The Relationship Between Switch-Term-Corrected Scattering-Parameters and Wave-Parameters Measured With a Two-Port Vector Network Analyzer

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***Abstract*— We explore the relationship between scattering-parameters (S-parameters), corrected for switch terms, and S-parameters calculated from wave-parameters measured with a two-port vector network analyzer. By deriving the equations for both representations using consistent notation, we clearly demonstrate their equivalence. This result proves that measuring wave-parameters enables us to obtain uniquely defined S-parameters for any changes of impedance behind the signal-separation hardware.**

***Index Terms*— Measurement, scattering parameters (S-parameters), switch terms, vector network analyzer (VNA), wave parameters.**

I. Introduction

SCATTERING parameters (S-parameters) describe high-frequency electrical behavior of linear networks having one or more ports. "Raw" S-parameters are usually measured with a vector network analyzer (VNA), and then calibrated to correct for systematic errors in the instrument, as well as losses and impedance mismatches in the cables and test fixtures [1].

In the case of two-port VNAs, many commercial models are equipped with four samplers to measure the incident (a_i) and reflected/ transmitted (b_i) waves at both ports ($i = 1$ and 2). These instruments can measure all four waves simultaneously, but typically measure only three at a time under normal operating conditions [2]. There are four "raw" (switch-term corrected, but not calibrated) S-parameters of a device measured by the VNA that relate the incident and reflected waves

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}. \tag{1}$$

Here, S_{11} and S_{22} refer to the reflection coefficients of ports 1 and 2, respectively, and S_{21} and S_{12} refer to the transmission coefficients from ports 1 to 2 and vice versa, respectively. We will repeatedly refer to (1) by insisting it is true for any incident waves a_1 and a_2 on the device and reflected waves b_1 and b_2 reflected by or transmitted through the device.

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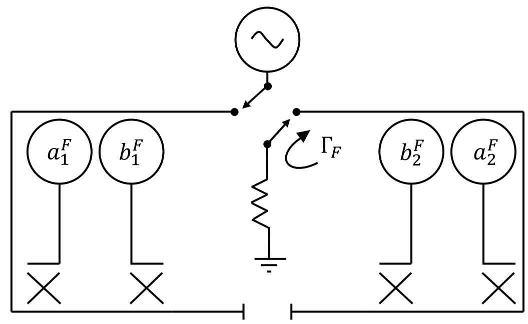

Fig. 1. Simplified schematic of a two-port VNA measuring the forward S-parameters or wave-parameters when the source is switched to port 1.

II. VNA Measurements

Fig. 1 illustrates a simplified schematic of a two-port VNA when the source is switched to port 1 for forward measurements. The two raw, forward S-parameters, S_{11}^F and S_{21}^F, are measured as ratios of the following reflected/transmitted and incident waves:

$$S_{11}^F \stackrel{\text{def}}{=} \frac{b_1^F}{a_1^F} \tag{2}$$

and

$$S_{21}^F \stackrel{\text{def}}{=} \frac{b_2^F}{a_1^F} \tag{3}$$

where the superscript "F" refers to the forward direction. Here, it is often assumed $a_2^F = 0$, but this is not generally true in practice due to the nonideal load terminating the unstimulated port. Likewise, when the source is switched to port 2, as shown in Fig. 2, the two raw, reverse S-parameters, S_{12}^R and S_{22}^R, are measured as ratios of the following reflected/transmitted and incident waves:

$$S_{12}^R \stackrel{\text{def}}{=} \frac{b_1^R}{a_2^R} \tag{4}$$

and

$$S_{22}^R \stackrel{\text{def}}{=} \frac{b_2^R}{a_2^R} \tag{5}$$

where the superscript "R" refers to the reverse direction. And here, it is also often assumed $a_1^R = 0$.

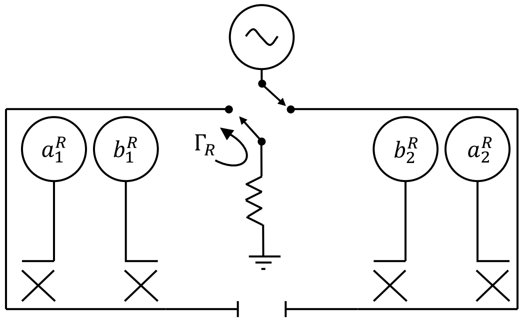

Fig. 2. Simplified schematic of a two-port VNA measuring the reverse S-parameters or wave parameters when the source is switched to port 2.

Note that the raw forward and reverse S-parameters, defined in (2)–(5), are not equivalent to the raw S-parameters in (1). This is because the load terminating the unstimulated ports, shown in Figs. 1 and 2, is not perfectly matched.

In (2) and (3), the reflection coefficient of the terminating load is calculated from

$$\Gamma_F = \frac{a_2^F}{b_2^F} \tag{6}$$

and a_2^F will not be exactly zero. Likewise, in (4) and (5), the reflection coefficient of the terminating load is calculated from

$$\Gamma_R = \frac{a_1^R}{b_1^R} \tag{7}$$

and a_1^R will not be exactly zero. Thus, a transformation must be performed to correct the forward and reverse S-parameters. The two reflection coefficients, Γ_F and Γ_R, are known as "switch-terms." They are typically measured only once during a calibration when the "thru" standard is connected and are assumed to be stable during the measurement process [3], [4].

As previously stated, some commercial two-port VNAs can measure all four waves simultaneously. In this mode, the incident and reflected waves a_1^F, b_1^F, a_2^F, and b_2^F are measured when the source is switched to port 1, as shown in Fig. 1. Likewise, the waves a_1^R, b_1^R, a_2^R, and b_2^R are measured when the source is switched to port 2, as shown in Fig. 2.

There are certain applications where measuring wave parameters rather than S-parameters are preferable, even when S-parameters are ultimately desired. For instance, when characterizing high-insertion loss devices, we can shift the dynamic range of the VNA by implementing additional hardware, including an amplifier [5], [6]. In such cases, the switch terms may either change when an amplifier is added to the system after the calibration is performed [5], or the switch terms may be difficult to accurately measure when the thru standard in the calibration is replaced with an attenuator [6].

In [5] and [6], we have postulated that S-parameters can be calculated directly from measured wave-parameters without the need for switch-term correction.

In Sections III and IV, we will explore the relationship between the two different forms of measurements—switch-term-corrected S-parameters versus wave-parameters.

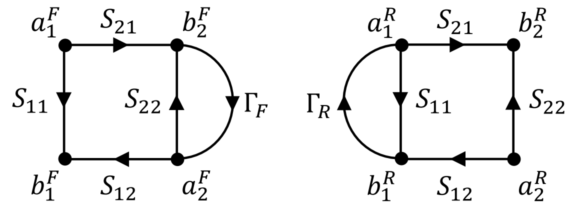

Fig. 3. Forward and reverse switch-term models.

First, we will review how to correct measured S-parameters for switch terms, and then we will derive the same equations for the case where we measure the four a and b waves simultaneously.

III. Forward and Reverse S-Parameters

Marks [2] presented the forward (source switched to port 1) and reverse (source switched to port 2) switch-term models, as shown in Fig. 3. These flow diagrams can be solved for the defined forward and reverse S-parameters of (2)–(5) in terms of the "raw" S-parameters in (1)

$$S_{11}^F \stackrel{\text{def}}{=} \frac{b_1^F}{a_1^F} = S_{11} + \frac{S_{12}S_{21}\Gamma_F}{1 - S_{22}\Gamma_F} \tag{8}$$

$$S_{21}^F \stackrel{\text{def}}{=} \frac{b_2^F}{a_1^F} = \frac{S_{21}}{1 - S_{22}\Gamma_F} \tag{9}$$

$$S_{12}^R \stackrel{\text{def}}{=} \frac{b_1^R}{a_2^R} = \frac{S_{12}}{1 - S_{11}\Gamma_R} \tag{10}$$

and

$$S_{22}^R \stackrel{\text{def}}{=} \frac{b_2^R}{a_2^R} = S_{22} + \frac{S_{12}S_{21}\Gamma_R}{1 - S_{11}\Gamma_R}. \tag{11}$$

Equations (8)–(11) can be inverted to yield the "raw" S-parameters corrected for switch terms

$$S_{11} = \frac{S_{11}^F - S_{12}^R S_{21}^F \Gamma_F}{1 - S_{12}^R S_{21}^F \Gamma_R \Gamma_F} \tag{12}$$

$$S_{21} = \frac{S_{21}^F - S_{22}^R S_{21}^F \Gamma_F}{1 - S_{12}^R S_{21}^F \Gamma_R \Gamma_F} \tag{13}$$

$$S_{12} = \frac{S_{12}^R - S_{11}^F S_{12}^R \Gamma_R}{1 - S_{12}^R S_{21}^F \Gamma_R \Gamma_F} \tag{14}$$

and

$$S_{22} = \frac{S_{22}^R - S_{12}^R S_{21}^F \Gamma_R}{1 - S_{12}^R S_{21}^F \Gamma_R \Gamma_F}. \tag{15}$$

Note that these switch-corrected S-parameters have yet to be calibrated. In other words, they are still "raw" measurements that have only been switch corrected. All the measured calibration standards and devices under test must be switch-corrected prior to a calibration being performed.

We do not address the various types of VNA calibrations in this paper. Suffice it to say, these switch-corrected S-parameters enable a variety of calibrations designed for an eight-term "error-box" model to be used, as opposed to the more restrictive twelve-term model, which requires separate terms for forward and reverse measurements.

IV. Wave-Parameters

In the case of wave-parameters [7], (1) can be solved as

$$\begin{pmatrix} b_1^F & b_1^R \\ b_2^F & b_2^R \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1^F & a_1^R \\ a_2^F & a_2^R \end{pmatrix}. \tag{16}$$

Solving this equation for the S-parameters gives

$$S_{11} = \frac{b_1^F a_2^R - b_1^R a_2^F}{a_1^F a_2^R - a_1^R a_2^F} \tag{17}$$

$$S_{21} = \frac{b_2^F a_2^R - b_2^R a_2^F}{a_1^F a_2^R - a_1^R a_2^F} \tag{18}$$

$$S_{12} = \frac{b_1^R a_1^F - b_1^F a_1^R}{a_1^F a_2^R - a_1^R a_2^F} \tag{19}$$

and

$$S_{22} = \frac{b_2^R a_1^F - b_2^F a_1^R}{a_1^F a_2^R - a_1^R a_2^F}. \tag{20}$$

We can manipulate (17)–(20) using the definitions described in (2)–(7) to yield results identical to (12)–(15). For clarity, we will explicitly show how (17) may be manipulated to yield (12). Similar procedures can be employed to transform (18)–(20) to (13)–(15), respectively. First, we divide the numerator and denominator of (17) by a_1^F and a_2^R, which gives

$$S_{11} = \frac{\frac{b_1^F}{a_1^F} - \frac{b_1^R}{a_2^R}\frac{a_2^F}{a_1^F}}{1 - \frac{a_1^R a_2^F}{a_1^F a_2^R}}. \tag{21}$$

Next, we multiply the second terms in the numerator and denominator of (21) by b_2^F/b_2^F and b_1^R/b_1^R, which gives

$$S_{11} = \frac{\frac{b_1^F}{a_1^F} - \frac{b_1^R}{a_2^R}\frac{b_2^F}{a_1^F}\frac{a_2^F}{b_2^F}}{1 - \frac{b_1^R}{a_2^R}\frac{b_2^F}{a_1^F}\frac{a_2^F}{b_2^F}\frac{a_1^R}{b_1^R}}. \tag{22}$$

And finally, (22) may be rearranged in terms of S_{11}^F, S_{21}^F, S_{12}^R, Γ_F, and Γ_R from (6)–(10), which gives

$$S_{11} = \frac{S_{11}^F - S_{12}^R S_{21}^F \Gamma_F}{1 - S_{12}^R S_{21}^F \Gamma_R \Gamma_F} \tag{23}$$

the same result as (12).

To clarify, only nonlinear VNAs (NVNAs) or large-signal network analyzers can provide the user with wave-parameters calibrated in terms of phase between the harmonics, as well as absolute amplitude [8]. In the case where ratios are taken, as in (22), for example, neither power nor phase calibrations are required.

V. Conclusion

We have shown the equivalence between switch-term-corrected S-parameters and S-parameters calculated from wave-parameters measured with a two-port VNA. Although the differences between the two formulations are minor, the authors have encountered numerous users who are perplexed by this equivalence. Thus, we derived the equations for both representations using consistent notation to clarify any existing confusion.

Calculating S-parameters from measured wave-parameters is valid without requiring switch-term correction, which is paramount when there are changes of impedance behind the signal-separation hardware of the VNA, or if the switch terms cannot be accurately measured.

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