Abstract—The power transmitted through a waveguide was determined using in-situ atom-based electric-field measurements. The field distribution in the waveguide was measured using Rydberg atoms to find the maximum field, which was used to determine the power. For a proof-of-concept, the power of radio frequency fields at 17.86, 19.63, 26.53, and 33.03 GHz were measured in a WR42 waveguide. A section of waveguide was sealed and filled with cesium atoms. Directional couplers allowed RF power to be coupled through the atom-filled section, while two lasers were used to probe the atom vapor.

I. Introduction

Accurately calibrated antennas, probes, and power meters are key to electromagnetic applications. In order to achieve high accuracy for next-generation technology, we are developing a calibration method for radio-frequency (RF) electric fields (E-fields) based on Rydberg states of alkali atoms that has a direct traceability path to the International System of Units. In this work, we extend this Rydberg atom-based approach to RF power measurements. This method has the potential to be a new international standard for RF E-field and power calibrations. The Rydberg atoms convert an RF amplitude measurement into an optical frequency measurement, which is directly proportional to the strength of the field through Planck’s constant. This technique allows for self-calibrated, directly SI-traceable E-field measurements over a large range of frequencies, from $0.1 \rightarrow 1000$ GHz. The spatial resolution of these measurements can be much smaller than the RF wavelength. This allows us to measure the field distribution inside a waveguide and determine the maximum field.

To use Rydberg atoms for RF power measurements, we have assembled an atomic vapor cell integrated inside a WR42 waveguide. The RF E-field distribution inside the rectangular waveguide can be measured by probing the atomic vapor with two lasers, as described in the next section. If the RF field is propagating in the fundamental transverse electromagnetic mode (TE$_{10}$), the maximum E-field can be used to determine the power $P$ inside the waveguide (in W) [1]

$$P = E_0^2 \frac{ab}{4} \sqrt{\frac{\epsilon_0}{\mu_0}} \sqrt{1 - \left(\frac{c}{2af}\right)^2},$$  

(1)

where $E_0$ is the field maximum (in V/m), $a$ and $b$ are the cross-sectional dimensions of the waveguide (in m), and $f$ is the frequency of the field (in Hz). The power determined using the atoms can be compared to the power output through the waveguide using a power meter. This system can provide a more direct traceability path, and potentially, lower uncertainties for RF power measurements and calibrations.

II. Atom-based RF Measurements

Recently, electromagnetically-induced transparency (EIT) in Rydberg atoms has been studied as a method for measuring RF E-field amplitudes [2]-[10]. The process requires a probe laser to excite a vapor of alkali atoms from the ground state to an excited state, and a coupling laser to couple the excited state to a high energy level Rydberg state (more details in [3], [4]). In this work we use cesium ($^{133}$Cs) as our alkali atoms due to its relatively higher vapor pressure as compared with other alkali atoms, which results in a strong EIT signal. The transmission of the probe laser through the atoms is measured as the optical frequency of the probe laser is scanned through the ground-state-to-excited-state resonance. With the coupling laser off, the probe laser is absorbed by the atoms when it is on resonance. When the coupling laser is turned on, the probe laser is transmitted through the atom over a narrow frequency window, known as EIT. An RF field (when it has a frequency corresponding to a transition to a nearby Rydberg state) can further modify the EIT, splitting the EIT peak into two (this is known as Autler-Townes (AT) splitting).

Figure 1 shows a typical EIT spectrum with no RF present (the single peak). When an RF E-field is applied, the EIT peak splits in two (the two peaks labeled by RF = 0.75 V/m). The amplitude of the applied RF E-
field \( |E| \) is related to the separation in optical frequency between the two peaks \( \Delta f_m \) by

\[
|E| = 2\pi \frac{\hbar}{\varphi} \Delta f_m,
\]

where \( \hbar \) is Planck’s constant and \( \varphi \) is the dipole moment of the Rydberg transition. The dipole moment can be calculated for a given Rydberg transition (for details see [3], [11], [12]). As the RF E-field increases, the AT splitting increases (shown by the red curve labeled by RF in Fig. 1).

III. Waveguide Vapor Cell

In order to obtain a power measurement from the E-field, a \(^{133}\)Cs atomic vapor is placed inside a waveguide to create a waveguide vapor cell. The waveguide vapor cell (Fig. 2) consists of a section of stainless steel WR42 rectangular waveguide with glass windows. The WR42 waveguide has cross-sectional dimensions of \( a = 10.668 \) mm and \( b = 4.318 \) mm. Two small holes in the sidewall allow the cell to be evacuated and filled with \(^{133}\)Cs atoms. A metal stem was attached to each of the holes in the sidewall, which were attached to glass tubes connected to a manifold that was used to supply the \(^{133}\)Cs. The glass tubes were pinched off to seal the cell. The glass windows allow two lasers to be counter-propagated through the cell to measure the E-field. Directional couplers were attached to either end of the waveguide cell to allow an RF field to be transmitted (Fig. 3) in the same direction as the lasers. The first directional coupler (on the left in the photo) was used to input the RF field, and the second (on the right) was used to monitor the power output. This setup allowed the lasers to travel unobstructed through the waveguide. Stub tuners were placed on either side of the vapor cell section to reduce reflections from the glass windows.

IV. Waveguide Measurements

A probe laser beam (852 nm) was sent through the waveguide along the direction of propagation of the RF field, and the transmission was measured using a photodetector outside the other end of the waveguide (Fig. 3). The coupling laser (510 nm) was counter-propagated and overlapped with the probe laser inside the waveguide. The probe laser power was 3.2 \( \mu \)W, the coupling laser power was 17.3 mW, and both lasers were polarized along the short dimension of the waveguide (b). The E-field inside the waveguide is measured only where the two lasers overlap inside the vapor cell section. The full-width at half maximum diameter of the lasers was 0.3 mm. This allows for a measurement of the E-field distribution across the waveguide. The waveguide section was placed on a translation stage to move it relative to the lasers. Measurements of the E-field were taken at steps of 0.2 mm along the long dimension (a) of the waveguide. Measurements were done at several different frequencies (17.86, 19.63, and 26.53 GHz) spanning the range of single-mode operation of the WR42 waveguide, as well as at one frequency above the cut-off (33.03 GHz). Table I shows the atomic transitions used for these different frequencies.

<table>
<thead>
<tr>
<th>(^{133})Cs transition</th>
<th>RF frequency (GHz)</th>
<th>( \lambda_0 ) (nm)</th>
<th>( \mu / e a_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 35D_5/2 \rightarrow 36P_5/2 )</td>
<td>17.86</td>
<td>510.974</td>
<td>770.05</td>
</tr>
<tr>
<td>( 34D_5/2 \rightarrow 35P_5/2 )</td>
<td>19.63</td>
<td>511.148</td>
<td>723.40</td>
</tr>
<tr>
<td>( 31D_5/2 \rightarrow 32P_5/2 )</td>
<td>26.53</td>
<td>511.787</td>
<td>592.16</td>
</tr>
<tr>
<td>( 29D_5/2 \rightarrow 30P_5/2 )</td>
<td>33.03</td>
<td>512.340</td>
<td>511.94</td>
</tr>
</tbody>
</table>

The E-field distribution of the TE_{10} mode in a rectangular waveguide along the long side (from \( x = 0 \) to \( x = a \)) is given by

\[
E = E_0 \sin \left( \frac{\pi x}{a} \right)
\]
where $E_0$ is the field maximum and $a$ is the length of the waveguide in the $\hat{x}$ direction. The field was input to the waveguide through the first directional coupler (10 dB coupler), and propagated through the first stub tuner, waveguide vapor cell, and second stub tuner. A portion of the field was split using the second directional coupler (10 dB coupler) and measured with a power meter.

The results of the atom-based E-field measurements versus position in the waveguide are shown in Figs. 4-7. The first three figures include fits of each cross-sectional scan to (3). In Fig. 4, a 19.63 GHz field is propagated through the waveguide at three different input powers ($-17.8$, $-20.8$, and $-23.8$ dBm). For each input power, we obtain a field distribution that follows (3), with errorbars that represent a 5% uncertainty. The uncertainty is mainly due to the effect of the glass windows (more detail on the uncertainties can be found in [6], [13]). The fit uses one free parameter $E_0$, which is the maximum field in the waveguide. For the three input powers, the maximum E-fields measured are $E_0 = 16.6$, $11.8$, and $8.1$ V/m, which gives power levels of $P_{\text{atom}} = -22.3$, $-25.3$, and $-28.5 \pm 0.4$ dBm using (1). This is compared to the output power from the directional coupler measured using a power meter, which was (after subtracting the attenuation from the coupler, measured by an S-parameter measurement on a vector network analyzer) $P_{\text{PM}} = -22.3$, $-25.2$, and $-28.2$ dBm. The average difference between $P_{\text{atom}}$ and $P_{\text{PM}}$ is 0.2 dBm. We now have a measurement of RF power determined using an atom-based E-field measurement.

We repeated these measurements at frequencies near the low and high cutoffs, 17.86 and 26.53 GHz. For 17.86 GHz in Figure 5, we used input powers of $-16.8$, $-19.7$, and $-22.7$ dBm. The power levels determined by the maximum E-fields were $P_{\text{atom}} = -23.7$, $-26.6$, and $-29.5 \pm 0.4$ dBm, which compared to the powers determined using the power meter ($P_{\text{PM}} = -23.0$, $-25.4$, and $-29.0$ dBm) give an average difference of 0.8 dBm. For 26.53 GHz in Figure 6, we used input powers of $-16.9$, $-20.4$, and $-23.3$ dBm. The power levels determined by the maximum E-fields were $P_{\text{atom}} = -24.9$, $-28.2$, and $-30.8 \pm 0.4$ dBm, which, when compared to the powers determined using the power meter ($P_{\text{PM}} = -23.6$, $-27.1$, and $-29.9$ dBm) give an average difference of 1.1 dBm. For both cases, we are still able to map the TE$_{10}$ mode and determine the maximum field and the power in the waveguide.

For each frequency, the stub tuners had to be adjusted to cancel the reflection of the RF field from the glass windows. Without this adjustment, the field inside the waveguide vapor cell was not the TE$_{10}$ mode. The signal from the atoms is very sensitive to this change, and a slight mis-adjustment of the stub tuner created a noisy signal.

We also took measurements of the field distribution in the waveguide for a frequency above the single-mode frequency (33.03 GHz), shown in Fig. 7. Here the field is no longer in the TE$_{10}$ mode, but is a combination of the TE$_{10}$ and TE$_{20}$ modes. We can see this structure in our E-field measurements, though it is difficult to fit this to
theory due to the imperfect cancellation of the reflections from the glass windows using the stub tuners at this dual mode.

V. Conclusions

We demonstrated a method to determine RF power in a waveguide using Rydberg atom-based RF E-field measurements. This proof-of-concept demonstrates the potential for self-calibrated, SI-traceable, in-situ waveguide power measurements. The atom-based field measurements are directly SI-traceable through Planck’s constant in (3), and are self-calibrated as they directly measure the amplitude of the field from atomic properties. As the power is determined by a field measurement, all of the power is available for use. This principle could be used to make a calibrated RF source. While more work must be done to reduce the uncertainties, this work is a step towards a quantum-based, SI-traceable RF power measurement technique.

References