lambda_n^2 \tau)]}{Nu^* + \lambda_n^2} \right\} \]
\[ (30) \]

Figure 4 shows the response curve of \( \{\theta(\xi = 0, \tau) - \theta(\xi = 1, \tau)\} \) as a function of \( \tau \) at various \( Nu^* \) and \( Q^* = 2 \) and \( \theta_\infty = 2 \). Within the range of \( Nu^* \) used in this study, the effect of convective heat transfer on the gage measurement is not significant.

As \( \tau \rightarrow \infty \) (steady state), Eq. (30) becomes

\[ \theta(\xi = 0, \tau \rightarrow \infty) - \theta(\xi = 1, \tau \rightarrow \infty) = 2[Q^* + Nu^*(\theta_\infty - \theta_0)] \sum_{n=1}^{\infty} \frac{1}{\lambda_n J_1(\lambda_n \xi)(Nu^* + \lambda_n^2)} \]
\[ (31) \]

The corresponding equation derived by Kuo and Kulkarni [9] and expressed in terms of the dimensionless variables used in the current work is

\[ \theta_{\xi=0} - \theta_{\xi=1} = \left[ \theta_0 - \frac{(Q^*)}{Nu^*} + \theta_\infty \right] \frac{1}{I_0(\sqrt{Nu^*})} - 1 \]
\[ (32) \]

Again, using the identity shown in Eq. (25), it can be easily shown that the steady-state temperature difference at the foil center and edge calculated using Eq. (31) and that of Kuo and Kulkarni [9], Eq. (32), are identical.

Figure 5 shows the steady-state dimensionless temperature difference at the foil center and edge calculated using Eq. (31) as a function of \( Q^* \) at various \( Nu^* \) and \( \theta_\infty = 2 \).
Remarks on Calibration

Garson gages are normally calibrated under steady-state conditions using prescribed incident radiative fluxes from a radiative heat source. If the gage is used in a purely convective or a mixed convective and radiative environment, correction to the calibration is needed. The correction can be obtained by equating the calibrated response of the purely radiative case to the measured response of the purely convective or the mixed cases.

Equating Eq. (21) to Eq. (26) and simplifying,

\[
\frac{\left[ Nu^*(\theta_\infty - \theta_0) \right]_{\text{measured}}}{\left[ Q^* \right]_{\text{calibrated}}} = \frac{1}{8} \sum_{n=1}^{\infty} \frac{1}{\lambda_n J_1(\lambda_n)} (Nu^* + \lambda_n^2) \tag{33}
\]

Equating Eq. (21) to Eq. (31) and simplifying,

\[
\frac{\left[ Q^* + Nu^*(\theta_\infty - \theta_0) \right]_{\text{measured}}}{\left[ Q^* \right]_{\text{calibrated}}} = \frac{1}{8} \sum_{n=1}^{\infty} \frac{1}{\lambda_n J_1(\lambda_n)} (Nu^* + \lambda_n^2) \tag{34}
\]

Similar to the results from Kuo and Kulkarni [9], the correction factors for the purely convective case, Eq. (33), and mixed convective and radiative case, Eq. (34), have the same form. Equation (34) is also analogous to the results given in Kuo and Kulkarni [9] and Fu et al. [10].

Conclusions

An expanded analysis of a thin-foil gage, which includes the transient effects, convective heat transfer, and an arbitrary time-varying boundary condition at the edge of the foil, has been performed. The general closed-form solution is obtained using Laplace transform. Specialized solutions under constant temperature at the edge of the foil and constant thermal radiative heat flux are provided under various modes of heat transfer. Steady-state solutions are also obtained as limiting cases. The transient and steady-steady solutions reduced to and matched the results reported in the literature under the less generalized conditions.
Nomenclature

\( c_p \)  
heat capacity of foil

\( f(t) \)  
heat sink temperature as a function of \( t \)

\( h \)  
convective heat transfer coefficient

\( H \)  
foil thickness

\( i \)  
imaginary number, \( \sqrt{-1} \)

\( I_0 \)  
modified Bessel function of the first kind of zero order

\( J_0 \)  
Bessel function of the first kind of zero order

\( J_1 \)  
Bessel function of the first kind of first order

\( k \)  
thermal conductivity of foil

\( k_f \)  
thermal conductivity of fluid

\( L \)  
Laplace transform operator

\( L^{-1} \)  
inverse Laplace transform operator

\( Nu \)  
Nusselt number ( \( Nu = hR / k_f \) )

\( Nu^* \)  
\( = \kappa \delta Nu \)

\( q_i \)  
radiative heat flux

\( Q^* \)  
\( = q_i R^2 / (kHT_0) \)

\( r \)  
radial direction

\( R \)  
radius of foil

\( s \)  
variable in Laplace transform domain

\( t \)  
time

\( T \)  
absolute temperature

\( T_0 \)  
initial foil temperature

\( T_\infty \)  
temperature of fluid flowing over the foil

\( Y_0 \)  
Bessel function of the second kind of zero order

\( \alpha \)  
thermal diffusivity of foil, \( = k / (\rho c_p) \)

\( \delta \)  
\( = R / H \)

\( \kappa \)  
\( = k_f / k \)
\( \lambda_n \) zeros of Bessel function of the first kind of zero order
\[ \theta = T / T_0 \]
\[ \theta_f = f(\tau) / T_0 \]
\[ \theta_0 = T_0 / T_0 = 1 \]
\[ \hat{\theta} = L\{\theta\}, \text{Laplace transform of } \theta \]
\[ \hat{\theta}_f = L\{\theta_f\}, \text{Laplace transform of } \theta_f \]
\[ \rho \] density of foil
\[ \tau = \alpha t / R^2 \]
\[ \tau^* \] integration dummy variable
\[ \xi = r / R \]
References


Figure 1. Cross section of a thin-foil gage.
Figure 2. Response curve showing dimensionless temperature difference between the center and edge of gage with no convective heat transfer ($N_u^* = 0$) at various $Q^*$. 
Figure 3. Comparison of response curves showing dimensionless temperature difference between the center and edge of gage with no thermal radiative heat transfer ($Q^* = 0$) at various $Nu^*$ and $\theta_\infty = 2$ from current work (lines) with that from Ash (1969) (symbols).
Figure 4. Response curve showing dimensionless temperature difference between the center and edge of gage with $Q^* = 2$ and $\theta_\infty = 2$ at various $Nu^*$. 
Figure 5. Steady state dimensionless temperature difference between the center and edge of gage as a function of $Q^*$ at various $Nu^*$ with $\theta_\infty = 2$. 